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A STUDY OF THE ERRORS IN SWING-BY DESIGN BY THE "PATCHED-CONICS" APPROACH APPLIED TO THE GALILEAN MOONS

Rodolfo Batista Negri
Instituto Nacional de Pesquisas Espaciais
rodolfo.negri@inpe.br
Antonio Fernando Bertachini de Almeida Prado
Instituto Nacional de Pesquisas Espaciais
antonio.prado@inpe.br

Abstract. *The simplest method to design a swing-by is the well known "patched-conics" approach. However this approach assumes some simplifications as: i) there is no perturbation on the trajectory during the maneuver; ii) the maneuver is instantaneous. For most of the cases these assumptions are appropriate and bring only small errors. However, there are some cases where the errors are considerably large. Some situations where a better understanding of the associated errors are desirable. Therefore, this work intend to attend this demand by mapping the errors in ΔVs given by the "patched-conics" approach. In order to do so, the values obtained by a three dimensional "patched-conics" approach is compared with values obtained from a circular restricted three-body problem simulation, considered as the real ones, for the same initial conditions. In view of a relatively great number of missions in the next years aiming to explore the Galilean moons of Jupiter - e.g., Europa Clipper, Juice and so on - this work will focus its analysis on them.*

Keywords: *patched-conics, swing-by, gravity-assisted, restricted three body problem, Jupiter system*

1. INTRODUCTION

The main aspects of a swing-by maneuver can be described using the "patched-conics" model, which splits the problem in a sequence of "two-body problems", as studied by Broucke (1988), for the planar motion, and Prado (2000), for the three dimensional maneuver. A first attempt to measure the differences between the two models, in terms of the variation of energy, is made by Prado (2007), which compared both approaches when applied to a planar maneuver. Negri *et al.* (2017) expanded this work to the three dimensional maneuver. There are also some studies of this problem related to astronomy, as the work of Greenberg *et al.* (1988). The goals are different, but the physical explanations are the same.

So, the present paper focus in appliying the work done by Negri *et al.* (2017) to the Galilean moons of Jupiter. A spacecraft is assumed to make a close approach with one of the moons, but its motion is not restricted to the orbital plane of the primaries. Hence, the location of the periapsis needs an extra angle to indicate the out-of-plane component of its position in space. The velocity at periapsis also needs a new variable, which can be an angle that defines its direction in space. So, the new number of parameters is increased to five, by adding those two new ones to the well known periapsis distance r_p, magnitude of the velocity at periapsis v_p and angle of approach α (Broucke, 1988).

Therefore, the strategy used in the present paper is to define a swing-by trajectory in the three-dimensional space by specifying those five parameters. But as it is done by Negri *et al.* (2017) for some selected systems of the solar system, it is calculated the ΔV instead of energy, using the "patched-conics" approach and the "restricted three-body problem". It is assumed that the results coming from the restricted problem represent the "real values" and the results coming from the "patched-conics" are the "estimated values". So, by varying the five parameters, it is possible to measure the effects of each parameter in the errors of the "estimated maneuver". The goal is to give estimates of the errors for the Galilean moons of Jupiter, since they will be very important in the next decades of space exploration (Grasset *et al.*, 2013; Phillips and Pappalardo, 2014; Bolton, 2010), so it is possible to know in advance if it is possible to use the "patched-conics" model, considering the system under study and the accuracy required by the maneuver. In order to do so, it is measured the errors as a function of all the parameters that define the maneuver. Minimum and maximum errors are shown as a function of each parameter and plotted.

2. MATHEMATICAL MODELS AND METHODS

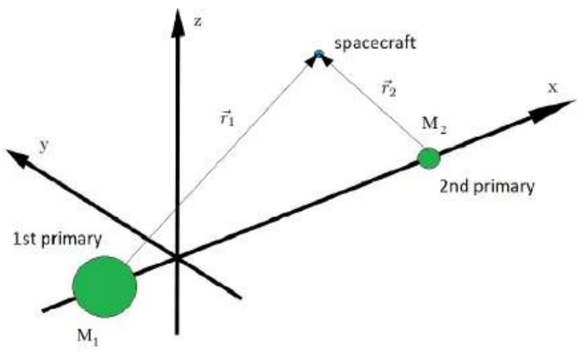
In this section are presented the mathematical models and the methods used in this work.

