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THE INFLUENCE OF QUANTUM VACUUM FRICTION ON PULSARS

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ABSTRACT

We first revisit the energy loss mechanism known as quantum vacuum friction (QVF), clarifying some of its subtleties. Then we investigate the observables that could easily differentiate QVF from the classical magnetic dipole radiation for pulsars with accurately measured braking indices (n). We show that this is particularly the case for the time evolution of a pulsar's magnetic dipole direction ($\dot{\phi}$) and surface magnetic field ($\dot{B}_0$). As is well known in the context of the classic magnetic dipole radiation, $n < 3$ would only be possible for positive ($\dot{B}_0/B_0 + \dot{\phi}/\tan\phi$), which, for instance, leads to $\dot{B}_0 > 0$ ($\dot{\phi} > 0$) when ϕ (B_0) is constant. On the other hand, we show that QVF can result in very different predictions with respect to those above. Finally, even if $\dot{B}_0$ has the same sign in both of the aforementioned models for a pulsar, then, for a given ϕ, we show that they give rise to different associated timescales, which could be another way to falsify QVF.

Key words: pulsars: general – stars: magnetic field – stars: neutron

1. INTRODUCTION

The important concept of the rotational energy of a neutron star (NS) as an energy reservoir for the pulsar's activity, put forward by Gold (1968) and Pacini (1968), is a way to explain its kinematical source of energy loss (Gunn & Ostriker 1969). A pulsar's surface magnetic field has since then been estimated by equating its temporal change of rotational energy,

$$\dot{E}_{\rm rot} = I\omega\dot{\omega}, \tag{1}$$

to the radiating power of a rotating magnetic point-like dipole in vacuum (see, e.g., Landau & Lifshitz 1975; Padmanabhan 2001),

$$P_{\rm dip} = -\frac{2}{3}\frac{m_\perp^2\omega^4}{c^3}, \tag{2}$$

where ω and $\dot{\omega}$ are the star's angular velocity and its derivative, respectively, I is its moment of inertia, $m_\perp = m_0 \sin\phi$ is the component of the magnetic dipole m_0 perpendicular to the axis of rotation (which is parallel to $\boldsymbol{\omega}$), and ϕ is the angle the magnetic dipole makes with $\boldsymbol{\omega}$.

One can readily show that

$$m_0 = \frac{B_0 R^3}{\sqrt{2}} \tag{3}$$

when B_0 is taken as the mean surface magnetic field (coming from a magnetic dipole) of a star of radius R. Thus, from Equations (1)–(3) we have

$$B_0 \sin\phi = \left(\frac{3c^3}{4\pi^2}\frac{I}{R^6}P\dot{P}\right)^{1/2}, \tag{4}$$

where $P = 2\pi/\omega$ and $\dot{P}$ are the rotational period and spin-down rate of a pulsar (observational parameters), while the star's moment of inertia and radius are model-dependent parameters. General considerations for the nature of pulsars have traditionally been obtained in the literature from the application of the above equations for systems with a representative mass $M = 1.4\,M_\odot$ and radius $R = 10$ km (fiducial parameters; see, e.g., Padmanabhan 2001), implying a moment of inertia of the order of $I \sim 10^{45}$ g cm^2. For instance, a class of NSs known as high magnetic field pulsars (high-B pulsars; Ng & Kaspi 2011; Zhu et al. 2011; Belvedere et al. 2015) would have B_0s higher than the scale field of the QED, namely, $B_c \doteq m_e^2 c^3/(e\hbar) \approx 4.4 \times 10^{13}$ G (Ruffini et al. 2010). Ordinary pulsars would have $B_0 \lesssim B_c$ (see, e.g., Shapiro & Teukolsky 1983; Camilo et al. 2000).

Nevertheless, whenever the magnetic field of a given system is close to B_c, quantum effects should play a significant role. Thus, one would expect that a more accurate description of pulsars, still from the classical point of view, could only be attained by using generalizations to the Maxwell Lagrangian, such as the Euler–Heisenberg Lagrangian for QED (Ruffini et al. 2010).

In this regard, it seems that the so-called quantum vacuum friction (QVF) effect, put forward by Dupays et al. (2008, 2012), has been overlooked in the literature. QVF can be understood as follows. In their seminal work, Born and Infeld (Born & Infeld 1934) showed that any nonlinear theory of electromagnetism in a vacuum described by a Lagrangian density $\mathcal{L}$ can be completely exchanged for the Maxwell theory in a convenient nonlinear medium. This is very important and powerful in the sense that one does not need to derive all of the involved properties and byproducts of $\mathcal{L}$, but rather one can work with Maxwell equations in continuous media. This means that for any $\mathcal{L}$, the concept of magnetization is present whenever non-null magnetic fields arise and its physical implications are as real as any tangible material medium. For the astrophysical case, it is well known that the magnetic dipole approximation already leads to the correct order of magnitude for the relevant physical quantities there, and thus should be the starting point of any model (Padmanabhan 2001). This is exactly where QVF comes into play: the effective magnetized medium (naturally outside the star) should interact with the magnetic dipole of the system (source of the magnetic field), leading it to eventually lose energy. Such an energy loss is due to the torque the magnetic field from the magnetization exerts on the rotating magnetic dipole. We will subsequently show in detail that such a resultant (time-averaged) torque is anti-parallel to the angular velocity of the star and linearly dependent on its norm (thus showing that the associated force is dissipative), which slows the star's rotation down while converting the rotational energy into heat. One could thus picture QVF as an energy loss mechanism caused by the "friction" between the magnetized vacuum and the rotating star, exactly as its name suggests (Dupays et al. 2008). The medium of the star itself is only important for determining the properties of the magnetic dipole (such as its magnitude and spatial orientation with respect to the axis of symmetry of the system) and does not directly contribute to QVF.

