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GRADIENT PATTERN ANALYSIS OF COUPLED MAP LATTICES

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Abstract: A modified computational operator for Gradient Pattern Analysis is introduced into the framework of 2D-Coupled Map Lattices (CML). The GPA coefficient is computed taking into account different sizes and boundary conditions for characterization of chaotic 2D-CML. In addition to quantifying concentric asymmetries, GPA measures the phase disorder associated with a given pattern. Simulations of chaotic CML using Gaussian and random initial conditions, provide interesting insights on the system gradual transition from order (concentric symmetries) to disorder (emergence of concentric asymmetries).

keywords: Chaos and Global Nonlinear Dynamics, Time Series Analysis , Discrete Dynamical Systems.

1. INTRODUCTION

Coupled Map Lattices (CMLs) are system discrete in time and space, composed by grid elements that interacts with its neighborhood, and can show chaotic behaviour [3]. The classical techniques to analyse dynamical system, as the Lyapunov exponent can characterize the evolution of these systems, as shown in [2]. However some proprieties such as symmetry breaking and regime are better described by techniques which analyse the matrices among its gradient pattern. An example of algorithm was proposed in [1], the Generalyzed Entropic Form (GEF) which was also applied to analyse the structure of CMLs, and determine flow regim. In order to characterize structural patterns of CMLs and the pattern evolution, we present a new version of Gradient Patttern Analysis (GPA). In this version of GPA we introduce a new technique for the symmetrical vector detection, which detect the radial symmetry of systems providing precise results.

2. COUPLED MAP LATTICE

CMLs are non-linear spatially extended systems, that represents the dynamical any dimensional system. Given a matrix A composed by elements $a^n_{x,y}$, where x and y represent the position of the matrix and n the matrix time step, a transition function extensively used in type of simulation is the global CML, that is show in equation 1, where ϵ is the coupling factor, k and l represent the neighborhood distance respectively in x direction and y direction, and $f(a)$ the application of a map.

$$a^{n+1}_{i,j} = \epsilon f(a^n_{i,j}) + \sum_{k,l} \frac{(1-\epsilon)}{N} f(a^n_{i+k,j+l}) \tag{1}$$

Different behaviours can be observed according to the grid dimensions, the type of boundary, the coupling factor and the map. With respect to the grid, coupling factor, and boundaries, in this work we have used the following proprieties:

- 128x128 lattice
- Toroidal boundaries
- Von Neumann neighborhood
- $\epsilon = 0.5$

An often map studied for model validation in CML is the logistic map, with the parameter a = 4.0. Although any map with chaotic behaviour can be used [3]. In this study we compared three different maps: the Logistic map with chaotic parameter a = 4.0, the Double map, and the Shobu Ose Mori (SOM) map [5] with parameters $\alpha = 0.6$, $\beta = 0.2$ which also has chaotic behaviour. We selected this maps with these parameters because every map show different behaviour near their bending point, as show in figure 1.

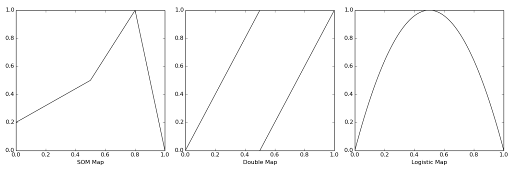

Figure 1 – Three maps with different behaviours

3. GRADIENT PATTERN ANALYSIS

The Gradient Pattern Analysis is a technique that detects the asymmetrical fragmentation of discrete systems, it was proposed in [4]. This operator is composed by four steps:

- Gradient field estimation
- Symmetrical vector removal
- Triangulation
- Coefficient Estimation

In the first step is commonly applied a finite central difference, in some cases a filtering method can provide better results. In this method is proposed the remotion of vectors with module smaller than 3% of the largest vector.

The analysis without the redundancies may provide better profile of the system, then in the second step is estimated the asymmetrical vector field, since the symmetrical gradient field is a redundant data. In order to estimate the asymmetrical vector field is defined as a symmetrical group, a group formed by pairs of vectors which the pair vector sum is a vector with length approximately zero, and both vector having opposite angle when related to an axis.

To define each group, usually applied the bilateral symmetry vector removal, because its a linear algorithm. Although this technique is very sensitive, and doesn't detect some symmetries in complex extended systems. The bilateral removal considers four axis of symmetry, comparing for each point its equivalent in each axis.

In order to provide an accurate and robust method we propose a new technique for the symmetrical vector removal, the radial symmetrical vector removal. The radial removal technique is a bilateral removal technique extension, which compares in many axis as possible. This process can be done comparing points equidistant, because for every pair of points in the same distance from the center there is an axis that part those points. Since we want a robust method, in this evaluation is not considered the angle of each vector in relation to the axis.

The third step is the triangulation where is applied the Delaunay triangulation, aiming to connect the remaining vector field, providing the spatial structure representation.

