Fundamental radio emission generated by a traveling Langmuir wave

José R. Abalde[1] and Abraham C.-L. Chian[2]

[1]Universidade do Vale do Paraíba-UNIVAP, Instituto de Pesquisas e Desenvolvimento-IP&D, Urbanova, 12224-000 São José dos Campos - SP, Brazil - E-mail: abalde@univap.br

[2] Instituto Nacional de Pesquisas Espaciais-INPE, P.O.Box 515, 12201-970 São José dos Campos SP, Brazil - E-mail: achian@dge.inpe.br

Abstract It is shown that a traveling Langmuir pump wave can nonlinearly convert into electromagnetic, Langmuir and ion-acoustic daughter waves via a new mechanism of hybrid stimulated modulational instability. The nonlinear temporal dynamics of this modulational process involving coupling to a resonant ion-acoustic wave is studied. The wave energy conservation relations are derived. The roles of frequency mismatch, dissipation and wave dispersion in the temporal evolution of nonlinear coupling of two wave triplets are analyzed.

---

1. Introduction The subject of excitation of fundamental plasma emissions is of great interest in plasma physics, space physics and astrophysics. In laboratory, plasma radiation near the fundamental plasma frequency has been observed in quiescent machines (Cheung *et al.* 1982), stellarators (Longinov *et al.* 1976), and tokamaks (Gandy *et al.* 1985) experiments. In space plasmas, it has been detected during active experiments in space (Thidé *et al.* 1982; Chian 1991), upstream of planetary bow shocks (Gurnett and Frank 1975; Chian and Abalde 1995), and in type-III events in the solar wind (Lin *et al.* 1986; Chian and Alves 1988; Abalde *et al.* 1998). In astrophysical plasmas, it may provide the source mechanism for emissions from flare stars, astrophysical jets and active galactic nuclei (Baker *et al.* 1988).

A number of nonlinear mechanisms involving the interaction of Langmuir and ion-acoustic waves have been proposed to explain the generation of fundamental plasma emissions. These include induced scattering of Langmuir waves off ion clouds; incoherent coalescense of Langmuir and ion-acoustic waves; collapse of nonlinear Langmuir wave packets; conversion of Langmuir waves by density fluctuations driven by strong turbulence (see e.g., Chian and Alves 1988 and Chian and Abalde 1997 for references). In particular, various types of parametric instabilities have been studied: three-wave electromagnetic decay (fusion) instability in which a Langmuir traveling pump wave produces a Stokes (anti-Stokes) electromagnetic wave via coupling to an ion-acoustic wave (Shukla *et al.* 1983; Chian 1991); electromagnetic modulational instabilities in which two counter-propagating Langmuir pump waves generate a pair of electromagnetic daughter waves via coupling to ion-acoustic waves (Lashmore-Davies 1974; Chian and Alves 1988); hybrid (electromagnetic-electrostatic) absolute modulational instability in which a Langmuir traveling pump wave excites an electromagnetic daughter wave and a Langmuir daughter wave via coupling to purely growing density fluctuations (Akimoto 1988); hybrid modulational instabilities in which two oppositely directed Langmuir pump waves emit a pair of electromagnetic daughter waves and two pairs of Langmuir daughter waves via coupling to ion-acoustic waves (Rizzato and Chian 1992).

In previous works on modulational instabilities the low-frequency (idler) mode is usually considered nonresonant, whereas the upper and lower-sidebands are resonant modes (Drake *et al.* 1974; Fried *et al.* 1976; Akimoto 1988). In contrast, we consider in the present paper the stimulated modulational instability in which all the daughter modes (including the idler mode) are resonant waves. The existence of the stimulated modulational instability in which the low-frequency mode is a resonant mode of plasma was mentioned in the general theory of parametric instabilities formulated by Mima and Nishikawa (1984).

A linear stability analysis of the purely electrostatic SMI induced by a traveling Langmuir wave (Bardwell and Goldman 1976), $L \rightarrow L + L + S$ (where S is a resonant ion-acoustic wave), shows that its maximum growth rate is comparable to the parametric decay instability (PDI) $L \rightarrow L + S$ as well as the oscillating two-stream instability (OTSI), $L \rightarrow L + L + S^*$ (where S^* is a nonresonant purely growing density perturbation). This result indicates that the SMI can compete effectively with other parametric processes when the Langmuir turbulence is excited in plasmas.