Based on the above reasoning, one can clearly see that QVF has an utterly different physical nature than that underlying the radiation of a rotating magnetic dipole. Therefore, it is meaningless to automatically assume that the former is smaller than the latter, even within the scope of small nonlinear corrections to the classical Maxwellian Lagrangian (known as weak field nonlinear Lagrangians). In this case, what does happen is that the corrections to the classic magnetic dipole radiation due to the nonlinearities of the Lagrangian are very small, and thus could be totally disregarded when other types of energy loss are also involved.

Besides modifying Equation (4), QVF also modifies the expression for the so-called braking index, with important consequences. Recall that this quantity is defined as (see, e.g., Padmanabhan 2001)

$$n = \frac{\omega\,\ddot{\omega}}{\dot{\omega}^2}, \tag{5}$$

where $\ddot{\omega}$ is the second time derivative of the angular velocity. It is well known from the literature that when energy losses are only related to the magnetic dipole radiation, $n = 3$; this fact is in disagreement with observations, which generally show $n < 3$. We will see later in this paper that $n < 3$ is naturally the case whenever QVF is also featured in the energy loss budget of pulsars, along with classic magnetic dipole radiation.

It is worth noting that there are several scenarios that challenge the magnetic dipole model, such as that involving the accretion of fall-back material via a circumstellar disk (Chen & Li 2016), relativistic particle winds (Xu & Qiao 2001; Wu et al. 2003), and modified canonical models used to explain the observed braking index ranges (see, e.g., Allen & Horvath 1997; Magalhaes et al. 2012 and references therein for further models). However, no model has yet been developed which can satisfactorily explain all of the measured braking indices, and none of the existing models have been ruled out by current data. Therefore, energy loss mechanisms for pulsars are still a topic of persistent debate.

Our aim in this work is to explore QVF in the context of pulsars (in particular, those that have accurately measured braking indices) solely with classic magnetic dipole radiation, since, as we will show, it can already explain several aspects of their phenomenology. Following this reasoning, we also explore the QVF model to perform evolutionary analyses of the pulsars' characteristic parameters, seeking for quantities that could easily provide a contrast of such model with classic magnetic dipole radiation and ultimately even falsify QVF.

This paper is organized as follows. In the next section, we revisit QVF within compact stars and derive its associated energy loss and rotational period evolution expression for weak field nonlinear Lagrangians, focusing mainly on QED. Section 3 is devoted to the investigation of the braking indices and the self-consistency of the model when both QVF and classic dipole radiation are responsible for the spin down of pulsars, for the simpler case in which only the evolution of P is of relevance. In Section 4, we elaborate on the evolution of other pulsars' characteristic parameters, such as B_0 and ϕ, in the context of QVF. Finally, in Section 5, we discuss the principal issues raised by QVF within the scope of pulsars. Here, we work with Gaussian units.

2. QVF IN STARS REVISITED

In this section we revisit in detail QVF as originally put forward by Dupays et al. (2008) in order to correct some misprints present there and to elucidate the physical ideas involved. The energy loss to be derived basically stems from a backreaction procedure, and thus is approximate. It would be of interest to contrast it with the result coming from direct analyses of the field equations for a nonlinear Lagrangian $\mathcal{L}$ (especially the effective nonlinear Lagrangian of QED), following the work of Deutsch (1955). We plan to do this elsewhere.

The phenomenon of QVF is basically an energy loss mechanism due to the interaction of a magnetic dipole ($\boldsymbol{m}$) with angular velocity $\boldsymbol{\omega}$ (taken to be in the z-direction) and the magnetization $\boldsymbol{M}_{qv}$ it produces in a surrounding medium. The associated induced magnetic field exerts a torque on the rotating magnetic dipole, leading the latter to lose energy. The infinitesimal version of such power is given by (Dupays et al. 2008)

$$d\dot{E}_{qv}(\boldsymbol{r}, t + r/c) \doteq \boldsymbol{m}(t + r/c) \times d\boldsymbol{B}_{qv}(\vec{0}, t + r/c) \cdot \boldsymbol{\omega}, \tag{6}$$

where r is the norm of the radial vector $\boldsymbol{r}$ connecting the volume element dV (which generates the infinitesimal magnetic field $d\boldsymbol{B}_{qv}$) to the origin of the system (where the magnetic dipole is supposed to be) and

$$d\boldsymbol{B}_{qv}(\vec{0}, t + r/c) = \frac{3\boldsymbol{r}\,[d\boldsymbol{m}_{qv}(\boldsymbol{r}, t)\cdot\boldsymbol{r}]}{r^5} - \frac{d\boldsymbol{m}_{qv}(\boldsymbol{r}, t)}{r^3}, \tag{7}$$

where $d\boldsymbol{m}_{qv} \doteq \boldsymbol{M}_{qv}dV$. Note from the above equations that retarded effects were considered and only the dipole approximation has been used for the determination of the magnetic fields. For completeness, we recall that the magnetic field generated by the magnetic dipole is given by

$$\boldsymbol{B}(\boldsymbol{r}, t) = \frac{3\boldsymbol{r}\,[\boldsymbol{m}(t - r/c)\cdot\boldsymbol{r}]}{r^5} - \frac{\boldsymbol{m}(t - r/c)}{r^3}. \tag{8}$$

In the following, we attempt to describe QVF related to the external region of a star of radius R that generates $\boldsymbol{m}$ and also rotates with angular velocity $\boldsymbol{\omega}$, assuming that $\boldsymbol{m}$ makes an angle ϕ with its axis of rotation. In other words, kinematically,

$$\boldsymbol{m}(t) = m_0[\hat{z}\cos\phi + \hat{x}\sin\phi\cos(\omega t) + \hat{y}\sin\phi\sin(\omega t)], \tag{9}$$

where m_0 is given by Equation (3).