The Gradient Asymmetric Coefficient (G_A), given the number of vectors (N_v), and the number of Delaunay connections (N_c) is estimated using the equation 2.

$$G_A = \frac{N_c - N_v}{N_v} \quad (2)$$

The values of G_A ranges from 0 to 2, where 0 represents an system totally symmetrical/ordered and 2 represents a system asymmetrical/disordered. One example of G_A estimation process is shown in the Figure 2.

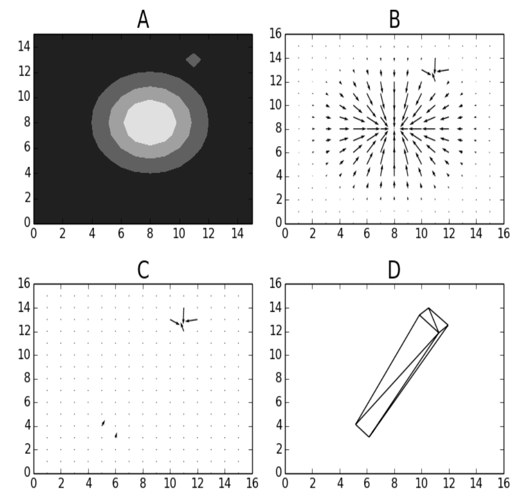

Figure 2 – Example of GPA operation sequence, starting from matrix image (A): the gradient field (B),the asymmetric gradient field (C), and the triangulation(D). In C we can see $N_v = 6$, and in D we can see $N_c = 10$, then according to the equation 2, $G_A = \frac{10-6}{6} \approx 0.6667$

4. PRELIMINAR RESULTS

We estimated G_A in 175 iterations of each type of map, starting from two initial conditions: a 2D-Gaussian which is totally symmetric, and a randomized matrix which is asymmetric. We want to see when the symmetry breaks in each type of map. The Figures 3 show 5 snapshots of each map iteration, while the Figure 4 show the G_A evolution in each map. As result we can see that SOM map and Double map converged to an asymmetric system near the 50th iteration, while the Logistic map converged near the 105th iteration.

The fluctuation observed in Logistic map iteration and in Double map iteration before the convergence point in many cases are caused by small structures formed near the boundaries, that can be causing the transition from symmetry to asymmetry in this systems.

5. CONCLUSION

We developed a new robust technique for discrete dynamical systems analysis, based on the system radial asymme-

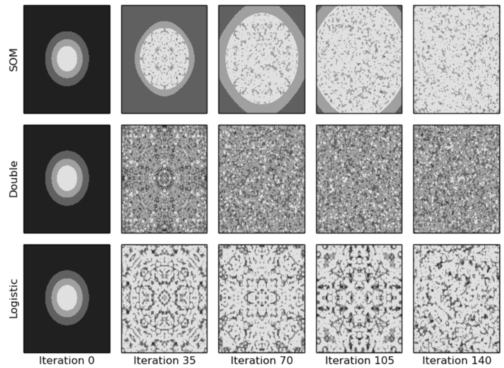

Figure 3 – Snapshots at every 35 iteration from SOM map, Double Map, and Logistic Map in matrices with 2D-Gaussian initial condition.

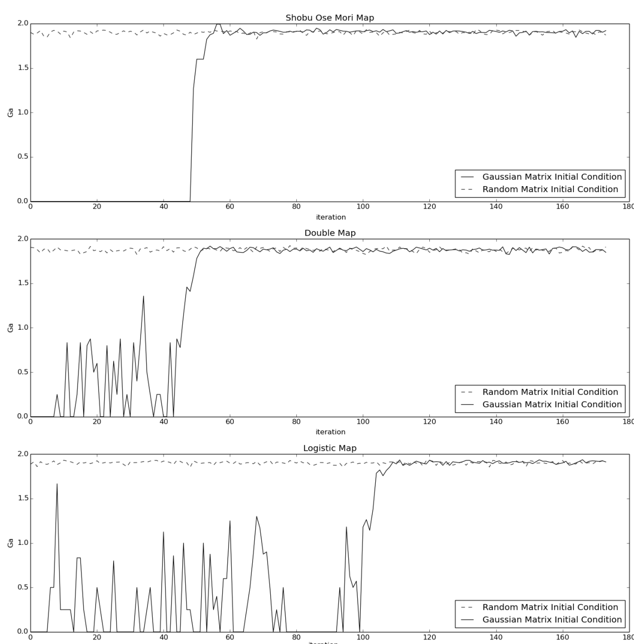

Figure 4 – Using G_A for tracking the pattern evolution from SOM map, Double Map, and Logistic Map, starting with a 2D-Gaussian matrix and randomized matrix

try/disorder. This technique provided a consistent result, especially in the determination of the symmetry breaking iteration. As result we also observed the gradual structural evolution of CMLs.

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