Four-wave hybrid processes driven by nonlinear coupling of Langmuir and ion-acoustic waves

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1. Introduction

In this paper, we report on a new nonlinear wave-wave process called the hybrid stimulated modulational instability (SMI) driven by a traveling Langmuir wave [1]. The nonlinear temporal behaviours of this parametric instability will be analyzed. This novel resonant wave-wave process may play a significant role in the generation of the fundamental plasma radiation in cosmic and laboratory plasmas.

A number of nonlinear mechanisms involving the interaction of Langmuir and ion-acoustic waves have been proposed to explain the generation of fundamental plasma emissions. These include induced scattering of Langmuir waves off ion clouds; incoherent coalescense of Langmuir and ion-acoustic waves; collapse of nonlinear Langmuir wave packets; conversion of Langmuir waves by density fluctuations driven by strong turbulence (se e.g. [2] for references). In particular, various types of parametric instabilities have been studied: three-wave electromagnetic decay (fusion) instability in which a Langmuir traveling pump wave produces a Stokes (anti-Stokes) electromagnetic wave via coupling to an ion-acoustic wave [3, 4]; electromagnetic modulational instabilities in which two counter-propagating Langmuir pump waves generate a pair of electromagnetic daughter waves via coupling to ion-acoustic waves [5, 2]; hybrid (electromagnetic-electrostatic) absolute modulational instability in which a Langmuir traveling pump wave excites an electromagnetic daughter wave and a Langmuir daughter wave via coupling to purely growing density fluctuations [6]; hybrid modulational instabilities in which two oppositely directed Langmuir pump waves emit a pair of electromagnetic daughter waves and two pairs of Langmuir daughter waves via coupling to ion-acoustic waves [7].

In previous works on modulational instabilities the low-frequency (idler) mode is usually considered non-resonant, whereas the upper and lower-sidebands are resonant modes [6, 8, 9]. In contrast, we consider in the present paper the stimulated modulational instability in which all the daughter modes (including the idler mode) are resonant waves.

A linear stability analysis of the purely electrostatic SMI induced by a traveling Langmuir wave [10], $L \rightarrow L + L + S$ (where S is a resonant ion-acoustic wave), shows that its maximum growth rate is comparable to the parametric decay instability (PDI) $L \rightarrow L + S$ as well as the oscillating two-stream instability (OTSI), $L \rightarrow L + L + S^*$ (where S^* is a nonresonant purely growing density perturbation). This result indicates that the SMI can compete effectively with other parametric processes when the Langmuir turbulence is excited in plasmas.

The novel *hybrid* (electrostatic and electromagnetic) SMI to be studied in this paper is $L \rightarrow T + L + S$. The Langmuir and electromagnetic waves have frequencies near the electron plasma frequency, whereas the ion-acoustic wave has frequency near the ion-acoustic frequency. The total wave electric field for this process can be written as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_T + \mathbf{E}_L + \mathbf{E}_S$, where $\mathbf{E}_0$ is a Langmuir traveling pump wave (L_0), $\mathbf{E}_T$ is an electromagnetic daughter wave (T), $\mathbf{E}_L$ is a Langmuir daughter wave (L), and $\mathbf{E}_S$ is a resonant ion-acoustic daughter wave (S). A previous study analyzed the linear theory of the hybrid nonresonant (absolute) modulational instability $L \rightarrow T + L + S^*$ wherein the low-frequency density fluctuations (S^*) are purely growing. In this paper, we present the nonlinear theory of the hybrid stimulated modulational processes $L \rightleftharpoons T + L + S$ which are convective processes involving interactions with resonant ion-acoustic waves.

