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PROPULSION PARAMETERS OF NOBLE-ABEL GASES

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Abstract. *Propulsive parameters of rockets are usually defined for perfect gases with constant properties. However propellants can reach high temperatures and pressures in the chamber and real gas effects and variable thermal properties can be relevant. This paper derives equations for propulsion parameters of rockets, including specific impulse, characteristic velocity, critical mass flow constant and thrust coefficient, assuming isentropic one-dimensional flow of Noble-Abel gases with variable thermal properties.*

Keywords: *Noble-Abel, specific impulse, characteristic velocity, thrust coefficient*

1. INTRODUCTION

Propulsive parameters are used to evaluate different performance aspects of rockets. Specific impulse is a parameter that relates instantaneous thrust and weight flow rate or, generally, relates the total impulse delivered with propellant weight spent during burning time. The characteristic velocity is a propulsive parameter which indicates the propellant performance and the motor design quality, whereas the thrust coefficient measures the nozzle design efficiency. The critical mass flow constant allows determining the mass flow rate from chamber stagnation conditions (Zucrow and Hoffmanm, 1976).

The classical equations for propulsive parameters are generally derived assuming one-dimensional isentropic flow of perfect gases and considering constant properties along the nozzle (Shapiro, 1953). However, perfect gases usually are not calorically perfect, i.e., they present specific heats varying significantly with temperature. In the case of real gases the specific heats are dependent on temperature and pressure. Propellants in chemical, electrothermal, laser, microwave and nuclear rockets can reach up to temperatures above 3000 K in the chamber, and, consequently, temperature variations along the nozzle are important, thus affecting strongly the specific heats of the flowing gases.

Several authors have considered the one-dimensional isentropic flows of thermally perfect gases or real gases. Donaldson (1946) evaluated the effects of imperfect gases and the variation of specific heats of diatomic gases; Tsien (1947) described the flow properties of Van der Waals gases with specific heats varying with temperature; Eggers (1948) examined the flow of diatomic gases obeying the Berthelot equation of state with specific heats varying with temperature; Johnson (1965) determined effects of real gases on the critical nozzle mass flow constant and on thermodynamic properties of isentropic flows; and Witte and Tatum (1994) developed a computer code to determine the thermally ideal gas properties, using specific heats approximated as NASA fourth order polynomials of the temperature. Ding et al. (2014) described flow characteristics of hydrogen gas through a critical nozzle using equations of state based on Helmholtz energy and compared the theoretical results with experimental data and CFD simulations with different equations of state. Costa et al. (2016) determined the main propulsion parameters of thermally perfect gases.

However, no reference was found describing equations for the propulsive parameters of rockets based on isentropic flow relations of real gases with variable properties. Therefore, this paper derives expressions for propulsive parameters of rockets, considering steady compressible one-dimensional isentropic flow of Noble-Abel and thermally perfect gases, i.e., with specific heats varying with temperature. Numerical results are obtained assuming a fourth degree polynomial variation of specific heats with temperature. The nozzle exit and throat conditions of Noble-Abel gases are also compared to the values found for calorically perfect gases.

2. METHODOLOGY

At high pressures, the effects of the volume of molecules and molecular attraction forces must be considered. In some applications, the high temperature of the propellant gases can make the effects of the attraction forces small because of the high molecular kinetic energy. Therefore, in these applications, the term relative to intermolecular forces can be disregarded without significant loss of precision. We can then use the Noble-Abel (*NA*) state equation:

$$P(v-b) = RT \tag{1}$$

where P is pressure, v is specific volume, T is temperature and R is the gas constant, and b is the covolume, which corresponds to about four times the volume occupied by the molecules.

2.1 Enthalpy, speed of sound and specific heats of a *NA* gas

The enthalpy of a NA gas is calculated from

$$h = h_{ref} + \int_{T_{ref}}^{T} c_p dT + b(P - P_{ref}) \tag{2}$$

where h is the specific enthalpy, c_p is the specific heat at constant pressure and subscript "*ref*" indicates reference conditions, usually P_{ref} = 1 atm and T_{ref} = 298 K.

The speed of sound of a *NA* gas is given by

$$c = \frac{v}{v-b}\sqrt{\gamma RT} \tag{3}$$

where $\gamma = c_p/c_v$. The specific heats at constant volume and constant pressure of a *NA* gas are given, respectively, by $c_v = c_v^0$ an $c_P = c_p^0$, where superscript "0" indicates low pressure condition or perfect gas property. Consequently, the difference of specific heats of a *NA* gas is $c_P - c_v = R$, similar to a perfect gas.

