Particle-in-cell Simulation of Waves in a Plasma Described by Kappa Velocity Distribution Function as Observed in the Saturn's Magnetosphere

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Introduction

Velocity distribution functions (VDF) are used to represent global characteristics of a set of particles that constitute a gas or a plasma. The most used distribution is the Maxwellian distribution, that represents system in thermal equilibrium. Kappa Velocity Distribution ($\kappa-$VDF) is a generalization of the Maxwellian distribution. This distribution describes better plasmas out of thermal equilibrium and is defined by the following equation [6]:

$$f_\kappa(v) = \frac{1}{(\pi\kappa\theta^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)} \left(1 + \frac{v^2}{\kappa\theta^2}\right)^{-\kappa-1}, \tag{1}$$

where $\theta^2 = (2\kappa-3)v_{th}^2/\kappa$, v_{th} is the thermal speed and κ is the index that defines how far the distribution is from the equilibrium. This index can assume values from $3/2 < \kappa < \infty$ and when κ goes to infinite the VDF tends to the Maxwellian distribution.

In Figure 1 are presented plots of the Equation 1 for some values of κ index, the green line being the Maxwellian VDF. This figure shows a prominent tail of electrons with high velocity.

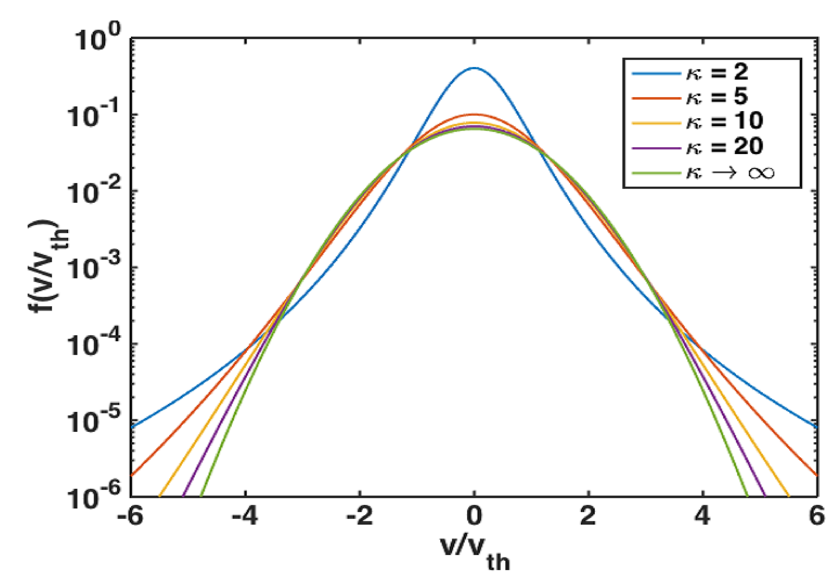

Figure 1: Kappa distribution function for different values of kappa index

Magnetosphere of Saturn

The electron population found in the Saturnian magnetosphere is constituted by two different species, both represented by κ-VDF but with different temperature and densities. This feature was described by [8], that used data from the Cassini mission to obtain the electron profile in different distances from the planet. A plasma constituted by two electron species can excite an electron-acoustic wave. This electrostatic wave has this name because it arrives from the interaction between the two electron species, one hot (here represented whit subscript h) and one cold (with subscript c), similar to what forms the ion-acoustic wave.

When the two species are represented by κ-VDF, the dispersion relation for this wave can be approximated by:

$$\omega_{EA}^2(k) = \omega_{pc}^2 \left[\frac{1 + 3k^2\lambda_{Dc}^2\left(1 + 1/k^2\lambda_{\kappa h}^2\right)}{1 + 1/k^2\lambda_{\kappa h}^2}\right], \tag{2}$$

where ω_{pc} is the plasma frequency of the cold electrons, $\lambda_{\kappa h}^2 = (\kappa_h - 3/2)(\kappa_h - 1/2)\lambda_{Dh}^2$, and $\lambda_{Di}$is the Debye length for cold ($i = c$) and hot electrons ($i = h$) [1].