2. Governing equations

The basic equations that govern the ponderomotive coupling of Langmuir waves with high-frequency electromagnetic and electrostatic waves, near the fundamental plasma frequency, and low-frequency ion-acoustic waves are the generalized

Zakharov equations;

$$(\partial_t^2 + \nu_e \partial_t + c^2 \nabla \times \nabla \times -\gamma_e v_{th}^2 \nabla(\nabla \cdot) + \omega_p^2) \times \mathbf{E} = -\frac{\omega_p^2}{n_0} n \mathbf{E}, \quad (1)$$

$$(\partial_t^2 + \nu_i \partial_t - v_S^2 \nabla^2) n = \frac{\varepsilon_0}{2m_i} \nabla^2 \langle E^2 \rangle. \quad (2)$$

where n is the ion density fluctuation, $\omega_p = (n_0 e^2/m_e \varepsilon_0)^{1/2}$ is the electron plasma frequency, $v_{th} = (KT_e/m_e)^{1/2}$ is the electron thermal velocity, $v_S = [K(\gamma_e T_e + \gamma_i T_i)/m_i]^{1/2}$ is the ion-acoustic velocity, ν_e (ν_i) is the phenomenological damping frequency for electrons (ions), γ_e (γ_i) is the ratio of the specific heats for electrons (ions), and the angular brackets denote the fast time average. It is evident from the wave operator in the LHS of (1) that the high-frequency wave field $\mathbf{E}$ can be *hybrid* (i.e., containing of both electromagnetic and electrostatic components).

Consider a traveling Langmuir pump wave $\mathbf{E}_0(\omega_0, \mathbf{k}_0)$ with dispersion relation $\omega_0^2 = \omega_p^2 + \gamma_e v_{th}^2 k_0^2$. The stimulated modulational process involves the coupling of two wave triplets which satisfy the following frequency and wavevector selection rules $\omega^- \approx \omega_0 - \omega^*, \omega^+ \approx \omega_0 + \omega; \mathbf{k}^\mp = \mathbf{k}_0 \mp \mathbf{k}_S$, where ω is a complex frequency. In this paper, we make the assumption of imperfect frequency matching but perfect wavevector matching, and focus on the *temporal* dynamics of the hybrid stimulated modulational processes. Two distinct hybrid SMI's can be generated: $L_0 \rightarrow T^+ + L^- + S$ and $L_0 \rightarrow T^- + L^+ + S$. In the first case, an anti-Stokes electromagnetic wave $\mathbf{E}_T^+(\omega_0 + \omega, \mathbf{k}_0 + \mathbf{k}_S)$ and a Stokes Langmuir wave $\mathbf{E}_L^-(\omega_0 - \omega^*, \mathbf{k}_0 - \mathbf{k}_S)$ are produced via coupling to an ion-acoustic wave $\mathbf{E}_S(\omega, \mathbf{k}_S)$. In the second case, a Stokes electromagnetic wave $\mathbf{E}_T^-(\omega_0 - \omega^*, \mathbf{k}_0 - \mathbf{k}_S)$ and an anti-Stokes Langmuir wave $\mathbf{E}_L^+(\omega_0 + \omega, \mathbf{k}_0 + \mathbf{k}_S)$ are generated. The linear dispersion relations are: $(\omega_T^\mp)^2 = \omega_p^2 + c^2(\mathbf{k}_0 \mp \mathbf{k})^2$ for electromagnetic waves, $(\omega_L^\mp)^2 = \omega_p^2 + \gamma_e v_{th}^2 (\mathbf{k}_0 \mp \mathbf{k})^2$ for Langmuir waves, and $\omega_S^2 = v_S^2 k_S^2$ for ion-acoustic waves.

3. Nonlinear solutions

As the result of nonlinear wave-wave interactions, slow spatio-temporal modulations of the wave fields appear and the pump depletion must be taken into account. Thus, we introduce the following modulational representation for the wave electric fields

$$\mathbf{E}_\alpha(\mathbf{r}, t) = \frac{1}{2} \mathcal{E}_\alpha(\mathbf{r}, t) \exp i\theta_\alpha + c.c., \quad (3)$$

where $\mathcal{E}_\alpha(\mathbf{r}, t)$ is a slowly varying complex envelope such that $|\partial_r^2 \mathcal{E}_\alpha| \ll |k_\alpha \partial_r \mathcal{E}_\alpha|$ and $|\partial_t^2 \mathcal{E}_\alpha| \ll |\omega_\alpha \partial_t \mathcal{E}_\alpha|$, $\theta_\alpha = \mathbf{k}_\alpha \cdot \mathbf{r} - \omega_\alpha t$ is a fast-varying phase. α refers to each interacting wave. ω_α and $\mathbf{k}_\alpha$ are the linear wave frequencies and wavevectors.