Our description is only meaningful when there is a medium for $r \geqslant R$ because that of the star does not contribute to QVF directly, but only to determining $\boldsymbol{m}$. As already mentioned, an effective medium is present whenever electromagnetism is nonlinear and its byproducts are as real as any physical medium. Its magnetization due to a nonlinear theory of electromagnetism $\mathcal{L}$ is (Gaussian units; Born & Infeld 1934; Jackson 1975)

$$\boldsymbol{M}_{qv} \doteq \frac{1}{4\pi}\left(\boldsymbol{B} + 4\pi\frac{\partial \mathcal{L}}{\partial \boldsymbol{B}}\right). \tag{10}$$

The functional form of the Lagrangians that we will be interested in this work is

$$\mathcal{L} = \frac{1}{16\pi}(-F + \mu F^2), \tag{11}$$

with $F \doteq F^{\mu\nu}F_{\mu\nu} = 2(B^2 - E^2)$, $F_{\mu\nu}$ being the electromagnetic field tensor (Landau & Lifshitz 1975), and for a given vector $\boldsymbol{X}$, $X^2 \doteq \boldsymbol{X}\cdot\boldsymbol{X}$. In addition, we assume that $|F| \ll 1/\mu$, which means that we work within the weak field limit for a nonlinear theory whose scale field is proportional to $1/\sqrt{\mu}$ (see its motivation in Section 1 in the scope of QED). Let us consider that $B^2 \gg E^2$, which is exacty the case for extended astrophysical bodies. Then, substituting Equation (11) into (10), we are left with

$$d\boldsymbol{m}_{qv}(\boldsymbol{r}, t) \doteq \boldsymbol{M}_{qv} dV = \frac{\mu}{\pi} B^2 \boldsymbol{B}(\boldsymbol{r}, t) dV. \tag{12}$$

From Equations (6)–(9) and (12), the infinitesimal mean value over a period $(2\pi/\omega)$ of energy loss due to QVF in a given polar direction θ and radial distance from the origin r is

$$\frac{\langle d\dot{E}_{qv}\rangle}{\mu dV} = \frac{m_0^4\omega\sin^2\phi}{128\pi r^{12}}\{156\cos(2\theta) + 81\cos(4\theta) - 557 + 3\cos(2\phi)[-84\cos(2\theta) + 45\cos(4\theta) - 25]\}\sin\left(\frac{2\omega r}{c}\right). \tag{13}$$

One can then integrate the above equation for $r \geqslant R$ and all angular directions, and after simple calculations obtain

$$\langle \dot{E}_{qv}\rangle \simeq -\frac{24\mu m_0^4\omega^2\sin^2\phi}{5cR^8}, \tag{14}$$

assuming that $\omega R/c \ll 1$. Note that the aforementioned integral can be solved exactly, and thus further powers of $R\omega/c$ can be readily obtained whenever necessary.

We stress that the electromagnetic properties of the star can be entirely summarized by its mean surface magnetic field for the dipole approximation (Equation (3)). Therefore, general relativistic corrections to this classical model could all be incorporated into B_0. Belvedere et al. (2015) have already shown that they mainly lead to a decrease of B_0 concerning its classical counterpart by a multiplicative factor related to the compactness of the star. Thus, classical analyses already suffice to obtain the main physical radiation aspects of pulsars.

As a realization of our analyses, let us consider the Lagrangian density of QED. In this case (Ruffini et al. 2010),

$$\mu = \frac{\alpha}{90\pi B_c^2}, \tag{15}$$

where α is the fine structure constant. Substituting Equation (3) into Equation (14) and taking into account Equation (15), we finally have

$$\langle \dot{E}_{qv}^{qed}\rangle = -\frac{4\alpha}{75}\frac{B_0^4 R^4 \pi\sin^2\phi}{B_c^2 cP^2}, \tag{16}$$

where we have considered, instead of the frequency of the star, its period P, $\omega = 2\pi/P$. Note that this result is half of that reported by Dupays et al. (2008). The main reasons behind this are believed to be the factor of 2 within the last multiplicative sinusoidal term on the right-hand side of Equation (13), obtained when dealing with retarded effects, and also the definition of the surface magnetic field due to a magnetic dipole, Equation (3), in terms of an area average procedure.