In the equatorial region of Saturn it is also found a banded emission. This emission can correspond to electron Bernstein waves, that propagates perpendicularly to the ambient magnetic field and occur in the harmonics of the electron cyclotron frequency, ω_{ce} [2].

Simulations

The simulations were made using the electromagnetic code called KEMPO, developed by Omura and Matsumoto [7], in which the particles can move only in one direction, but the velocities and the fields are calculated in all three dimensions. The code was modified to introduce as initial condition the velocities of the particles following a κ-VDF for the two electron species. The κ-VDF used here have $\kappa_c = \kappa_h = 4$, the plasma frequencies, normalized by ω_{ce}, are defined by the upper hybrid frequency $\omega_{UH}/\omega_{ce} = \left(\omega_{pc}^2 + \omega_{ph}^2 + \omega_{ce}^2\right)^{1/2}/\omega_{ce} = 5.099$ and $\omega_{pc} = \omega_{ph}$. The distributions, however, have different thermal speeds, obtained by the temperature of the plasma in the region of 9 Saturn radii [8].

To study the behavior of the electron-acoustic waves, several simulation were performed for different angles of propagation in respect to the ambient magnetic field, B_0. For the propagation perpendicular to B_0, the simulations were run also with the VDF being Maxwellian, for comparison, since the Bernstein modes are more sensitive to changes in the distribution function [5]. In all the simulations using κ-VDF both electron species have the same κ index, $\kappa_c = \kappa_h = 4$.

Evolution of VDF

The distribution of electrons evolves in the simulation due to the interaction between the particles. As shown in Figure 2, for the parallel propagation, the colder electrons (smaller thermal speed) have a decrease in the peak at $v = 0$ and the width of the distribution increase, showing that the cold electrons gained energy.