The coupled mode equations derived from equations (1)-(2) for the process $L_0 = T^+ + L^- + S$ are

$$(\partial_t + \nu_L/2)\mathcal{E}_0 = c^- \omega_0 (\mathcal{E}_S \mathcal{E}_L^- \exp i\delta^- t - r \mathcal{E}_S^* \mathcal{E}_T^+ \exp i\delta^+ t), \quad (4)$$

$$(\partial_t + \nu_S/2)\mathcal{E}_S = -c^- (m_e\, \omega_p^2 / 2\, m_i\, \omega_S) \times (\mathcal{E}_0 \mathcal{E}_L^{-*} \exp -i\delta^- t + r \mathcal{E}_0^* \mathcal{E}_T^+ \exp i\delta^+ t), \quad (5)$$

$$(\partial_t + \nu_L/2)\mathcal{E}_L^- = -c^- \omega_L^- \mathcal{E}_S^* \mathcal{E}_0 \exp -i\delta^- t, \quad (6)$$

$$(\partial_t + \nu_T/2)\mathcal{E}_T^+ = c^+ \omega_T^+ \mathcal{E}_S \mathcal{E}_0 \exp -i\delta^+ t, \quad (7)$$

where ν_T is the electron-ion collisional frequency, ν_L is the sum of electron Landau damping frequency and electron-ion collisional frequency, and ν_S is the ion Landau damping frequency [6]; the nonlinear coupling coefficients are given by $c^- = (ek_S)/(4m_e \omega_0 \omega_L^-)$ and $c^+ = (ek_S)/(4m_e \omega_0 \omega_T^+)$; $\delta^- = \omega_0 - \omega_S - \omega_L^-$ and $\delta^+ = \omega_0 + \omega_S - \omega_T^+$ are the linear frequency mismatch parameters; and $r = c^+/c^-$ is a measure of the relative coupling strength of the anti-Stokes and the Stokes waves.

For the sake of simplicity, we did not specify the frequency of the high-frequency fields in the RHS of the generalized Zakharov equations (1) and (2). Although the frequencies of the Langmuir and electromagnetic sidebands are close to the electron plasma frequency, they need to be differentiated because the quiver electron velocities in the Langmuir and electromagnetic fields are different, as specified now in the RHS of equations (4)-(7). We only included the dominant nonlinear coupling terms in the RHS of the high- frequency wave equations. In particular, we only considered the nonlinear current arising from the fast component of the electron quiver velocity but ignored the nonlinear current arising from the slow component of the electron quiver velocity which may give additional (but negligible) contributions.

The nonlinear solutions of (4)-(7) are facilitated by using the polar notation, $\mathcal{E}_\alpha = \beta_\alpha F_\alpha^{1/2} \exp i\phi_\alpha$ where F_α and ϕ_α are real variables. With this notation, (4)-(7) can be rewritten identically as

$$\dot{F}_1 = 2(F_1 F_2 F_3)^{1/2} \cos\phi^-$$

$$-2r(F_1F_2F_4)^{1/2}\cos\phi^+ - \nu_1'F_1, \quad (8)$$

$$\dot{F}_2 = -2(F_1F_2F_3)^{1/2}\cos\phi^- -2r(F_1F_2F_4)^{1/2}\cos\phi^+ - \nu_2'F_2, \quad (9)$$

