Chaotic dynamics of planetary electromagnetic waves emissions

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Abstract A dynamical theory of nonlinear three-wave interaction involving Langmuir, whistler and Alfvén waves in the planetary magnetospheres is developed. By assuming linear growth for the Langmuir wave and linear damping for both whistler and Alfvén waves, the wave triplet is shown to evolve temporally from order to chaos via the period doubling route. Numerical solutions of this dynamical system are presented, showing the time series of the wave amplitude and the corresponding power spectra. The characterization of orderly and chaotic states is performed by plotting the Poincaré maps and calculating the Lyapunov exponents. The relevance of this theory for the observation of chaos in nonthermal planetary radio emissions is discussed.

Introduction The nonthermal planetary electromagnetic emissions are intense and have brightness temperature greater than the plasma thermal temperature. These are emitted by Earth, Jupiter, Saturn, Uranus and Neptune (Kaiser, 1989; Kurth, 1991; Chian, 1993).The radiation mechanisms of these radio waves are related to linear and nonlinear plasma processes of the source regions in the planetary magnetospheres. In particular, various nonlinear mode-mode coupling processes involving interaction of plasma waves have been proposed as generation mechanisms of the nonthermal planetary radio emissions (Chian, 1993). Hence, the theoretical and observational studies of planetary radio emissions are essential for improving our understanding of the nonlinear dynamical processes occurring in the planetary magnetospheres.

Traditionally, it is believed that nonlinear interaction of electron-beam driven high-frequency electron plasma waves with the low-frequency density fluctuations, such as ion-acoustic waves, is the main mechanism for beam-driven turbulence to produce high-frequency electromagnetic waves (Chian and Alves, 1988; Chian and Abalde, 1995; Abalde et al., 1998). Recently, it has been shown that the nonlinear interaction of high-frequency Langmuir (L) waves with low-frequency magnetic field fluctuactions, such as Alfvén (A) waves, may also provide an efficient mechanism for generating high-frequency electromagnetic waves in space plasmas (Chian et al., 1994a,b; Lopes and Chian, 1996; Chian et al., 1997). This novel plasma emission mechanism may account for the production of whistler-mode (W) planetary radio emission in the Earth's and Jupiter's auroral acceleration regions.

In this paper, we develop a dynamical theory to describe the transition from order to chaos in nonlinear three-wave coupling involving Langmuir waves, electromagnetic whistler waves and Alfvén waves. The aim is to demonstrate that planetary radio emissions may exhibit chaotic behavior. We treat the nonlinear parametric coupling of three waves (L, W, A), traveling along the ambient magnetic field $\mathbf{B}=B_0\,\hat{\mathbf{z}}$, which fulfill the following phase-matching conditions: $\omega_L \approx \omega_W + \omega_A, \mathbf{k}_L = \mathbf{k}_W + \mathbf{k}_A$, where we allow frequency mismatch but assume perfect wavevector match. In addition to the wave frequency and wavevector matching conditions, the wave triplet (L, W, A) must also satisfy the conservation of wave helicity. Since the electromagnetic whistler wave is right-hand circularly polarized, the Alfvén wave in $L \rightleftharpoons W + A$ is left-hand circularly polarized (i.e., shear Alfvén mode).

The nonlinear system of coupled wave equations governing the three-wave process $L \rightleftharpoons W + A$ is given by

$$D_L \mathbf{E}_L = -ic_{WA} E_W E_A \hat{\mathbf{z}}, \tag{1}$$

$$D_W \mathbf{E}_W = ic_{LA} E_L \mathbf{E}_A^*, \tag{2}$$

$$D_A \mathbf{E}_A = ic_{LW} E_L \mathbf{E}_W^*, \tag{3}$$

the dispersion operators are given by

$$D_L = -\omega_L^2 + \omega_{pe}^2 + 3v_{th}^2 k_L^2 - i\nu_L \omega_L, \tag{4}$$

$$D_W = -\omega_W^2 + c^2 k_W^2 + \frac{\omega_{pe}^2 \omega_W}{\omega_W - \omega_{ce}} - i\nu_W \omega_W, \tag{5}$$