2.1 Stating the problem

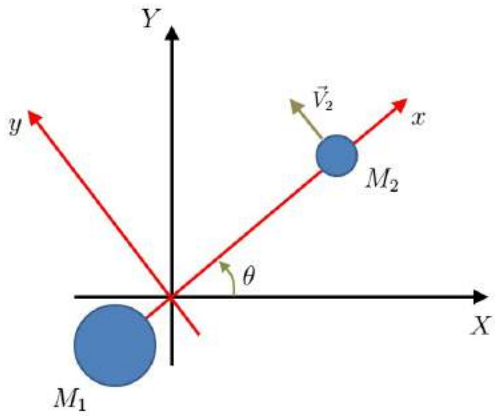
The system is composed by three point masses, where two of them are called M_1 and M_2. The first body, M_1, has the largest mass, while M_2 is the second largest body in mass. Both are from now on called primaries, and each of them perform a circular orbit around their center of mass. The third body is the spacecraft, which has negligible mass if compared to the other two.

For a better understanding of the problem is convenient to define three reference frames. They are shown in Fig. 1 and called: synodic reference frame, or xyz; inertial reference frame, or XYZ; and the pseudo-inertial reference frame, or $X'Y'Z'$.

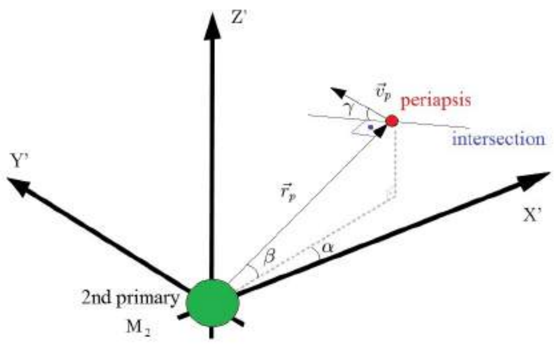
The synodic reference frame xyz, showed in Fig. 1(a), is the reference frame usually used in the circular restricted three-body problem. As the name points out, it is a rotating system, and has a constant angular speed represented by n. Its origin is in the center of mass of the system. Its x axis matches exactly with the line connecting the primaries, while its y axis lies on the orbital plane of the primaries, perpendicular to the x axis. Lastly, the z axis completes the right-handed coordinate system.



(a) Synodic reference system, xyz.



(b) Inertial reference system, XYZ.



(c) Representation of the parameters that define the swing-by in the pseudo-inertial reference system $X'Y'Z'$.

Figure 1. The used reference systems.

We can move now to the inertial coordinate system XYZ, presented in Fig. 1(b). This is a fixed reference frame with origin in the center of mass of the system, it is defined in such a way that its axis are coincident with the synodic system axis at the initial time (t=0).

The third reference system to be presented is the pseudo-inertial $X'Y'Z'$, as shown by Fig. 1(c). Its axis points to the same direction of the XYZ system axis, however the system is displaced and moves coupled to M_2, that is why it is called pseudo-inertial. The swing-by is performed on the second primary (M_2), thus the $X'Y'Z'$ system is convenient to define the five parameters which univocally define the swing-by.

These parameters define the position and velocity of the spacecraft at the periapsis of the spacecraft approach with M_2, and they are also presented in Fig. 1(c), where: r_p is the distance between the periapsis and the center of the second primary; α and β are the angles defining the postion $\vec{r_p}$ in the three dimensional space; v_p is the speed of the spacecraft at the periapsis with respect to M_2; and γ is the angle between the velocity $\vec{v}_p$ and the intersection between a plane that contains the periapsis and is parallel to the $X'Y'$ plane and a plane perpendicular to r_p that contains v_p. Therefore, the components of position and velocity in the $X'Y'Z'$ reference system can be written as (Prado and de Felipe, 2007):

$$\begin{aligned}
X' &= r_p \cos\beta \cos\alpha, && (1)\\
Y' &= r_p \cos\beta \sin\alpha, && (2)\\
Z' &= r_p \sin\beta, && (3)\\
\dot{X}' &= -v_p \sin\gamma \sin\beta \cos\alpha - v_p \cos\gamma \sin\alpha, && (4)\\
\dot{Y}' &= -v_p \sin\gamma \sin\beta \sin\alpha + v_p \cos\gamma \cos\alpha, && (5)\\
\dot{Z}' &= v_p \cos\beta \sin\gamma. && (6)
\end{aligned}$$

2.2 The restricted problem

Firstly, it is necessary to write the Eq. 1 to Eq. 6 in the xyz reference system, they are:

$$\begin{aligned}
x &= 1 - \mu + r_p \cos\beta \cos\alpha, && (7)\\
y &= r_p \cos\beta \sin\alpha, && (8)\\
z &= r_p \sin\beta, && (9)\\
\dot{x} &= -v_p \sin\gamma \sin\beta \cos\alpha - v_p \cos\gamma \sin\alpha + r_p \cos\beta \sin\alpha, && (10)\\
\dot{y} &= -v_p \sin\gamma \sin\beta \sin\alpha + v_p \cos\gamma \cos\alpha - r_p \cos\beta \cos\alpha, && (11)\\
\dot{z} &= v_p \cos\beta \sin\gamma. && (12)
\end{aligned}$$