$$\dot{F}_3 = -2(F_1F_2F_3)^{1/2}\cos\phi^- - \nu_3'F_3, \quad (10)$$

$$\dot{F}_4 = 2r(F_1F_2F_4)^{1/2}\cos\phi^+ - \nu_4'F_4, \quad (11)$$

$$\dot{\phi}_1 = -(F_2F_3/F_1)^{1/2}\sin\phi^- + r(F_2F_4/F_1)^{1/2}\sin\phi^+. \quad (12)$$

$$\dot{\phi}_2 = -(F_1F_3/F_2)^{1/2}\sin\phi^- + r(F_1F_4/F_2)^{1/2}\sin\phi^+. \quad (13)$$

$$\dot{\phi}_3 = -(F_1F_2/F_3)^{1/2}\sin\phi^- + \delta'^-. \quad (14)$$

$$\dot{\phi}_4 = r(F_1F_2/F_4)^{1/2}\sin\phi^+ + \delta'^+, \quad (15)$$

where for the process $L_0 \Rightarrow T^+ + L^- + S$ ($L_0 \Rightarrow T^- + L^+ + S$) the suscript 1 denotes the pump Langmuir wave, 2 denotes the idler ion-acoustic wave, 3 denotes the Stokes Langmuir (electromagnetic) wave, and 4 denotes the anti-Stokes electromagnetic (Langmuir) wave; the dot denotes differentiation with respect to $\tau = \omega_p t$; $\phi^- = \phi_1 - \phi_2 - \phi_3$ and $\phi^+ = \phi_1 + \phi_2 - \phi_4$; $\delta'^{\mp} = \delta^{\mp}/\omega_p$; $\nu_\alpha' = \nu_\alpha/2\omega_p$; the normalization parameters β_α are given by $\beta_1 = (1/c^-)(2m_i\omega_S/m_e\omega^-)^{1/2}$, $\beta_2 = (\omega_p/c^-)(\omega_0\omega^-)^{-1/2}$, $\beta_3 = (1/c^-)(2m_i\omega_S/m_e\omega_0)^{1/2} \times \exp i\delta'^-\tau$, and $\beta_4 = (1/c^-)(2m_i\omega_S/m_e\omega_0)^{1/2} \times \exp i\delta'^+\tau$. Note that (8)-(15) can be reduced to six equations by rewriting (12)-(15) in terms of ϕ^- and ϕ^+.

In the absence of dissipation ($\nu_\alpha' = 0$), a number of constant of motion can be derived from (8)-(15):

$$H = 2(F_1F_2)^{1/2}(F_3^{1/2}\sin\phi^- - rF_4^{1/2}\sin\phi^+) -\delta'^-F_3 - \delta'^+F_4. \quad (16)$$

$$c_1 = F_1 + F_3 + F_4, \quad c_2 = F_2 - F_3 + F_4, \quad (17)$$

where H is the Hamiltonian of the system and (17) are the Manley-Rowe relations.

4. Wave energy conservation

The physics of nonlinear interaction of the hybrid stimulated modulational process can be elucidated by the conservation relations (16)-(17). Noting that the dimensionless real variables F_α, introduced to simplify the derivation of nonlinear solutions in section 3. are normalized wave energies. Therefore, the Hamiltonian H in (16) describes the wave energy conservation of all the interacting waves: the first term in the RHS demonstrates the nonlinear coupling of the two wave triplets 1-2-3 (pump - idler - Stokes modes) and 1-2-4 (pump - idler - anti-Stokes modes); the second and third terms in the RHS represent the effects of frequency mismatch (δ'^- and δ'^+) of the two wave triplets.