On the other hand, as is well known and has already been mentioned in the previous section, pulsars also lose energy via magnetic dipole radiation, $\dot{E}_d \doteq P_{\rm dip}$, i.e., (see Equations (2) and (3); Landau & Lifshitz 1975; Padmanabhan 2001)

$$\dot{E}_d = -\frac{2}{3c^3}|\ddot{\boldsymbol{m}}|^2 = -\frac{16\pi^4 B_0^2 R^6\sin^2\phi}{3P^4c^3}. \tag{17}$$

In this work, we surmise that the total energy of the star is provided by its rotational counterpart, $E_{\rm rot} = I\omega^2/2$, and its change is attributed to both $\langle \dot{E}_{qv}\rangle$ and $\dot{E}_r$. Therefore,

$$\dot{E}_{\rm rot} \equiv \langle \dot{E}_{qv}\rangle + \dot{E}_d. \tag{18}$$

Thus, from Equations (1) and (16)–(18), the evolution of the period of a star is given by

$$\dot{P} = \frac{4\pi^2 B_0^2 R^6\sin^2\phi}{3IPc^3} + \frac{\alpha B_0^4 R^4 P\sin^2\phi}{75 I\pi c B_c^2}. \tag{19}$$

From the above equation, one clearly sees that its period of rotation tends to increase with time (it slows down as time goes on) and that the first term on the right-hand side is predominant for systems with small periods, while the opposite is true for its second term. Therefore, one would expect that in magnetized white dwarfs (see, e.g., Ferrario et al. 2015), super-Chandrasekhar White Dwarfs (Das & Mukhopadhyay 2013), Soft Gamma-Ray Repeaters, and Anomalous X-ray Pulsars (see McGill Magnetar Catalog (Olausen & Kaspi 2014)), the effect of QVF (more likely its generalization by means of the insertion of higher powers of F into the Euler–Heisenberg Lagrangian density in order to describe supercritical magnetic fields) could be significant. This will be investigated elsewhere.

3. QVF BRAKING INDEX FOR CONSTANT I, B_0, AND ϕ

Now we turn our attention to the braking indices. Typically, n is associated with pulsars and it is a measure of the spin-down's slope curve. It can be used to determine how close a rotationally powered pulsar is from the magnetic dipole model pertaining to its energy losses, namely, 3. Among the known radio pulsars, only young pulsars have accurately measured braking indices. We emphasize that nearly all of the reported braking indices have values smaller than 3 (see Table 1 and Figure 1).

This quantity has a special relevance for compact stars, since it is a direct observable. Using Equations (16), (17), and (19), it is simple to show that for the model given by Equation (18) in the case where B_0, I, R, and ϕ are all constants (physically equivalent to having $\dot{P}/P \gg (|\dot{B}_0|/B_0$, $|\dot{I}|/I$ and $|\dot{\phi}|/\tan\phi)$), n

Table 1
Estimates of ϕ for Pulsars with Known Braking Index

Pulsar	P (s)	$\dot{P}$ (10^{-13} s s^{-1})	n	References	ϕ
PSR B0833−45 (Vela)	0.089	1.25	1.4 ± 0.2	Lyne et al. (1996)	∼6°.0
PSR B0540−69	0.050	4.79	2.140 ± 0.009	Livingstone et al. (2007)	∼19°.6
PSR J1846−0258	0.324	71	2.19 ± 0.03	Archibald et al. (2015)	⋯
PSR B0531+21 (Crab)	0.033	4.21	2.51 ± 0.01	Lyne et al. (1993)	∼17°.3
PSR J1119−6127	0.408	40.2	2.684 ± 0.002	Weltevrede et al. (2011)	⋯
PSR B1509−58	0.151	15.3	2.839 ± 0.001	Livingstone et al. (2007)	⋯
PSR J1833−1034	0.062	2.02	1.8569 ± 0.0006	Roy et al. (2012)	∼11°.3
PSR J1734−3333	1.17	22.8	0.9 ± 0.2[a]	Espinoza et al. (2011)	∼28°.6
PSR J1640−4631	0.207	9.72	3.15 ± 0.03	Archibald et al. (2016)	⋯

Note.
[a] We adopted $n = 1.01$ to calculate ϕ.

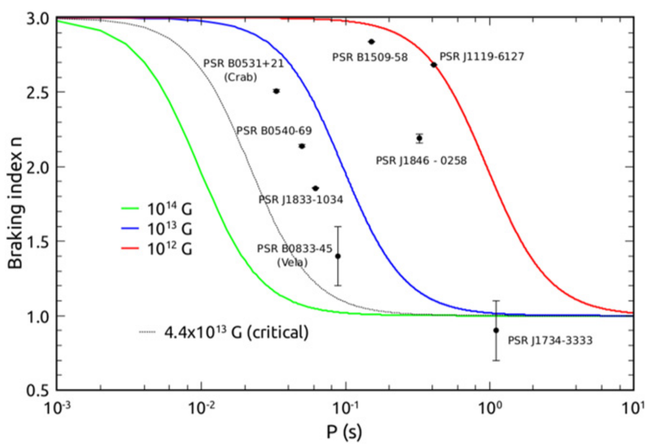

Figure 1. Braking index n (see Equation (20)) for some pulsars when both the classic dipole and QVF models are taken into account.

is given by

$$n = n_0 \doteq 3 - \frac{2}{1 + \frac{\dot{E}_d}{\langle \dot{E}_{qv} \rangle}} = 3 - \frac{2\alpha B_0^2 c^2 P^2}{B_0^2 c^2 P^2 \alpha + 100\pi^3 B_c^2 R^2}. \tag{20}$$