The novel *hybrid* (electrostatic and electromagnetic) SMI to be studied in this paper is $L \rightarrow T + L + S$. The Langmuir and electromagnetic waves have frequencies near the electron plasma frequency, whereas the ion-acoustic wave has frequency near the ion-acoustic frequency. The total wave electric field for this process can be written as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_T + \mathbf{E}_L + \mathbf{E}_S$, where $\mathbf{E}_0$ is a Langmuir traveling pump wave (L_0), $\mathbf{E}_T$ is an electromagnetic daughter wave (T), $\mathbf{E}_L$ is a Langmuir daughter wave (L), and $\mathbf{E}_S$ is a resonant ion-acoustic daughter wave (S). A previous study analyzed the linear theory of the hybrid nonresonant (absolute) modulational instability $L \rightarrow T + L + S^*$ wherein the low-frequency density fluctuations (S^*) are purely growing (Akimoto 1988). In this paper, we present the linear and nonlinear theories of the hybrid stimulated modulational processes $L \rightleftharpoons T + L + S$ which are convective processes involving interactions with resonant ion-acoustic waves.

2. Governing equations The basic equations that govern the ponderomotive coupling of Langmuir waves with high-frequency electromagnetic and electrostatic waves, near the fundamental plasma frequency, and low-frequency ion-acoustic waves are the generalized Zakharov equations (Akimoto 1988; Rizzato and Chian 1992; Chian and Rizzato 1994; Chian and Abalde 1997)

$$(\partial_t^2 + \nu_e\partial_t + c^2\nabla\times\nabla\times - \gamma_e v_{th}^2\nabla(\nabla\cdot) + \omega_p^2)\mathbf{E} = = -\frac{\omega_p^2}{n_0} n\mathbf{E}, \quad (1)$$

$$(\partial_t^2 + \nu_i\partial_t - v_S^2\nabla^2)n = \frac{\varepsilon_0}{2m_i}\nabla^2\langle E^2\rangle, \quad (2)$$

where n is the ion density fluctuation, $\omega_p = (n_0e^2/m_e\varepsilon_0)^{1/2}$ is the electron plasma frequency, $v_{th} = (KT_e/m_e)^{1/2}$ is the electron thermal velocity, $v_S = [K(\gamma_eT_e + \gamma_iT_i)/m_i]^{1/2}$ is the ion-acoustic velocity, ν_e (ν_i) is the phenomenological damping frequency for electrons (ions), γ_e (γ_i) is the ratio of the specific heats for electrons (ions), and the angular brackets denote the fast time average. It is evident from the wave operator in the LHS of (1) that the high-frequency wave field $\mathbf{E}$ can be *hybrid* (i.e., containing of both electromagnetic and electrostatic components).

3. Nonlinear solutions As the result of nonlinear wave-wave interactions, slow spatio-temporal modulations of the wave fields appear and the pump depletion must be taken into account. Thus, we introduce the following modulational representation for the wave electric fields $\mathbf{E}_\alpha(\mathbf{r},t) = \frac{1}{2}\mathcal{E}_\alpha(\mathbf{r},t)\exp i\theta_\alpha + c.c.$, where $\mathcal{E}_\alpha(\mathbf{r},t)$ is a slowly varying complex envelope such that $|\partial_r^2\mathcal{E}_\alpha| \ll |k_\alpha\partial_r\mathcal{E}_\alpha|$ and $|\partial_t^2\mathcal{E}_\alpha| \ll |\omega_\alpha\partial_t\mathcal{E}_\alpha|$, $\theta_\alpha = \mathbf{k}_\alpha\cdot\mathbf{r} - \omega_\alpha t$ is a fast-varying phase, α refers to each interacting wave, ω_α and $\mathbf{k}_\alpha$ are the linear wave frequencies and wavevectors.