2.2 Isentropic relations

Combining first and second laws of thermodynamics yields Gibbs relation, $Tds = dh - vdP$, where s is gas entropy. Assuming an isentropic process, replacing equations 1 and 2 and integrating, provides pressure and specific volume in terms of temperature:

$$\frac{P}{P_c} = \exp\left(-\int_{T}^{T_c} \frac{c_P}{R}\frac{dT}{T}\right) \tag{4}$$

$$\frac{v-b}{v_c-b} = \frac{T}{T_c}\exp\left(\int_{T}^{T_c} \frac{c_P}{R}\frac{dT}{T}\right) \tag{5}$$

where subscript "*c*" indicates chamber or stagnation condition. Equations 4 and 5 were derived for a closed system however they are also valid in the case of steady one-dimensional adiabatic frictionless flow, since energy and momentum equations in differential form yield $dh - vdP = 0$.

2.3 Energy equation

Energy equation in integral form for steady one-dimensional adiabatic nozzle flow with no friction is $h_c = h + u^2/2$ where u is the flow speed. Therefore, after replacing enthalpy Eq. 2, the flow velocity of a *NA* gas along a nozzle is given by:

$$u = \sqrt{2\left(\int_{T}^{T_c} c_p dT + b(P_c - P)\right)} \tag{6}$$

Since Mach number in a point within the flow is defined by $M = u/c$, it follows that

$$M = \frac{v-b}{v}\sqrt{\frac{2}{\gamma T}\left(\int_{T}^{T_c}\frac{c_p}{R}dT + \frac{b}{R}(P_c - P)\right)} \quad (7)$$

2.4 Throat temperature of a *NA* gas

The flow velocity at the nozzle throat is

$$u_t = 2\left(\int_{T_t}^{T_c} c_p dT + b(P_c - P_t)\right)^{1/2} \quad (8)$$

where subscript "*t*" denotes throat conditions. It can be shown that $M_t = 1$, consequently, $u_t = c_t$, yielding

$$T_t = \frac{2}{\gamma_t}\frac{(v_t - b)^2}{v_t^2}\left(\int_{T_t}^{T_c}\frac{c_p}{R}dT + \frac{b}{R}(P_c - P_t)\right) \quad (9)$$

2.5 Critical mass flow rate of a *NA* gas

The mass flow rate at the throat is $\dot{m} = u_t A_t / v_t$ where A_t is the throat cross-section area. Replacing v_t and u_t from Eqs. 5 and 8, respectively, and rearranging, it follows that

$$\dot{m}_{NA} = \Gamma_{NA} A_t \frac{P_c}{\sqrt{RT_c}} \quad (10)$$

where

$$\Gamma_{NA} = \sqrt{\gamma_{t,NA}\frac{T_c}{T_{t,NA}}}\exp\left(-\int_{T_{t,NA}}^{T_c}\frac{c_p}{R}\frac{dT}{T}\right) \quad (11)$$

is the critical mass flow rate constant of a *NA* gas.

2.6 Characteristic velocity

Characteristic velocity is defined by $c^* = P_c A_t / \dot{m}$, therefore

$$c^*_{NA} = \frac{\sqrt{RT_c}}{\Gamma_{NA}} \quad (12)$$

2.7 Thrust Coefficient

The thrust coefficient is defined by $c_F = F/(P_c A_t)$ where $F = \dot{m}u_e + (P_e - P_a)A_e$ is the thrust, and subscript "*e*" denotes exit conditions, for example, A_e is the nozzle exit area. Replacing exit velocity and mass flow rate from Eqs. 6 and 10, respectively, it gives

$$c_{F,NA} = \Gamma_{NA}\sqrt{\frac{2}{T_c}\left[\int_{T_e}^{T_c}\frac{c_p}{R}dT + \frac{bP_c}{R}\left(1 - \frac{P_e}{P_c}\right)\right]} + \left(\frac{P_e}{P_c} - \frac{P_a}{P_c}\right)\frac{A_e}{A_t} \quad (13)$$

where $\dfrac{P_e}{P_c} = \exp\left(-\int_{T_e}^{T_c}\frac{c_p}{R}\frac{dT}{T}\right)$.