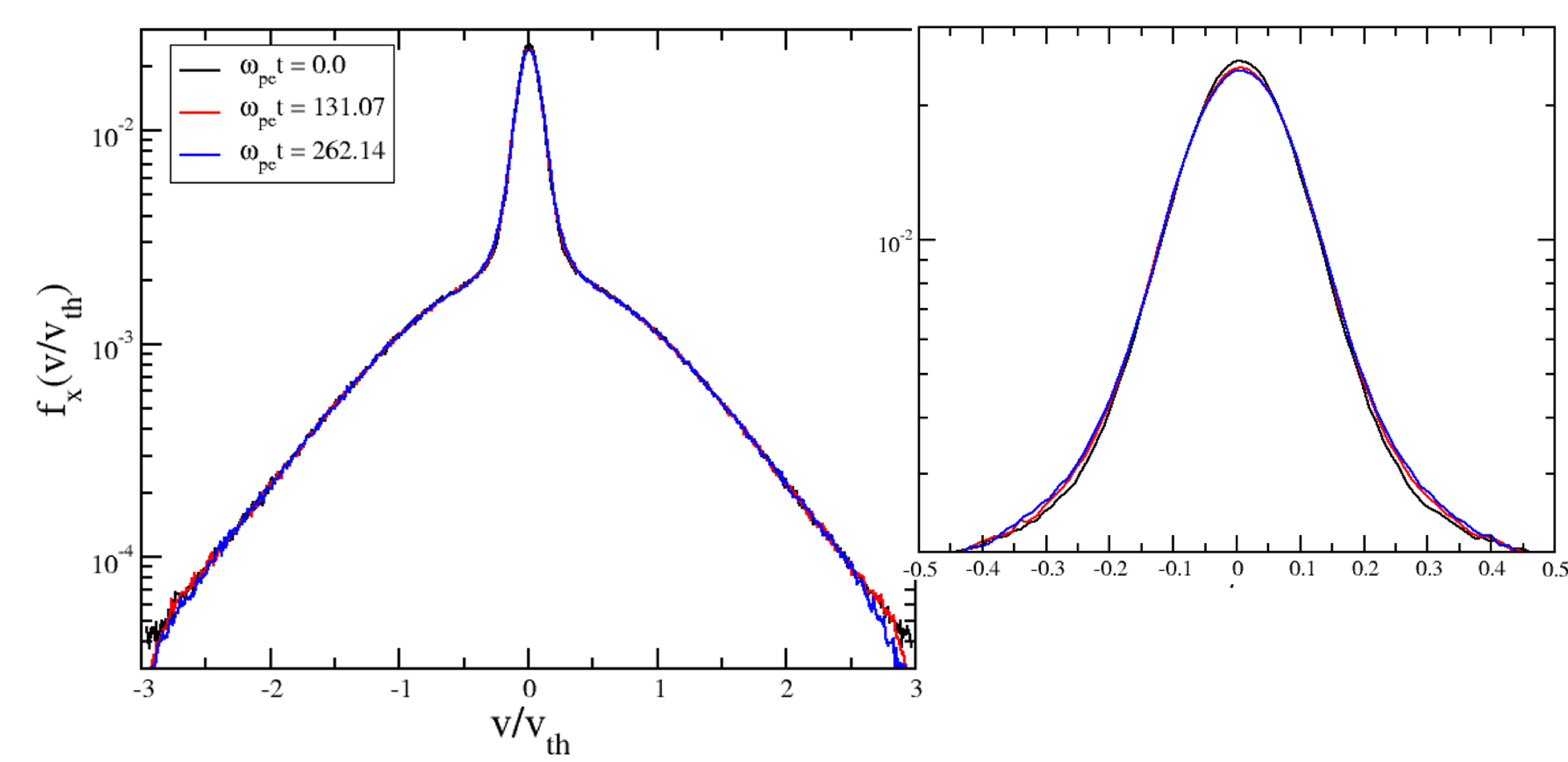

Figure 2: Time evolution of the VDF for parallel propagation, with two electron populations with same density and different thermal speed.

Plasma Waves

Figure 3 presents the dispersion relation for the electric field in the direction of propagation. The bottom panel presents the result for parallel propagation, showing not only the electron-acoustic wave (following the dotted line, given by equation 2), but also the Langmuir wave beginning at the local plasma frequency, $\omega_{pe} = \sqrt{\omega_{pc}^2 + \omega_{ph}^2} = 4.999$. The upper panel has the oblique propagation at an angle of $60°$ in respect to B_0, where we observe the whistler wave for small wave number k (normalized by the gyroradius, ρ_h) and small frequency, $\omega < \omega_{ce}$ [3]. For oblique propagation the electron-acoustic wave is absorbed in the small wave number region, and also is more intense for a frequency a little higher than the analytic expression.