The coupled mode equations derived from equations (1)-(2) for the process $L_0 \rightleftharpoons T^+ + L^- + S$ are

$$(\partial_t + \nu_L/2)\mathcal{E}_0 = = c^-\ \omega_0(\mathcal{E}_S\mathcal{E}_L^-\exp i\delta^- t - r\mathcal{E}_S^*\mathcal{E}_T^+\exp i\delta^+ t), \quad (3)$$

$$(\partial_t + \nu_S/2)\mathcal{E}_S = -c^-(m_e\ \omega_p^2/2\ m_i\ \omega_S)\times \times(\mathcal{E}_0\mathcal{E}_L^{-*}\exp -i\delta^- t + r\mathcal{E}_0^*\mathcal{E}_T^+\exp i\delta^+ t), \quad (4)$$

$$(\partial_t + \nu_L/2)\mathcal{E}_L^- = -c^-\ \omega_L^-\ \mathcal{E}_S^*\mathcal{E}_0\exp -i\delta^- t, \quad (5)$$

$$(\partial_t + \nu_T/2)\mathcal{E}_T^+ = c^+\ \omega_T^+\ \mathcal{E}_S\mathcal{E}_0\exp -i\delta^+ t, \quad (6)$$

where ν_T is the electron-ion collisional frequency, ν_L is the sum of electron Landau damping frequency and electron-ion collisional frequency, and ν_S is the ion Landau damping frequency (Akimoto 1988); the nonlinear coupling coefficients are given by $c^- = (ek_S)/(4m_e\omega_0\omega_L^-)$ and $c^+ = (ek_S)/(4m_e\omega_0\omega_T^+)$; $\delta^- = \omega_0 - \omega_S - \omega_L^-$ and $\delta^+ = \omega_0 + \omega_S - \omega_T^+$ are the linear frequency mismatch parameters; and $r = c^+/c^-$, is a measure of the relative coupling strength of the anti-Stokes and the Stokes waves.

For the sake of simplicity, we did not specify the frequency of the high-frequency fields in the RHS of the generalized Zakharov equations (1) and (2). Although the frequencies of the Langmuir and electromagnetic sidebands are close to the electron plasma frequency, they need to be differentiated because the quiver electron velocities in the Langmuir and electromagnetic fields are different, as specified now in the RHS of equations (3)-(6). Following Thornhill and ter Haar (1978) and Bingham and Lashmore-Davies (1979), we only included the dominant nonlinear coupling terms in the RHS of the high- frequency wave equations. In particular, we only considered the nonlinear current arising from the fast component of the electron quiver velocity but ignored the nonlinear current arising from the slow component of the electron quiver velocity which may give additional (but negligible) contributions.

The nonlinear solutions of (3)-(6) are facilitated by using the polar notation, $\mathcal{E}_\alpha = \beta_\alpha F_\alpha^{1/2}\exp i\phi_\alpha$, where F_α and ϕ_α are real variables. With this notation, (3)-(6) can be rewritten identically as

$$\dot{F}_1 = 2(F_1F_2F_3)^{1/2}\cos\phi^- - 2r(F_1F_2F_4)^{1/2}\cos\phi^+ - \nu_1'F_1, \quad (7)$$

$$\dot{F}_2 = -2(F_1F_2F_3)^{1/2}\cos\phi^- - 2r(F_1F_2F_4)^{1/2}\cos\phi^+ - \nu_2'F_2, \quad (8)$$

$$\dot{F}_3 = -2(F_1F_2F_3)^{1/2}\cos\phi^- - \nu_3'F_3, \quad (9)$$

$$\dot{F}_4 = 2r(F_1F_2F_4)^{1/2}\cos\phi^+ - \nu_4'F_4, \quad (10)$$

$$\dot{\phi}_1 = -(F_2F_3/F_1)^{1/2}\sin\phi^- + r(F_2F_4/F_1)^{1/2}\sin\phi^+, \quad (11)$$

$$\dot{\phi}_2 = -(F_1F_3/F_2)^{1/2}\sin\phi^- + r(F_1F_4/F_2)^{1/2}\sin\phi^+, \quad (12)$$