Nozzle area expansion rate A_e / A_t can be related to the exit pressure ratio P_e / P_c, as showed next.

The mass flow rate can be written as $\dot{m} = u_e A_e / v_e = \Gamma_{NA} A_t P_c / \sqrt{RT_c}$. Replacing exit specific volume $v_e = b + RT_e / P_e$ and exhaustion velocity $u_e = \sqrt{2\left(\int_{T_e}^{T_c} c_p dT + b(P_c - P_e)\right)}$, the area ratio equation for a *NA* gas is obtained:

$$\frac{A_e}{A_t} = \frac{\Gamma_{NA}\left(\frac{bP_c}{RT_c} + \frac{T_e}{T_c}\frac{P_c}{P_e}\right)}{\sqrt{\frac{2}{T_c}\int_{T_e}^{T_c}\frac{c_p}{R}dT + 2\frac{bP_c}{RT_c}\left(1 - \frac{P_e}{P_c}\right)}} \tag{14}$$

Nozzle area expansion ratio depends only on exit pressure ratio since the exit temperature ratio is related to the exit pressure ratio. Consequently the opposite is valid, i.e., the exit pressure ratio depends only on the nozzle area expansion rate. As seen, the thrust coefficient depends mainly on nozzle geometry while the characteristic velocity depends on propellant performance.

2.8 Specific impulse

The specific impulse, for constant thrust and constant mass flow rate, is defined by $Isp = F / \dot{m} g_0$ where g_o=9,80665 m/s^2 is the standard gravity acceleration. Then, $Isp_{NA} = c^*_{NA} c_{F,NA} / g_0$.

2.9 Optimum conditions

For perfect expansion in vacuum, P_a = 0, then P_e ~ 0 and T_e ~ 0, and the optimum thrust coefficient and optimum specific impulse of *NA* gases are obtained:

$$c_{F,NA,opt} = \Gamma_{NA}\sqrt{\frac{2}{T_c}\left[\int_0^{T_c}\frac{c_p}{R}dT + \frac{bP_c}{R}\right]} \tag{15}$$

$$Isp_{NA,opt} = \frac{c^*_{NA} c_{F,NA,opt}}{g_0} = \frac{1}{g_0}\sqrt{2\left[\int_0^{T_c} c_p dT + bP_c\right]} \tag{16}$$

3. RESULTS

Simulations of isentropic one-dimensional flows of CO_2, H_2O and N_2 were performed versus chamber temperature.

Figures 1 to 6 show the calculated propulsive parameters, flow properties at throat and nozzle exit sections, and percent errors, assuming vacuum expansion of Noble-Abel gases with specific heats varying with temperature (*NA*,*TP*) and perfect gases with constant specific heats (*PG*,*CP*). It was considered a nozzle with expansion rate equal to 60, chamber pressure 50 bar and chamber temperatures varying from 1000 K to 4000K.

The percent error of $\phi = Isp$, Γ, etc is defined as:

$$error = 100\frac{\phi_{PG,CP} - \phi_{NA,TP}}{\phi_{NA,TP}} \tag{17}$$

Specific heats obeying fourth order polynomials of temperature were considered, as described in NASA TP-2002-211556:

$$\frac{c_p^0}{R} = a_1 T^{-2} + a_2 T^{-1} + a_3 + a_4 T + a_5 T^2 + a_6 T^3 + a_7 T^4 \tag{18}$$

The temperature coefficients $a_{i=1,..,7}$ in different temperature ranges are presented on Table 1.

It was assumed $\gamma = \gamma(T_c)$ in the case of *PG*, *CP* gases, however other values, for example, $\gamma = \gamma(T_{av})$, where T_{av} = $(T_c + T_e)/2$, could be adopted.

Table 1 – Polynomial coefficients of specific heats. Source: NASA TP-2002-211556.