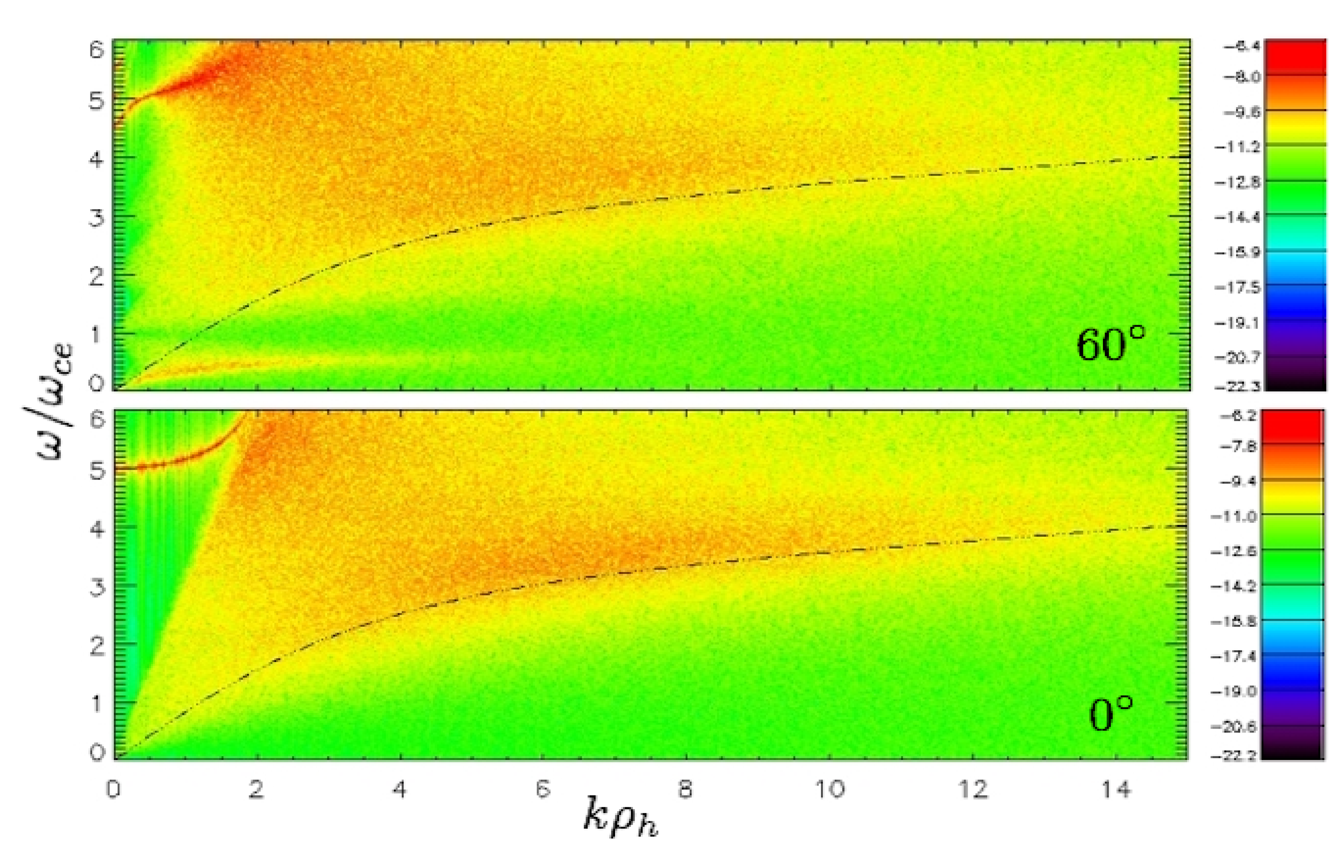

Figure 3: $\omega - k$ diagram of E_x for oblique propagation (top panel) and parallel propagation (bottom panel), in respect to B_0.

The two simulations made with perpendicular propagation are shown in Figure 4. In the left panel is presented the dispersion relation for the perpendicular propagation for the same plasma of the previous results, but now with both electron species following a Maxwellian VDF. In the right panel the results with the electron species represented by κ-VDF. Here the dotted lines show that the peak of the Bernstein waves (region were $\partial\omega/\partial k = 0$) change with the κ-VDF in comparison with a Maxwellian VDF. Henning et. al. [5] have shown that changes in the κ index of the hot electrons affect the region of large wave numbers, while the cold electrons modify the dispersion diagram for small wave numbers.