By defining the wave energy of each wave as

$$\epsilon_{1,3,4} = \frac{\varepsilon_0}{2}|\mathcal{E}_{1,3,4}|^2\left(\frac{\omega_{1,3,4}^2}{\omega_0^2}\right),$$

$$\epsilon_2 = \frac{\varepsilon_0}{2}|\mathcal{E}_2|^2\left(\frac{m_i}{m_e}\frac{\omega_2^2}{\omega_0^2}\right), \quad (18)$$

the Manley-Rowe relations (17) can be rewritten, respectively, as

$$\epsilon_1/\omega_1 + \epsilon_3/\omega_3 + \epsilon_4/\omega_4 = constant, \quad (19)$$

$$\epsilon_2/\omega_2 - \epsilon_3/\omega_3 + \epsilon_4/\omega_4 = constant, \quad (20)$$

which represent the wave action conservations since $\epsilon_\alpha/\omega_\alpha$ is the wave action. It follows from (19) and (20) that

$$\partial_t(\epsilon_1 + \epsilon_3 + \epsilon_4) = 0, \quad (21)$$

$$\partial_t(\epsilon_2 - \epsilon_3 + \epsilon_4) = 0, \quad (22)$$

which are the wave energy conservation relations for the two sets of waves 1-3-4 (pump - Stokes - anti-Stokes modes) and 2-3-4 (idler - Stokes - anti-Stokes modes), respectively. In quantum mechanical language, (21) implies that two pump quanta are required to produce two daughter quanta (one Stokes and one anti-Stokes); whereas, (22) indicates that a Stokes quantum emits (the minus sign) an idler quantum and an anti-Stokes quantum absorbs (the plus sign) an idler quantum, in agreement with the frequency and wavevector selection rules.

5. Discussion

In the absence of frequency mismatch and dissipation ($\delta'^- = \delta'^+ = \nu_\alpha' = 0$), some analytical periodic solutions of (8)-(15) can be obtained [11, 12]. An example of the periodic nonlinear saturated state of the hybrid SMI is given in 1.

The effect of the finite linear frequency mismatch is illustrated in figure 2 for the process $L_0 = T^+ + L^- + S$. Figures 2 indicate that the frequency mismatch ($\delta'^{\pm} \neq 0$) reduces the efficiency of the energy transfer among waves. When $\delta'^{\pm} = 0$, figure 1 shows that the Langmuir pump is fully

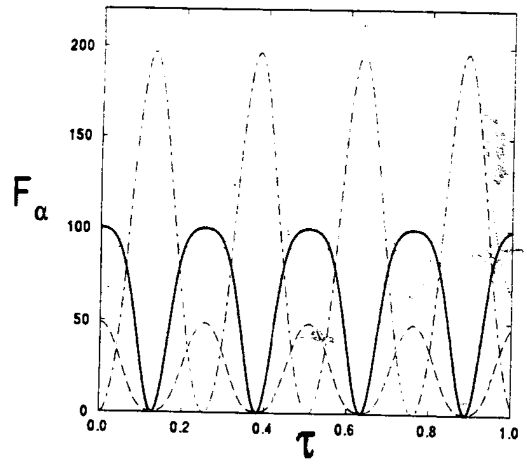

Figure 1: The plot of $F_\alpha(\tau)$ for $L_0 \rightleftharpoons T+L+S$: $r = 0.95$, and $\delta'^- = \delta'^+ = \nu'_\alpha = 0$: the initial conditions are $F_1(0) = 100.01$, $F_2(0) = 0$, $F_3(0) = 64$ and $F_4(0) = 49$. The solid curve is $F_1(\tau)$, the dot-dashed curve is $F_2(\tau)$, the dotted curve is $F_3(\tau)$, and the long dashed curve is $F_4(\tau)$.