CO_2	a_1	a_2	a_3	a_4
200 ≤ T < 1000	4.943650540D+04	-6.264116010D+02	5.301725240D+00	2.503813816D-03
	a_5	a_6	a_7	
	-2.127308728D-07	-7.689988780D-10	2.849677801D-13	
1000 ≤ T < 6000	a_1	a_2	a_3	a_4
	1.176962419D+05	-1.788791477D+03	8.291523190D+00	-9.223156780D-05
	a_5	a_6	a_7	
	4.863676880D-09	-1.891053312D-12	6.330036590D-16	
H_2O	a_1	a_2	a_3	a_4
200 ≤ T < 1000	-3.947960830D+04	5.755731020D+02	9.317826530D-01	7.222712860D-03
	a_5	a_6	a_7	
	-7.342557370D-06	4.955043490D-09	-1.336933246D-12	
1000 ≤ T < 6000	a_1	a_2	a_3	a_4
	1.034972096D+06	-2.412698562D+03	4.646110780D+00	2.291998307D-03
	a_5	a_6	a_7	
	-6.836830480D-07	9.426468930D-11	-4.822380530D-15	
N_2	a_1	a_2	a_3	a_4
200 ≤ T < 1000	2.210371497D+04	-3.818461820D+02	6.082738360D+00	-8.530914410D-03
	a_5	a_6	a_7	
	1.384646189D-05	-9.625793620D-09	2.519705809D-12	
1000 ≤ T < 6000	a_1	a_2	a_3	a_4
	5.877124060D+05	-2.239249073D+03	6.066949220D+00	-6.139685500D-04
	a_5	a_6	a_7	
	1.491806679D-07	-1.923105485D-11	1.061954386D-15	

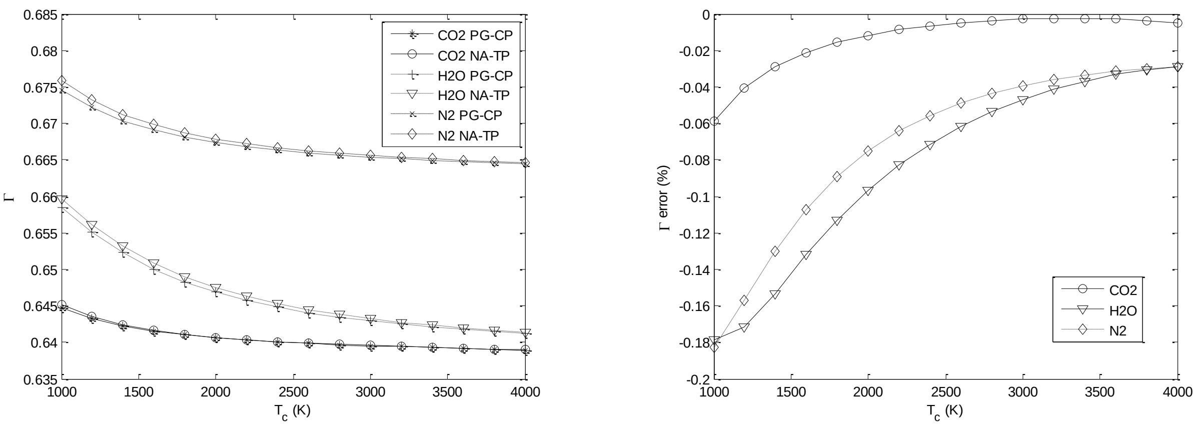

Figure 1. Critical mass flow constant and percent error with P_c = 50 bar.

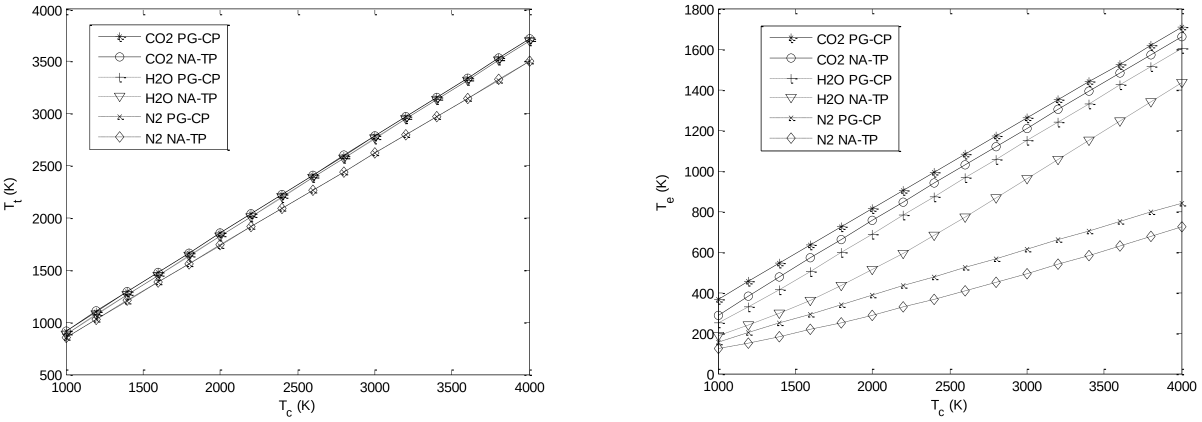

Figure 2. Throat and exit temperatures with $\varepsilon = 60$ and $P_c = 50$ bar.