$$\dot{\phi}_3 = -(F_1F_2/F_3)^{1/2}\sin\phi^- + \delta'^-, \quad (13)$$

$$\dot{\phi}_4 = r(F_1F_2/F_4)^{1/2}\sin\phi^+ + \delta'^+, \quad (14)$$

where for the process $L_0 \rightleftharpoons T^+ + L^- + S$ ($L_0 \rightleftharpoons T^- + L^+ + S$) the suscript 1 denotes the pump Langmuir wave, 2 denotes the idler ion-acoustic wave, 3 denotes the Stokes Langmuir (electromagnetic) wave, and 4 denotes the anti-Stokes electromagnetic (Langmuir) wave; the dot denotes differentiation with respect to $\tau = \omega_p t$; $\phi^- = \phi_1 - \phi_2 - \phi_3$ and $\phi^+ = \phi_1 + \phi_2 - \phi_4$; $\delta'^{\mp} = \delta^{\mp}/\omega_p$; $\nu_\alpha' = \nu_\alpha/2\omega_p$; the normalization parameters β_α are given by $\beta_1 = (1/c^-)(2m_i\omega_S/m_e\omega^-)^{1/2}$, $\beta_2 = (\omega_p/c^-)(\omega_0\omega^-)^{-1/2}$,

$\beta_3 = (1/c^-)(2m_i\omega_S/m_e\omega_0)^{1/2}\exp i\delta'^-\tau$, and $\beta_4 = (1/c^-)(2m_i\omega_S/m_e\omega_0)^{1/2}\exp i\delta'^+\tau$. Note that (7)-(14) can be reduced to six equations by rewriting (11)-(14) in terms of ϕ^- and ϕ^+.

In the absence of dissipation ($\nu'_\alpha = 0$), a number of constant of motion can be derived from (7)-(14):

$$H = 2(F_1F_2)^{1/2}(F_3^{1/2}\sin\phi^- - rF_4^{1/2}\sin\phi^+) - \delta'^-F_3 - \delta'^+F_4, \quad (15)$$

$$c_1 = F_1 + F_3 + F_4, \qquad c_2 = F_2 - F_3 + F_4, \quad (16)$$

where H is the Hamiltonian of the system, and (16) are the Manley-Rowe relations.

4. Wave energy conservation The physics of nonlinear interaction of the hybrid stimulated modulational process can be elucidated by the conservation relations (15)-(16). Noting that the dimensionless real variables F_α, introduced in the polar notation to simplify the derivation of nonlinear solutions, are normalized wave energies. Therefore, the Hamiltonian H in (15) describes the wave energy conservation of all the interacting waves: the first term in the RHS demonstrates the nonlinear coupling of the two wave triplets 1-2-3 (pump - idler - Stokes modes) and 1-2-4 (pump - idler - anti-Stokes modes); the second and third terms in the RHS represent the effects of frequency mismatch (δ'^- and δ'^+) of the two wave triplets.

By defining the wave energy of each wave as

$$\epsilon_{1,3,4} = \frac{\varepsilon_0}{2}|\mathcal{E}_{1,3,4}|^2\left(\frac{\omega_{1,3,4}^2}{\omega_0^2}\right), \quad (17)$$

$$\epsilon_2 = \frac{\varepsilon_0}{2}|\mathcal{E}_2|^2\left(\frac{m_i}{m_e}\frac{\omega_2^2}{\omega_0^2}\right), \quad (18)$$

the Manley-Rowe relations (16) can be rewritten, respectively, as

$$\epsilon_1/\omega_1 + \epsilon_3/\omega_3 + \epsilon_4/\omega_4 = constant, \quad (19)$$

$$\epsilon_2/\omega_2 - \epsilon_3/\omega_3 + \epsilon_4/\omega_4 = constant, \quad (20)$$

which represent the wave action conservations since $\epsilon_\alpha/\omega_\alpha$ is the wave action. It follows from (19) and (20) that

$$\partial_t(\epsilon_1 + \epsilon_3 + \epsilon_4) = 0, \quad (21)$$

$$\partial_t(\epsilon_2 - \epsilon_3 + \epsilon_4) = 0, \quad (22)$$

which are the wave energy conservation relations for the two sets of waves 1-3-4 (pump - Stokes - anti-Stokes modes) and 2-3-4 (idler - Stokes - anti-Stokes modes), respectively. In quantum mechanical language, (21) implies that two pump quanta are required to produce two daughter quanta (one Stokes and one anti-Stokes); whereas, (22) indicates that a Stokes quantum emits (the minus sign) an idler quantum and an anti-Stokes quantum absorbs (the plus sign) an idler quantum, in agreement with the frequency and wavevector selection rules.

5. Discussion In the absence of frequency mismatch and dissipation ($\delta'^- = \delta'^+ = \nu'_\alpha = 0$), some analytical periodic solutions of (7)-(14) can be obtained (Walters & Lewak 1977; Romeiras 1983). An example of the periodic nonlinear saturated state of the hybrid SMI is given in figure 1.