Note from Equation (20) that when $\dot{E}_{qv} \gg \dot{E}_r$ $[B_0 \gg 5\sqrt{\pi/\alpha}\, B_c (R\omega/c)]$, $n \to 1$. When the classical radiation term is much larger than QVF, the braking index tends to 3. Since the second term on the right-hand side of the above equation is never larger than 2, we conclude that $1 < n < 3$. Besides, given a value of n in such an interval, one shows that its corresponding B_0 is

$$B_0 = \frac{5\sqrt{(3-n)\pi}}{\sqrt{\alpha(n-1)}} \left(\frac{R\omega}{c}\right) B_c. \tag{21}$$

We recall that Equation (20) is only physically relevant in the context of QVF when $B_0 \ll \sqrt{90\pi/\alpha}\, B_c \approx 200 B_c$. From Equation (21), this means that

$$\sqrt{\frac{3-n}{n-1}} \left(\frac{R\omega}{c}\right) \ll 1. \tag{22}$$

The proximity of n to unity (from above) is dictated solely by the system's kinematic aspects (naturally $R\omega/c < 1$). For a typical pulsar, for instance, $R\omega/c \approx 10^{-3}$ and one sees that any $n \gtrsim 1.001$ leads the above inequality being fulfilled. This means that within QVF, surface magnetic fields for pulsars with $1 < n < 3$ can be at most of the order of the critical magnetic field of QED. Figure 1 shows that this is exactly the case for all of the associated pulsars (the case $n > 3$ is clearly not contemplated in the simple model analyzed here and will be investigated in the next section; as one physically expects, subcritical magnetic fields will also be raised there, but non-null $\dot{B}_0$ or $\dot{\phi}$ will be required). Note that the pulsar PSR J1734−3333 seems to be a very special case. It has a braking index of $n = 0.9 \pm 0.2$ (see Espinoza et al. 2011 for details). This value is well below 3 and, in light of our analyses, it indicates that QVF could be the most relevant mechanism for the energy loss in the system.

As in the case of the classic magnetic dipole radiation model, one can solve Equation (19) for B_0 by assuming that all of the other quantities are given, and thus the result is

$$B_0^2 = \frac{50\pi^3 B_c^2 R^2}{c^2 P^2 \alpha} \left[\sqrt{1 + \frac{3\dot{P} I P^3 c^5 \alpha}{100 B_c^2 R^8 \pi^5 \sin^2\phi}} - 1\right]. \tag{23}$$

The consistency of QVF with observational quantities demands that the averaged surface magnetic fields given by Equations (23) and (21) agree. This can be achieved by fixing some of the free parameters of the model. One of the most primitive parameters in this regard is the angle a magnetic dipole makes with the axis of rotation of the star. Thus, from the previously noted equations, one shows that

$$\sin^2\phi = \frac{3c^5 \dot{P} I (n-1)^2 P^3 \alpha}{800\pi^5 B_c^2 (3-n) R^8}. \tag{24}$$

An immediate outgrowth of the above equation is

$$\frac{M}{R^6} < \frac{2000\pi^5 B_c^2}{3c^5 \alpha P^3 \dot{P}} \frac{(3-n)}{(n-1)^2}, \tag{25}$$

where we assumed that $I = 2MR^2/5$, i.e., the moment of inertia of a homogeneous sphere. Care should be taken here concerning the physical interpretation of Equation (25). It is not necessary for the mass and radius of a pulsar to satisfy this constraint.It is solely a byproduct of the assumptions underlying Equation (20). Table 1 shows the ϕs associated with Equation (24) for all of the pulsars with known braking indices. For those cases where they cannot be found, it is simple to conclude that the associated changes which need to be made to the fiducial parameters are unrealistic. Indeed, using Equation (25) for $M = 1.4\,M_\odot$, the radii of PSR J1846−0258, PSR J1119−6127, and PSR B1509−58 would have to be larger than 29 km, 38 km, and 29 km, respectively, which are utterly improbable for pulsars. This implies that one should actually take into account the evolution of other parameters for the braking index, such as those that were ignored to obtain Equation (24). We will come back to this issue in the next section.

4. PULSAR'S EVOLUTIONARY ASPECTS WITHIN THE SCOPE OF QVF

It is very likely that pulsars, due to their dynamic nature, should always present important temporal changes in quantities other than P. This signifies that Equation (24) may not represent the inclination of the magnetic moments of realistic pulsars (which is equivalent to saying that Equation (20) is not the most adequate equation for the braking index). Therefore, more complex scenarios should be investigated, generalizing the results of the previous section.

Let us start with the situation in which both ϕ and B_0 are time-dependent. The case $I = I(t)$ seems unrealistic for the isolated pulsars we are investigating, or, at least, is less relevant than the time dependence of B_0 and ϕ. From Equations (5) and (19), one can readily show in this case that

$$n = n_0 - 2\frac{P}{\dot{P}}\left[\frac{(5 - n_0)}{2}\frac{\dot{B}_0}{B_0} + \dot{\phi}\cot\phi\right], \tag{26}$$

where n_0 is the braking index for the case where both B_0 and ϕ are constants, Equation (20), and B_0 will be assumed to be given by Equation (23). (In this case, as self-consistency naturally demands, $\dot{\phi}$ will be the same as from Equation (26), given $\dot{B}_0$ and n, or direct analyses of Equation (23).)