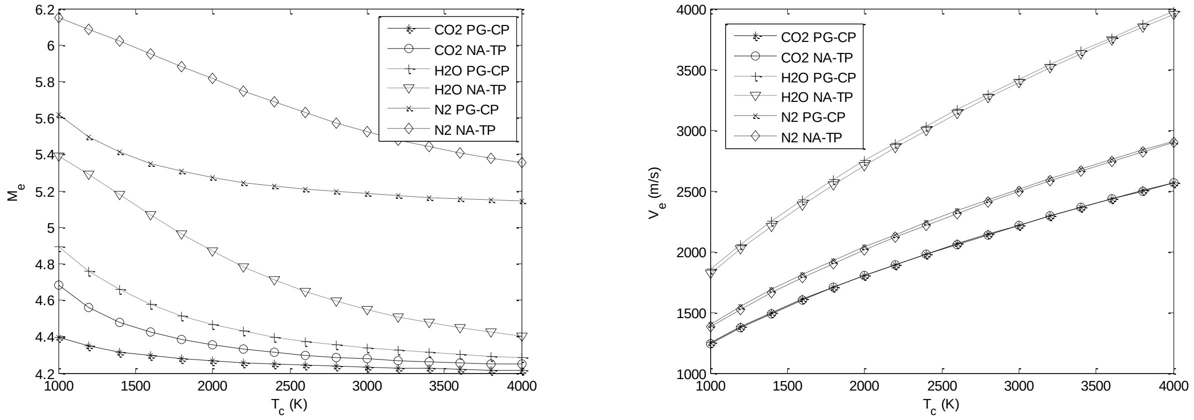

Figure 3. Mach number and flow velocity at nozzle exit with $\varepsilon = 60$ and $P_c = 50$ bar.

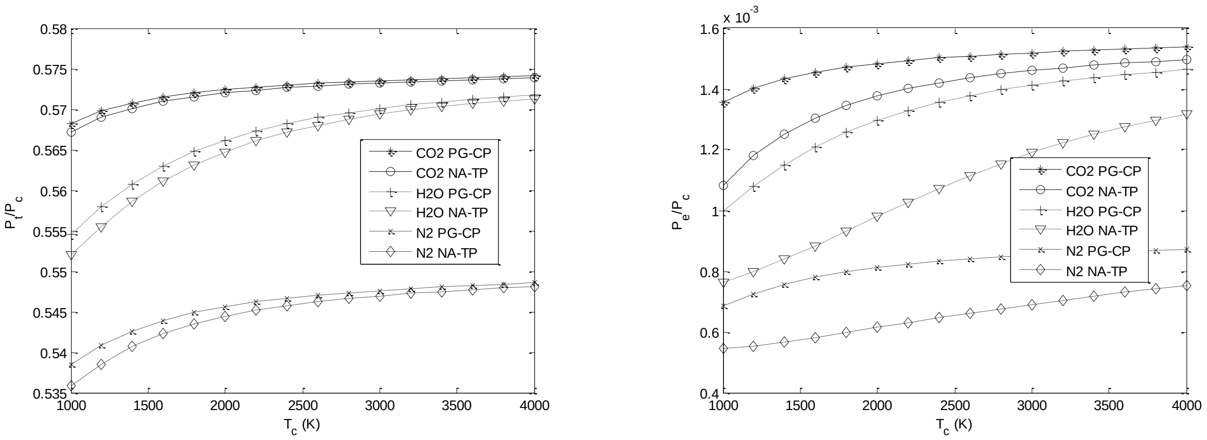

Figure 4. Throat and exit pressure ratios with $\varepsilon = 60$ and $P_c = 50$ bar.