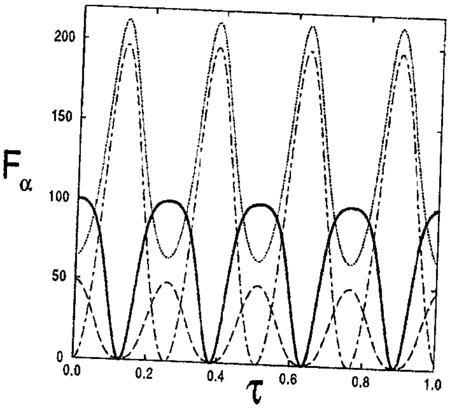

Figure 1: The plot of $F_\alpha(\tau)$ for $L_0 \rightleftharpoons T + L + S$; $r = 0.95$, and $\delta'^- = \delta'^+ = \nu'_\alpha = 0$; the initial conditions are $F_1(0) = 100.01$, $F_2(0) = 0$, $F_3(0) = 64$ and $F_4(0) = 49$. The solid curve is $F_1(\tau)$, the dot-dashed curve is $F_2(\tau)$, the dotted curve is $F_3(\tau)$, and the long dashed curve is $F_4(\tau)$.

The effect of the finite linear frequency mismatch is illustrated in figure 2 for the process $L_0 \rightleftharpoons T^+ + L^- + S$, and in figure 3 for the process $L_0 \rightleftharpoons T^- + L^+ + S$. Figures 2 and 3 indicate that the frequency mismatch ($\delta'^\pm \neq 0$) reduces the efficiency of the energy transfer among waves. When $\delta'^\pm = 0$, figure 1 shows that the Langmuir pump is fully depleted with $(F_1)_{min} = 0$. However, a finite frequency mismatch prevents a complete depletion of the pump energy so that $(F_1)_{min} > 0$, as seen in figures 2 and 3.

The effect of dissipation can be identified by comparing figures 2 and 3 with figure 1. In the absence of dissipation, all four interacting waves are strictly periodic as shown in figure 1. The dissipation causes the gradual damping of wave amplitudes as shown in figures 2 and 3. The damping rate of each wave depends on the nature of wave-particle and particle-particle interactions. In general, the damping of Langmuir waves is due to the combined action of electron Landau damping and electron-ion collision; the damping of electromagnetic waves is due to electron-ion collision; and the damping of ion-acoustic waves is due to ion Landau damping (Akimoto 1988). Wave damping converts the wave energies into the kinetic energies of particles, resulting in plasma heating.

The effect of wave dispersion is contained in the nonlinear coupling coefficients $c^\mp$ as well as the parameter r. In fact, the ratio r as defined reduces to $\omega^-(\mathbf{k}^-)/\omega^+(\mathbf{k}^+)$. Therefore, the relative coupling strength of the two coupled wave triplets in the hybrid stimulated modulational processes is determined by the dispersive properties of Stokes and anti-Stokes waves.

The authors wish to thank M. V. Alves , S. R. Lopes and F. J. Romeiras for discussion and the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for the support received through grants 98/12493-0, 98/09892-0 and 98/02841-0.

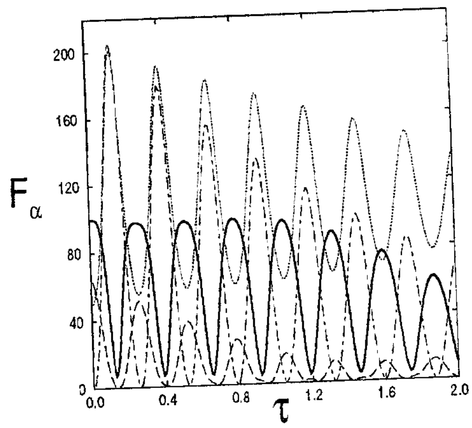

Figure 2: The plot of $F_\alpha(\tau)$ for $L_0 \rightleftharpoons T^+ + L^- + S$; $r = 0.95$; $\nu_1' = \nu_3' = 0.1, \nu_2' = 0.5$ and $\nu_4' = 0.001$; $\delta'^- = 5$ and $\delta'^+ = -5$; the initial conditions are $F_1(0) = 100.01$, $F_2(0) = 0$, $F_3(0) = 49$ and $F_4(0) = 64$; the notations are the same as in figure 1.