$$D_A = -\omega_A^2 + c_A^2 k_A^2 - i\nu_A \omega_A, \tag{6}$$

the coupling coefficients are

$$c_{WA} = \frac{e\omega_{pe}^2}{2m_e(\omega_W - \omega_{ce})}\left[\frac{k_A}{\omega_A} + \frac{k_W(\omega_W - \omega_{ce})}{\omega_W(\omega_A + \omega_{ce})}\right], \tag{7}$$

$$c_{LA} = \left(\frac{\omega_W^2}{\omega_L^2}\right) c_{WA}, \tag{8}$$

$$c_{LW} = \left(\frac{\omega_A^2 c_A^2}{\omega_L^2 c^2}\right) c_{WA}, \tag{9}$$

and the wave growth/damping parameters are

$$\nu_L = \frac{\omega_{pe}^2}{\omega_L^2}\nu_e, \tag{10}$$

$$\nu_W = \frac{\omega_{pe}^2 \nu_e}{(\omega_W - \omega_{ce})^2}, \tag{11}$$

$$\nu_A = \frac{\omega_{pe}^2 c_A^2 \nu_e}{c^2(\omega_A + \omega_{ce})^2} + \frac{\omega_{pi}^2 c_A^2 \nu_i}{c^2(\omega_A - \omega_{ci})^2}, \tag{12}$$

where $\omega_{pe}^2 = (n_0 e^2/m_e \varepsilon_0)^{1/2}$ is the electron plasma frequency, $\omega_{ce} = eB_0/m_e$ is the electron cyclotron frequency, $\omega_{ci} = eB_0/m_i$ is the ion cyclotron frequency, $v_{th} = (KT_e/m_e)^{1/2}$ is the electron thermal velocity, $c_A = B_0/(\mu_0\rho_0)^{1/2}$ is the Alfvén velocity, and $\nu_e(\nu_i)$ is the electron (ion) growth/damping rate.

As the result of nonlinear wave interaction, slow spatiotemporal modulation of wave fields appear, which can be represented by

$$\mathbf{E}_\alpha(z,t) = \frac{1}{2}\mathcal{E}_\alpha(z,t)\ exp\ i\theta_\alpha + c.c., \tag{13}$$

where $\mathcal{E}(z,t)$ is a slowly varying complex envelope such that $|\partial_t^2\mathcal{E}_\alpha| \ll |\omega_\alpha \partial_t \mathcal{E}_\alpha|$ and $|\partial_z^2\mathcal{E}_\alpha| \ll |k_\alpha \partial_z \mathcal{E}_\alpha|$, $\theta_\alpha = k_\alpha z - \omega_\alpha t$ is the fast-varying phase, and $\alpha = (L, W, A)$. A substitution of equation (13) and the spatial and temporal operators, $\partial_z \rightarrow ik$ and $\partial_t \rightarrow -i\omega$, into equations (1)-(3) yields

$$(\partial_t + v_{gL}\partial_z + \nu_L')\mathbf{E}_L = -\frac{c_{WA}}{2\partial D_L/\partial\omega_L}\mathcal{E}_W\mathcal{E}_A\ exp\ i\Delta t, \tag{14}$$

$$(\partial_t + v_{gW}\partial_z + \nu_W')\mathbf{E}_W = \frac{c_{LA}}{2\partial D_W/\partial\omega_W}\mathcal{E}_L\mathcal{E}_A^*\ exp\ -i\Delta t, \tag{15}$$

$$(\partial_t + v_{gA}\partial_z + \nu_A')\mathbf{E}_A = \frac{c_{LW}}{2\partial D_A/\partial\omega_A}\mathcal{E}_L\mathcal{E}_W^*\ exp\ -i\Delta t, \tag{16}$$

where $v_{g\alpha} = \partial\omega_\alpha/\partial k_\alpha$ is the wave group velocity, $\nu_\alpha' = -\nu_\alpha\omega_\alpha/\partial_{\omega_\alpha}D_\alpha$ and the linear frequency mismatch $\Delta = \omega_L - \omega_W - \omega_A$, with $\partial D_L/\partial\omega_L = -2(\omega_{pe} + 3v_{th}^2 k_L^2)^{1/2}$, $\partial D_W/\partial\omega_W = -\omega_{pe}/\omega_{ce}$, and $\partial D_A/\partial\omega_A = -2c_A k_A$. Finally, the system of coupled wave equations (14)-(16) can be rewritten in the normalized form