It is believed that magnetic fields should decay in pulsars (usually due to the Ohmic decay, Hall drift, and ambipolar diffusion; Jones 1988; Goldreich & Reisenegger 1992) on timescales of the order of (10^6–10^7) years (see, e.g., Goldreich & Reisenegger 1992; Graber et al. 2015 and references therein). Nevertheless, there are also suggestions that the timescales for B_0 could actually be smaller, on the order of 10^5 years (Igoshev & Popov 2014, 2015). Thus, bearing in mind that magnetic fields in the context of QVF for pulsars are of the order of (10^{12}–10^{13}) G (see Figure 1), let us assume in what follows that $\dot{B}_0 < 0$ and $|\dot{B}_0|$ is of the order of (10^{-2}–10^{-1}) G s^{-1}. (This is estimated directly from the above-mentioned usual timescales T_B, such that $|\dot{B}_0| \sim B_0/T_B$.) Our analyses for this case concerning $\dot{\phi}$, taking into account the braking indices of the relevant pulsars, are summarized in Table 2 for the representative angle $\phi = \pi/4$ (see Equation (26)). Since the physically relevant values of $\dot{B}_0$ are small, the conclusions that ensue are essentially the same as for the case of constant B_0. Note that some pulsars have positive $\dot{\phi}$s while others have negative, and all of them present subcritical magnetic fields (thus also clearly showing the self-consistency of QVF in this more complex scenario). Special attention should be paid to the Crab pulsar. The value $\dot{\phi} \simeq 3 \times 10^{-12}$ rad s^{-1} has been observationally inferred for this pulsar (Lyne et al. 2013, 2015; Yi & Zhang 2015), which has the same sign and magnitude as that predicted by QVF, and thus could always be related to a specific angle ϕ there. We emphasize that the same analyses as those above could be performed in the scope of the classic magnetic dipole model. In this case, one can easily verify that all of the pulsars in Table 1 with $n < 3$ are such that $\dot{\phi} > 0$ and it is of the order of 10^{-12} rad s^{-1} (see Equation (26) for the formal case $\alpha \to 0$). Therefore, measurements of $\dot{\phi}$ for other pulsars (especially those with $\dot{\phi} < 0$ in the context of QVF) could easily falsify any of these models for the given mechanisms of magnetic field decay and evidence their underlying physics (e.g., NS precessions could lead to $\dot{\phi} > 0$; Zanazzi & Lai 2015; Kerr et al. 2016). For the special pulsar PSR J1640-4631, both QVF and the classic magnetic dipole model result in $\dot{\phi} < 0$, but the former model predicts a faster rate of change than the latter. Finally, note that all $\dot{\phi}$s in Table 2 are positive only when $\dot{B}_0 \lesssim -10^2$ G s^{-1}, always leading to values larger than their classical counterparts. The difficulty in this case, however, would be the physical explanation of timescales at least three orders of magnitude smaller than those from known mechanisms of magnetic field decay.

Table 2
Estimates of $\dot{\phi}$ for the same Pulsars as in Table 1, with $\dot{B}_0 = -0.05$ G s^{-1}, for the Representative Inclination Angle $\phi = \pi/4$

Pulsar	$\dot{\phi}$ (10^{-12} rad s^{-1})	B_0 (10^{12} G)
PSR B0833−45 (Vela)	0.8	6.2
PSR B0540−69	2.3	9.3
PSR J1846−0258	−12	17
PSR B0531+21 (Crab)	2.3	7.7
PSR J1119−6127	−8	14
PSR B1509−58	−7.5	14
PSR J1833−1034	1.3	6.9
PSR J1734−3333	0.02[a]	9.5
PSR J1640−4631	−4.4	11

Note.
[a] We have adopted $n = 1$ here.

Since the Crab pulsar has an observationally inferred $\dot{\phi}$, let us study more precisely the implications of QVF and the classic magnetic dipole for this pulsar. Figure 2 depicts the behavior of $\dot{\phi}$ (see Equation (26)) for the Crab pulsar when $\dot{B}_0 = -0.05$ G s^{-1} for both of the above-mentioned models. Note that for angles $\phi \lesssim 5°$, QVF analyses are not trustworthy because we are approaching its threshold of validity (see Figure 3 and Equation (23) for this case). Besides, $\dot{\phi}$s related to the classic dipolar model are always larger than those from QVF, which means that in the latter model, for a given $\dot{\phi}$, the actual (instantaneous) ϕ is always larger than that from the former model. For instance, for $\dot{\phi} \simeq 3 \times 10^{-12}$ rad s^{-1}, the classic dipole model implies $\phi \approx 45°$, while QVF predicts $\phi \approx 51°$. For completeness, in Figure 3, we plot the instantaneous surface magnetic field for the Crab pulsar as a function of ϕ. Here, it is evident that for angles larger than $\phi \gtrsim 15°$, only subcritical magnetic fields rise. In Figure 2, one can also see that for the QVF model, there is a nontrivial angle such that $\dot{\phi} = 0$. This is already expected due to the existence of a ϕ satisfying the simpler case given by Equation (24) (see Table 1). Their proximity is simply due to the smallness of $\dot{B}_0$. Finally, in the case of the Crab pulsar, QVF can only be differentiated from the classic magnetic dipole model if precise measurements of its $\dot{\phi}$ are available, which is still not the case.