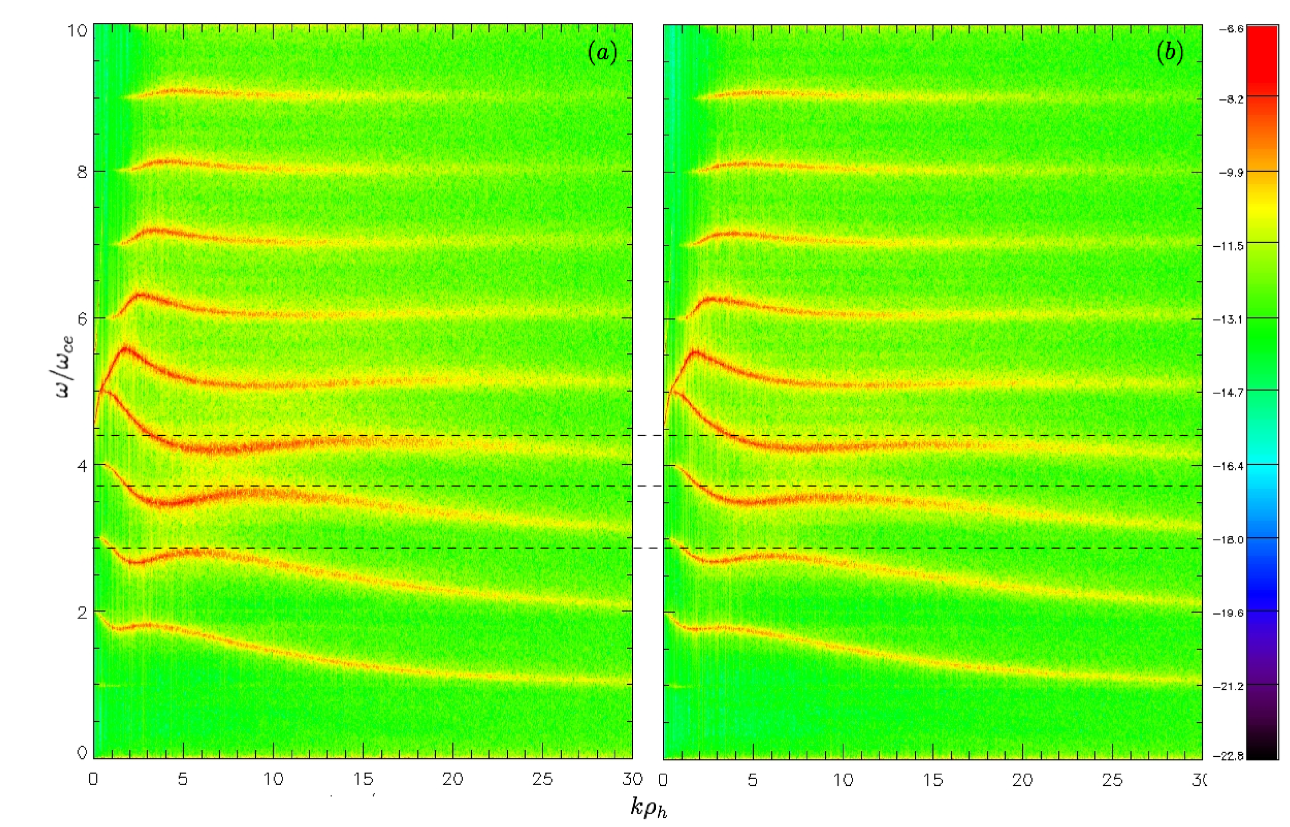

Figure 4: $\omega - k$ diagram for perpendicular propagation whit both electron species represented by Maxwellian VDF (left) and κ-VDF (right).

Conclusions

The results for the Bernstein waves are in good agreement with the theoretical ones [5]. However, since this simulation has user an electromagnetic code the ordinary and extraordinary modes can also be seen here. In the parallel propagation, the existence of the electron acoustic wave can be also found in the literature [1, 4], but simulations of this wave is a new result.

This well know waves haven been heavily studied in the past, and now with the new tool that is the κ-VDF. It is possible to observe that in the cases presented here, the Maxwellian distribution well represents the plasma process that can occur. For less understood plasma phenomena, such as coronal heating, this distribution can be very essential, since this plasmas contain a larger number of electrons with high velocity.

References

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Acknowledgements

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