$$\dot{A}_L = \nu_L'' A_L + A_W A_A, \tag{17}$$

$$\dot{A}_W = \nu_W'' A_W - A_L A_A^*, \tag{18}$$

$$\dot{A}_A = i\delta A_A + \nu_A'' A_A - A_L A_W^*, \tag{19}$$

with

$$A_L = \left[\frac{c_{LW}c_{LA}}{4k^2(v_{gA} - v)(v_{gW} - v)\frac{\partial D_A}{\partial\omega_A}\frac{\partial D_W}{\partial\omega_W}}\right]^{1/2}\mathcal{E}_L, \tag{20}$$

$$A_W = \left[\frac{c_{WA}c_{LW}}{4k^2(v_{gL} - v)(v_{gA} - v)\frac{\partial D_L}{\partial\omega_L}\frac{\partial D_A}{\partial\omega_A}}\right]^{1/2}\mathcal{E}_W, \tag{21}$$

$$A_A = \left[\frac{c_{WA}c_{LA}}{4k^2(v_{gL} - v)(v_{gW} - v)\frac{\partial D_L}{\partial\omega_L}\frac{\partial D_W}{\partial\omega_W}}\right]^{1/2} \times \mathcal{E}_A\ exp\ i\Delta t, \tag{22}$$

where the dot denotes differentiation with respect to the 'temporal' phase variable $\tau = k(z - vt)$, v and k are arbitrary wave velocity and wave vector, respectively, $\nu_\alpha'' = \nu_\alpha'/[k(v_{g\alpha} - v)]$ and $\delta = \Delta/[k(v_{gA} - v)]$.

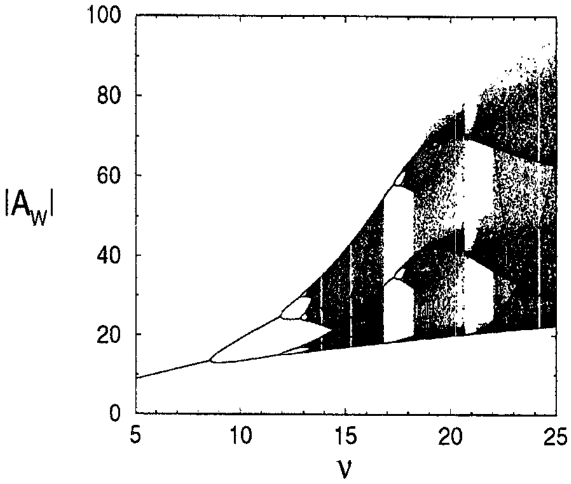

Figure 1: Bifurcation diagram $|A_W(\nu)|$ for $\delta = 2$.

Transition from order to chaos Equations (17)-(19) contain a source term, $\nu_L'' \equiv 1$, which represents the linear growth of the Langmuir wave. We assume whistler wave and Alfvén wave linearly damped with the same damping rate, $\nu_W'' = \nu_A'' \equiv -\nu < 0$. Under these assumptions, the system of equations (17)-(19) presents a wealth of dynamical behaviors: divergence, fixed point, limit cycle, and strange attractor (Wersinger et al., 1980; Meunier et al., 1982; Lopes and Chian, 1996).

The transition from order to chaos in equations (17)-(19) can follow different routes, depending on the value of two control parameters: the damping parameter ν and the linear frequency mismatch parameter δ. The overall system dynamics can be studied by constructing a bifurcation diagram, as shown in Fig. 1, where we fix $\delta = 2$ and vary the parameter ν.

Figure 1 shows that at $\nu = 5$ the solutions are periodic with period one. As ν increases beyond a certain value the period doubling cascade sets in, leading to periodic solutions with period two, four, eight, sixteen and thirty-two, respectively, until chaos appears. This is the route to chaos via period doubling (Wersinger et al., 1980; Lopes and Chian, 1996). Examples of time series for this route are plotted in Fig. 2. Figure 2a shows a periodic solution with period two ($\nu = 10.0$); Fig. 2b shows a periodic solution with period eight ($\nu = 13.0$); and Fig. 2c shows a chaotic solution ($\nu = 13.197$).