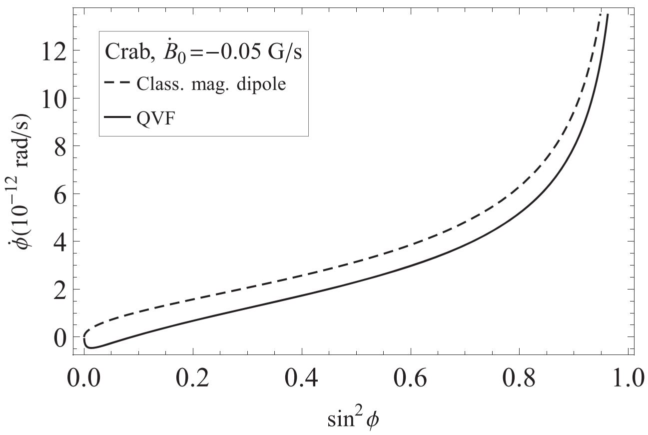

Figure 2. Instantaneous evolution of $\dot{\phi}$ as a function of the inclination angle ϕ for the Crab pulsar (see Equation (26) and Table 1) with $\dot{B}_0 = -0.05$ G s^{-1} for both QVF and magnetic dipole ($\alpha \to 0$) models. Note that for $\phi \lesssim 5°$, QVF analyses are not reliable because we are close to the threshold of its validity (see Figure 3). For the parameters in this figure, one sees, for instance, that $\dot{\phi} \simeq 3 \times 10^{-12}$ rad s^{-1} (Crab's inferred inclination rate), which would imply $\phi \approx 45°$ for the classic magnetic dipole model, while $\dot{\phi}$ in the context of QVF would be approximately 51°.

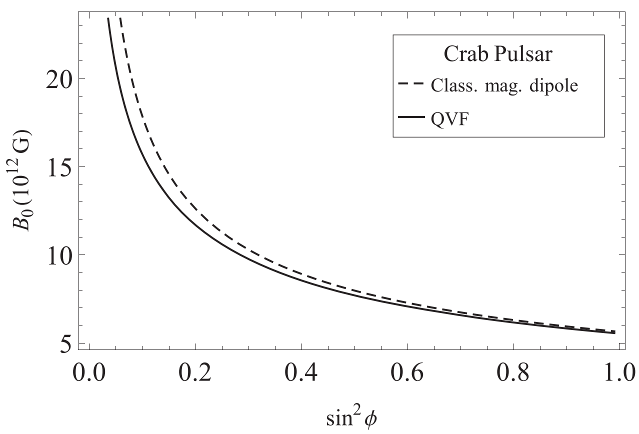

Figure 3. Instantaneous surface magnetic field as a function of the inclination angle ϕ for the Crab pulsar (see Equation (26) and Table 1) in the scope of QVF and the classic dipolar model. Note that fields of the order of the critical one rise for $\phi \lesssim 5°$ (see Equation (23)), indicating the limit of validity of QVF analyses. Besides, QVF always leads to smaller magnetic fields when compared to the classical model.

We now consider the case in which only B_0 is allowed to change with time. This would be a natural consequence of the existence of equilibrium angles to the directions of the pulsars' magnetic dipoles. Using Equation (26) one can verify that (self-consistent) ϕs could only be found for the pulsars with $n < 3$ that do not satisfy Equation (24) in Table 1 when $\dot{B}_0 < 0$ and $|\dot{B}_0| \sim (10^2\text{–}10^3)$ G s^{-1}. Hence, when QVF is taken into account for these pulsars, it would lead surface magnetic fields to *decrease* with time, and in a such a way that their rotational energy always overwhelms their magnetic energy. From previous results, one also determines that the characteristic timescales for the surface magnetic fields of the pulsars under discussion are $(10^3\text{–}10^4)$ years. It is interesting to note that such timescales are in agreement with those from $P/\dot{P}$ for the same pulsars. This suggests that the evolution of B_0s in pulsars with known braking indices and not associated ϕ in Table 1 should be connected to their spindown, pointing to the relevance of the mechanisms where this takes place, such as in Ruderman's neutron vortices (which will drag along protons, and thus also influence the magnetic field of a pulsar; Ruderman 1970, 1972). We emphasize that the above timescales for surface magnetic fields obtained within QVF clearly contrast with those related to a purely magnetic dipole radiation model for pulsars where surface magnetic fields should *increase* with time, having timescales of $(10^2\text{–}10^3)$ years (Muslimov & Page 1996). The fact that the magnetic timescales found within the scope of QVF for $n < 3$ are much smaller than those coming from Ohmic decay and Hall drift, for instance, suggests that the associated pulsars are currently experiencing transient periods. This would be supported by PSR J1846−0258, which had $n = 2.65$ six years ago (Archibald et al. 2015). Another natural conclusion would be that the assumption of having constant ϕ is incorrect, as suggested by Yi & Zhang (2015). Only further observations could settle this ambiguity. For completeness, for the pulsars in Table 1 that already have associated angles, one can check that QVF leads to positive $\dot{B}_0$ of the order of 10 G s^{-1} whenever the chosen ϕs are larger than those satisfying Equation (24). Due to a simple continuity argument, $\dot{B}_0 < 0$ when they are smaller. For the former case, one sees that the associated timescales of magnetic field growth are of the order of 10^4 years, which is larger than their classical counterparts. In summary, measurements of $\dot{B}_0$ for $n < 3$ could also easily falsify QVF, since it leads to both positive and negative values of such a quantity, which is not the case for the classic magnetic dipole model. Besides, if magnetic fields increase, then the above-mentioned models predict that they will have very different timescales. The same ensues for the case $n > 3$, where now in both models the magnetic fields should decrease with time.