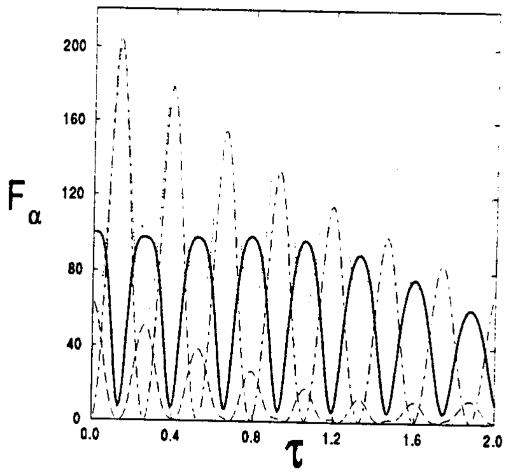

Figure 2: The plot of $F_\alpha(\tau)$ for $L_0 \rightleftharpoons T^+ + L^- + S$; $r = 0.95$; $\nu'_1 = \nu'_3 = 0.1$, $\nu'_2 = 0.5$ and $\nu'_4 = 0.001$; $\delta'^- = 5$ and $\delta'^+ = -5$; the initial conditions are $F_1(0) = 100.01$, $F_2(0) = 0$, $F_3(0) = 49$ and $F_4(0) = 64$; the notations are the same as in figure 5.

depleted with $(F_1)_{min} = 0$. However, a finite frequency mismatch prevents a complete depletion of the pump energy so that $(F_1)_{min} > 0$, as seen in figures 2.

The effect of dissipation can be identified by comparing figures 2 with figure 1. In the absence of dissipation, all four interacting waves are strictly periodic as shown in figure 1. The dissipation causes the gradual damping of wave amplitudes as shown in figure 2. The damping rate of each wave depends on the nature of wave-particle and particle-particle interactions. Wave damping converts the wave energies into the kinetic energies of particles, resulting in plasma heating.

The effect of wave dispersion is contained in the nonlinear coupling coefficients $c^{\mp}$ as well as the parameter r. In fact, the ratio r reduces to $\omega^-(\mathbf{k}^-)/\omega^+(\mathbf{k}^+)$. Therefore, the relative coupling strength of the two coupled wave triplets in the hybrid stimulated modulational processes is determined by the dispersive properties of Stokes and anti-Stokes waves.

6. Conclusion

In conclusion, we have developed a nonlinear theory of the hybrid stimulated modulational processes. It is shown that the fundamental plasma radiation can be driven by a traveling Langmuir wave via the ponderomotive coupling to the induced Langmuir and resonant ion-acoustic waves. In general, the process $L_0 \rightleftharpoons T^+ + L^- + S$ is more efficient than the process $L_0 \rightleftharpoons T^- + L^+ + S$ in generating the escaping radiation in plasmas since the frequency of the induced electromagnetic wave is upconverted to $\omega_0 + \omega_S$ and can readily leave the source region, whereas in the latter process the electromagnetic wave frequency is downconverted to $\omega_0 - \omega_S$ and can be easily absorbed by plasmas. Hence, it is plausible that the hybrid stimulated modulational process, $L_0 \rightleftharpoons T^+ + L^- + S$, may contribute effectively to the generation of nonthermal plasma emissions in cosmic and laboratory plasmas.

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References

[1] A. C.-L. Chian and J. R. Abalde, J. Plasma Phys. *in press* (1996).

[2] A. C.-L. Chian and M. V. Alves, Astrophys. J. **330**, L77 (1988).

[3] P. K. Shukla, M. Y. Yu, M. Mohan, R. K. Varma and K. H. Spatschek, Phys. Rev. **A27**, 552 (1983).

[4] A. C.-L. Chian, Planet. Space Sci. **39**, 1217 (1991).

[5] C. N. Lashmore-Davies, Phys. Rev. Lett. **32**, 289 (1974).

[6] K. Akimoto, Phys. Fluids **31**, 538 (1988).

[7] F. B. Rizzato and A. C.-L. Chian, J. Plasma Phys. **48**, 71 (1992).

[8] J. F. Drake, P. K. Kaw, Y. C. Lee, G. Schmidt, C. S. Liu, and M. N. Rosenbluth, Phys. Fluids **17**, 778 (1974).

[9] B. D. Fried, T. Ikemura, K. Nishikawa and G. Schmidt, Phys. Fluids **19**, 1975 (1976).

[10] S. Bardwell and M. Goldman, Astrophys. J. **209**, 912 (1976).

[11] D. Walters and G. Lewak, J. Plasma Phys. **18**, 525 (1977).

[12] F. J. Romeiras, Phys. Lett. **93A**, 227 (1983).