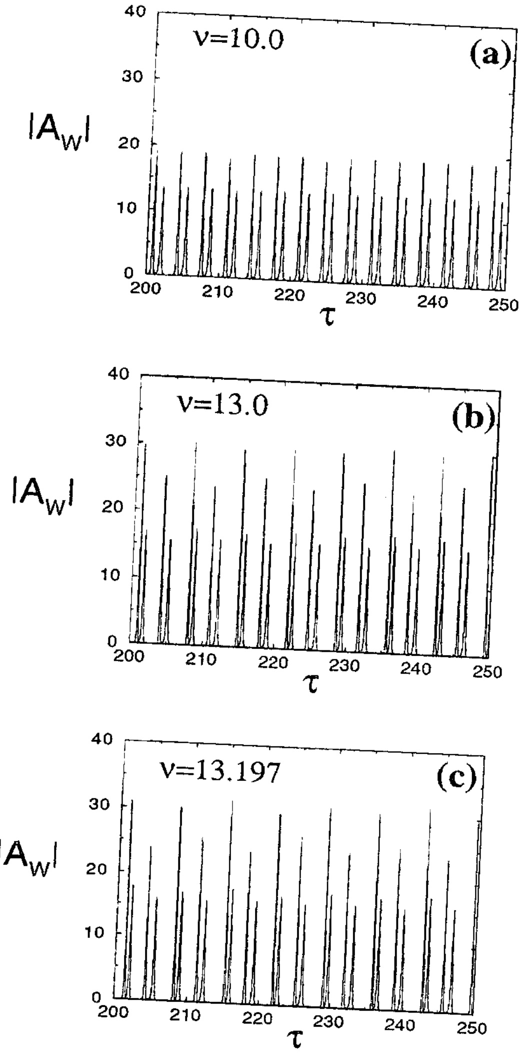

Figure 2: Time series $|A_W(\tau)|$ of the period doubling route to chaos for $\delta = 2$ and $\nu = 10.0$ (a), $\nu = 13.0$ (b) and $\nu = 13.197$ (c).

The characterization of the dynamical behavior can be done by the Poincaré map and the Lyapunov exponent (Ott, 1993). The Poincaré maps for the three time series of Fig. 2 are shown in Fig. 3a, wherein the Poincaré points correspond to the minima of A_W (Meunier et al., 1982). In contrast to the periodic solutions ($\nu = 10.0$ and 13.0) for which the Poincaré points have discrete values (two and eight, respectively), the Poincaré points for the chaotic solution ($\nu = 13.197$) are continuous forming a strange attractor with self-similar fractal features. Figure 3b shows the asymptotic temporal behavior of the leading Lyapunov exponent (λ) of the three dynamical states of Fig. 2. For the chaotic state, λ tends to a positive value as $\tau \to \infty$; whereas for the periodic state, λ tends to zero as $\tau \to \infty$. The corresponding power spectra are shown in Fig. 4, which illustrate how the system dynamics changes from order to chaos via period doubling.

Discussion The nonlinear three-wave process $L \rightleftharpoons W + A$ discussed in this paper may explain the generation of auroral whistler radio waves near the electron plasma frequency in the Earth's and Jupiter's magnetospheres. This process can only take place inside the auroral density depletion region where the electron plasma frequency is smaller than the electron cyclotron frequency ($f_{pe} \leq f_{ce}$).

The auroral LAW events detected by Boehm et al. (1990) during rocket experiments provide observational evidence in support of the process $L \rightleftharpoons W + A$. This process may account for the sharp peak around the electron plasma frequency in the auroral whistler wave spectrum measured by the satellite in the Earth's magnetosphere (Gurnett et al., 1983). It can explain the excitation of the leaked auroral kilometric radiation seen by satellite, rocket and ground receivers (Benson et al., 1988; Chian et al., 1994b).