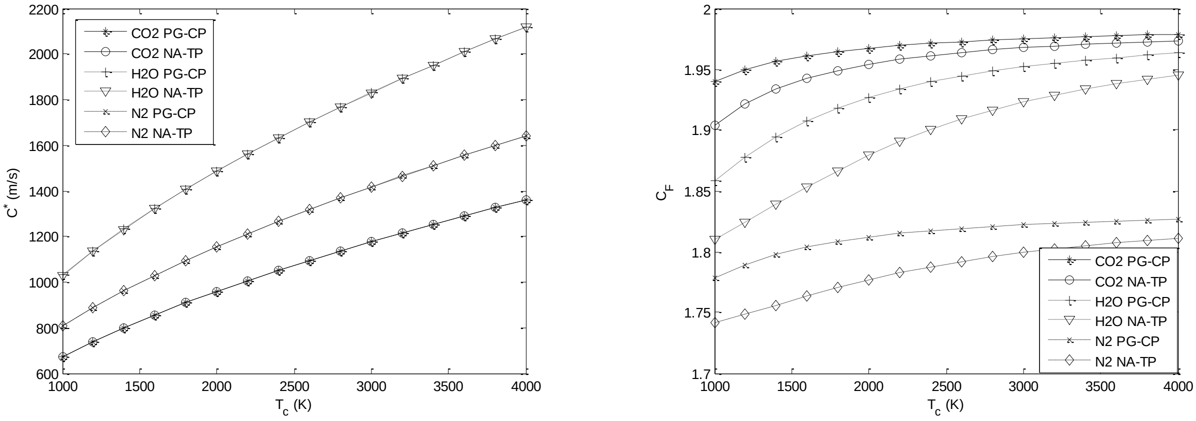

Figure 5. Characteristic velocity and thrust coefficient in vacuum with ε = 60 and P_c = 50 bar..

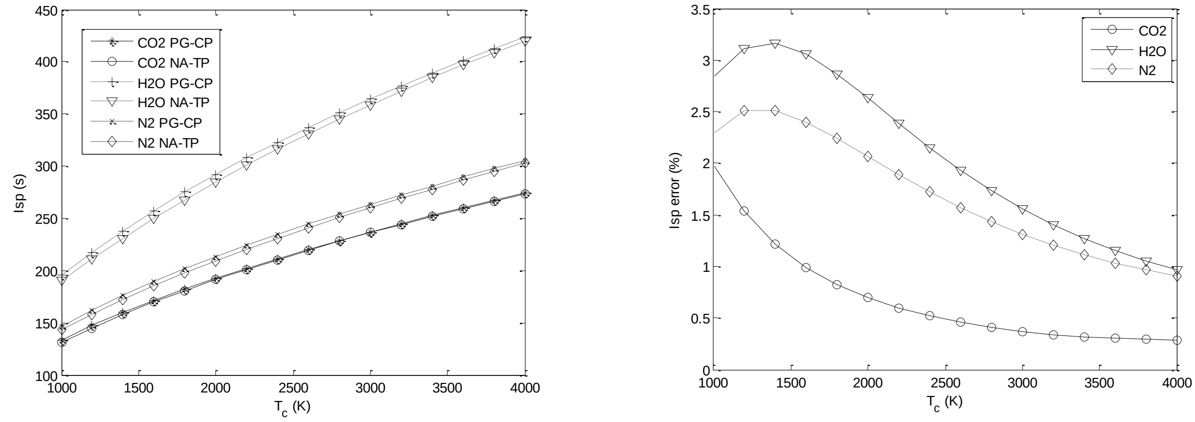

Figure 6. Vacuum specific impulse and percent error for ε = 60 and P_c = 50 bar.

4. CONCLUSIONS

Propulsive parameters of rockets and nozzle flow properties were determined for the adiabatic frictionless one-dimensional flow of Noble-Abel gases. Calculations were performed for CO_2, H_2O and N_2 with chamber temperatures between 1000 and 4000 K and specific heats varying according to NASA fourth degree polynomials of temperature. The vacuum specific impulses, thrust coefficients, critical mass flow constant, exit Mach number and exit pressures presented significant variations, whereas characteristic velocities, exit temperatures and exit velocities of Noble-Abel gases showed no significant variations as compared to perfect gases with constant specific heats. Temperature dependent specific heats presented significant influence on propulsion parameters while covolume and chamber pressure did not show significant effects. Further investigation can be made considering real gases with different equations of state.

5. ACKNOWLEDGEMENTS

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