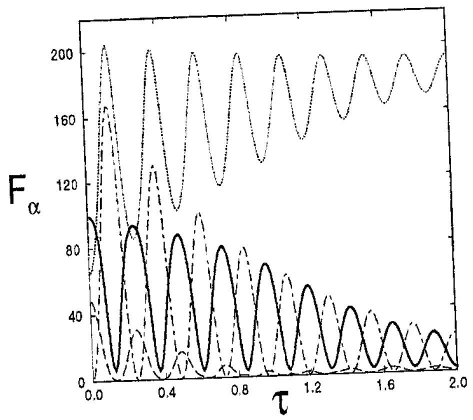

Figure 3: The plot of $F_\alpha(\tau)$ for $L_0 \rightleftharpoons T^- + L^+ + S$; $r = 0.95$; $\nu_1' = \nu_4' = 0.1$, $\nu_2' = 1.0$ and $\nu_3' = 0.001$; $\delta'^- = 5$ and $\delta'^+ = -5$; the initial conditions and the notations are the same as in figure 1.

References

Abalde, J. R., Chian, A. C.-L. and Alves, M. V. *Astron. Astrophys.* **331**, L21, 1998.

Akimoto, K. *Phys. Fluids* **31**, 538, 1988.

Baker, D. N., Borovsky, J. E., Bendford, G. and Eilek, J. A. *Astrophys. J.* **326**, 110, 1988.

Bardwell, S. and Goldman, M. V. *Astrophys. J.* **209**, 912, 1976.

Bingham, R. and Lashmore-Davies, C. N. *J. Plasma Phys.* **21**, 51, 1979.

Cheung, P. Y., Wong, A. Y., Darrow, C. B. and Qian, S. J. *Phys. Rev. Lett.* **48**, 1348, 1982.

Chian, A. C. -L. *Planet. Space Sci.* **39**, 1217, 1991.

Chian, A. C. -L. and Abalde, J. R. *Astron. Astrophys.* **298**, L9, 1995.

Chian, A. C. -L. and Abalde, J. R. *J. Plasma Phys.* **57**, 753, 1997.

Chian, A. C. -L. and Alves, M. V. *Astrophys. J.* **330**, L77, 1988.

Chian, A. C. -L. and Rizzato, F. B. *J. Plasma Phys.* **51**, 61, 1994.

Drake, J. F., Kaw, P. K., Lee, Y. C., Schmidt, G., Liu, C. S. and Rosenbluth, M. N. *Phys. Fluids* **17**, 778, 1974.

Fried, B. D., Ikemura, T., Nishikawa, K. and Schmidt, G. *Phys. Fluids* **19**, 1975, 1976.

Gandy, R. F., Hutchinson, I. H. and Yates, D. H. *Phys. Rev. Lett.* **54**, 800, 1985.

Gurnett, D. A. and Frank, L. A. *Solar Phys.* **45**, 477, 1975.

Lashmore-Davies C. N. *Phys. Rev. Lett.* **32**, 289, 1974.

Lin, R. P., Levedahl, W. K., Lotko, W., Gurnett, D. A. and Scarf, F. L. *Astrophys. J.* **308**, 954, 1986.

Longinov, A. V. Perepelkin, N. F. and Suprunenko, V. A. *Sov. J. Plasma Phys.* **2**, 344, 1976.

Mima, K. and Nishikawa, K. *Handbook of Plasma Physics, Vol. 2* (ed. A. A. Galeev & R. N. Sudan), p. 461, North-Holland, 1974.

Rizzato, F. B. and Chian, A. C. -L. *J. Plasma Phys.* **48**, 71, 1992.

Romeiras, F. J. *Phys. Lett.* **93A**, 227, 1983.

Shukla, P. K., Yu, M. Y., Mohan, M., Varma, R. K. and Spatschek, K. H. *Phys. Rev.* **A27**, 552, 1983.

Thidé, B., Kopka, H. and Stubbe, P. *Phys. Rev. Lett.* **49**, 1561, 1982.

Thornhill, S. G. and ter Haar, D. *Phys. Reports* **43**, 45, 1978.

Walters, D. and Lewak G. *J. Plasma Phys.* **18**, 525, 1977.