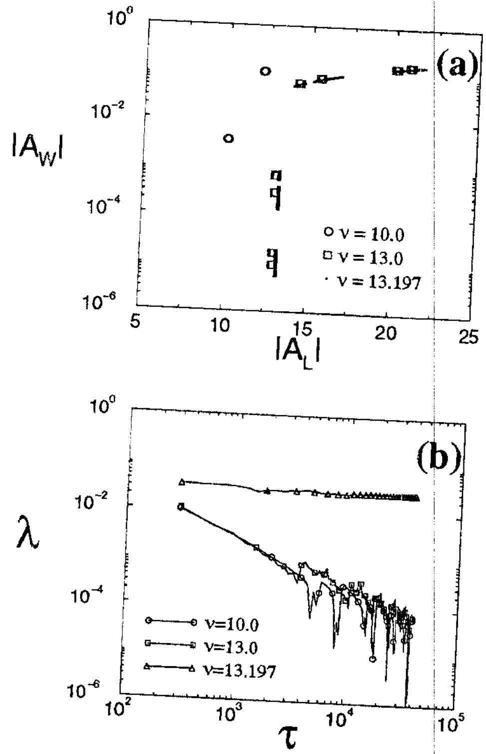

Figure 3: Characterization of the period doubling route to chaos: (a) Poincaré maps, (b) the leading Lyapunov exponents as a function of τ.

This theory can be used to interpret the fine structures of auroral radio emissions as well as the amplitude modulation and localized structures of auroral Langmuir waves (Lopes and Chian, 1996).

The nonlinear three-wave coupling can be a source mechanism of planetary radio emissions such as auroral whistler radio waves through interaction of Langmuir waves and shear Alfvén waves. This nonlinear process may evolve from order to chaos via period doubling route. The theoretical results presented in this paper suggest strongly that chaotic behaviors are intrinsic properties of nonthermal planetary radio emissions. Evidence of chaos in nonthermal radio emissions from the sun (Kurths et al., 1991; Isliker and Benz, 1994) and pulsars (Zhuravlev and Popov, 1990; Romani et al., 1992) has been reported. It is likely that a systematic data analysis of planetary radio emissions will also identify the presence of chaos in planetary radio waves.

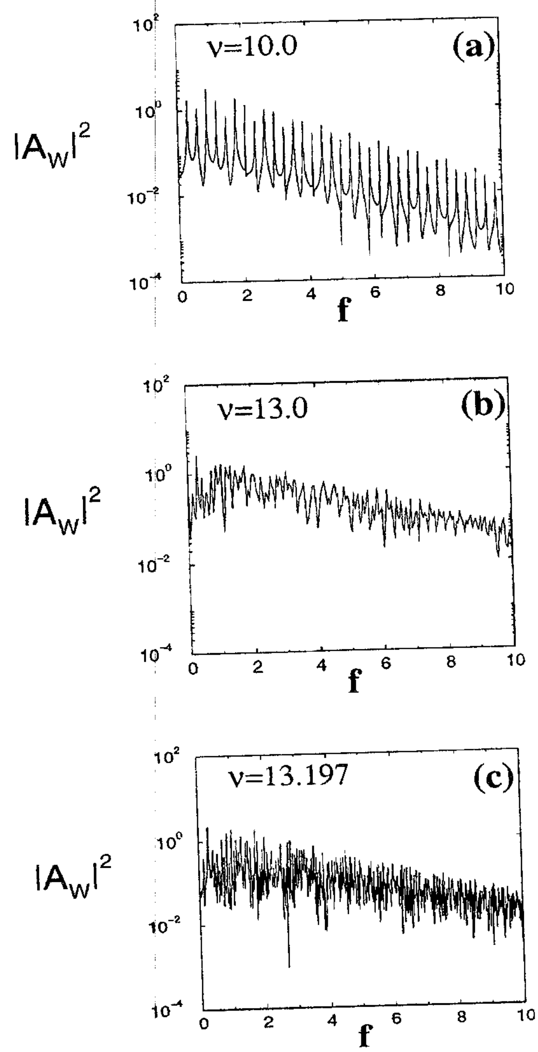

Figure 4: Power spectrum for $\delta = 2$ and $\nu = 10.0$ (a), $\nu = 13.0$ (b) and $\nu = 13.197$ (c).

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