5. DISCUSSION AND CONCLUSIONS

Since the stars we analyzed are rotating, the physically relevant quantities should be time averaged (per period of rotation), as in Section 2. This is particularly the case for the resultant mean torque per cycle $\langle \boldsymbol{N} \rangle$ on the star's surface due to the whole effective magnetized medium surrounding it. As is evident from the symmetry of the problem, $\langle \boldsymbol{N} \rangle$ must be colinear with the rotation axis of the star. The simplest way to obtain it is from Equation (16) along with the definition of the power associated with any torque ($\boldsymbol{N} \cdot \boldsymbol{\omega}$), which leads to

$$\langle \boldsymbol{N} \rangle = -\frac{\alpha}{75} \frac{B_0^4 R^4 \sin^2\phi}{B_c^2 \pi c} \boldsymbol{\omega}. \quad (27)$$

(One can also obtain $\langle \boldsymbol{N} \rangle$ as above by a direct computation, starting from the definition of the infinitesimal torque related to Equation (6) and then following the same procedure that led us to Equation (16).) One sees from Equation (27) that the resultant time-averaged torque is anti-parallel to $\boldsymbol{\omega}$, intrinsically associated with a force proportional to the negative of the velocity. Thus, QVF leads the decrease of rotational energy of the star to be converted into heat. This explains the energy balance related to QVF. It clearly contrasts with magnetic dipole radiation in which the star's slowdown is due to the emission of electromagnetic radiation. The byproducts of this heat are beyond the scope of this work and will be investigated elsewhere.

It should be stressed that QVF is an intrinsically quantum effect related to the backreaction of the vacuum polarization on a classical magnetic dipole, leading the star to lose energy by means of a torque. There is no reason for it to be much smaller than the radiation associated with a classic magnetic dipole simply because the effects do not have the same nature. Only corrections to the classical magnetic dipole radiation due to the nonlinearites of the Lagrangian density are automatically small within the QVF model, see Equation (11), and for just this reason they were disregarded in our analyses.

Let us quickly discuss some evolutionary aspects of the braking indices of pulsars in light of QVF. Since the energy loss due to QVF decreases with P^2 while the classic magnetic dipole radiation decreases with P^4, see Equations (16) and (17), QVF should be predominant only at later evolutionary times of a pulsar, making its braking index tend to unity if B_0 and ϕ are asymptotically stationary (see Equation (26)). In this regard, one could tentatively state that this could be the case for (or supported by) the pulsar PSR J1734−3333, due to its relatively large value of P and to its measured braking index. At the same time, the aforementioned pulsar could also be an example that falsifies QVF in the case studied in Section 3. This naturally motivates further studies concerning PSR J1734−3333 in order to decrease the uncertainty present in its braking index (the same can be said of the pulsar PSR J0537–6910, whose normally associated braking index of −1.5 (see, e.g., Ho 2015) is not at all accurate due to the large dispersion in $\ddot{P}$ it presents; Middleditch et al. 2006), as well as to restrict the evolutionary aspects of its B_0 and ϕ. Whenever $\dot{B}_0$ and $\dot{\phi}$ are not asymptotically stationary, one can clearly see from Equation (26) that several scenarios arise within QVF, even that in which $n < 1$, which can be obtained when $\dot{B}_0/B_0 \geqslant -2\dot{\phi}/[(5-n)\tan\phi]$. (Here, similar to what happens in the classic model, one notes that when $\dot{B}_0 < 0$, $\dot{\phi}$ must be positive for $0 < \phi < \pi/2$ and negative for $\pi/2 < \phi < \pi$, thus indicating that $\phi = \pi/2$ is an attractor to the magnetic dipole direction.) Another example would be that studied in Section 4 in which, for the time being, $1 < n < 3$. In such a case, QVF could only constrain the temporal evolution of some pulsars' parameters (as we have done for $\dot{\phi}$ and $\dot{B}_0$) and only their measurement could rule it out. Thus, generically speaking, observations of the braking index alone cannot fully falsify QFV, but only constrain it.

QVF can in principle be easily distinguished from the classic magnetic dipole radiation. As we have shown in Section 4, it predicts in such a scenario that ϕ should change with time, being either negative or positive for different pulsars, which is quite different from the classic magnetic dipole model. Similar conclusions can be drawn for the evolution of B_0. For the Crab pulsar, both of the above-mentioned models lead to current inclination angles which differ from each other by some degrees. This motivates further analyses in this direction.

We point out that a simplified model has been chosen in order to assess more transparently the relevance of QVF. Actually, it is known that a pulsar should have a plasma atmosphere (see, e.g., Michel 1974 and references therein) and it is not simply surrounded by vacuum. Besides modifying the standard magnetic dipole radiation model (with an extra torque), it is also expected to influence QVF due to the following reason: this plasma region would influence the resultant magnetic field felt in the (outer) vacuous region, which would directly influence the vacuum magnetization (see Equation (12)), which would lead, in turn, the quantum vacuum to exert a different torque on the star, which would change its slowdown. Clearly, this is a more elaborated scenario, and we will discuss it more precisely elsewhere.

In summary, in this work, we have taken QVF as a fundamental energy loss mechanism and have tried to assess its relevance to the description of pulsars. In its simplest form, it leads magnetic fields to automatically be subcritical (in plain contrast, for instance, with high-B pulsars in the context of the classic magnetic dipole). In addition, measurements of quantities other than P and its derivatives for pulsars (such as $\dot{\phi}$ and $\dot{B}_0$) could easily falsify QVF for the case $n < 3$. Finally, it seems that QVF should be a relevant source of energy loss for the pulsar PSR J1734−3333.

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