Now is presented the numerical steps utilized to solve the circular restricted three-body equations of motion[1]. The equations of motion are (Szebehely, 1967):

$$\begin{aligned}
\ddot{x} - 2\dot{y} &= x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3}, && (13)\\
\ddot{y} + 2\dot{x} &= y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3}, && (14)\\
\ddot{z} &= -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3}; && (15)
\end{aligned}$$

where r_1 and r_2 are, respectively, the distance between the spacecraft and the first and second primary, as shown in Fig. 1(a).

In order to find the variation of velocity given by the restricted problem (ΔV_{RP}), the following algorithm is used:

- By using Eq. 7 to Eq. 12, with initial values for α, β, r_p, γ and v_p, the initial conditions are calculated;
- The equations of motion are integrated forward in time until the spacecraft reaches a distance r_2 equal to the sphere of influence (SOI) of M_2 (r_{SOI}), which the radius is given by (Araujo *et al.*, 2008):

$$r_{SOI} = \left(\frac{\mu}{1-\mu}\right)^{2/5}. \qquad (16)$$

- Once the integration is performed, the speed after the maneuver V_{RP}^{+} is calculated by saving the velocity in the xyz reference system, transforming it to the XYZ reference system (Murray and Dermott, 1999) and obtaining its modulus;
- The spacecraft is then positioned again at its initial conditions, and the equations of motion are integrated backward in time, until $r_2$reaches the sphere of influence of M_2 one more time;
- The speed V_{RP}^{-} before the swing-by is then calculated;
- Finally, the ΔV for the restricted problem is calculated as:

$$\Delta V_{RP} = V_{RP}^{+} - V_{RP}^{-}. \qquad (17)$$

[1] A forth-order Runge-Kutta method with local truncation error to fifth-order was used to assure the precision and to adjust the step size.

2.3 The patched-conics approach

For the patched-conics approach, the pseudo-inertial $X'Y'Z'$ reference system is more convenient to describe the trajectory of the spacecraft as it performs the maneuver. Here, this system is assumed to be inertial, which means that no M_2 movement is taken into account. As a direct consequence, it is supposed that the maneuver is taken instantaneously.

Another assumption of this approach is that the restricted three-body problem may be reduced to a sequence of three two-body problems (spacecraft-M_1, spacecraft-M_2 and spacecraft-M_1 once again) that are patched together at its extremities (Minovitch, 2010). This means that the perturbation effects of M_2 in the orbit of the spacecraft around M_1 and the perturbation effects of M_1 in the orbit of the spacecraft around M_2 are not considered in this model. Therefore, there are three conic orbits. The points where the orbits are patched are determined by the SOI (Araujo *et al.*, 2008) of M_2, which is given by Eq. 16.

By using this approach, Prado (2000) shows that the velocity after the maneuver ($\vec{V}_i$) and before it ($\vec{V}_o$), with respect to the XYZ reference frame, are calculated as:

$$\vec{V}_i = v_\infty \sin\delta \begin{bmatrix} \cos\beta\cos\alpha \\ \cos\beta\sin\alpha \\ \sin\beta \end{bmatrix} + v_\infty \cos\delta \begin{bmatrix} -\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha \\ -\sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha \\ \cos\beta\sin\gamma \end{bmatrix} + \begin{bmatrix} 0 \\ V_2 \\ 0 \end{bmatrix}, \tag{18}$$

$$\vec{V}_o = -v_\infty \sin\delta \begin{bmatrix} \cos\beta\cos\alpha \\ \cos\beta\sin\alpha \\ \sin\beta \end{bmatrix} + v_\infty \cos\delta \begin{bmatrix} -\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha \\ -\sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha \\ \cos\beta\sin\gamma \end{bmatrix} + \begin{bmatrix} 0 \\ V_2 \\ 0 \end{bmatrix}. \tag{19}$$

With some algebra is possible to show that the modulus of the vectors $\vec{V}_i$ and $\vec{V}_o$ are:

$$V_i = [v_\infty^2 + V_2^2 + 2v_\infty V_2(\cos\alpha\cos\delta\cos\gamma + \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma)]^{1/2}, \tag{20}$$

$$V_o = [v_\infty^2 + V_2^2 + 2v_\infty V_2(\cos\alpha\cos\delta\cos\gamma - \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma)]^{1/2}. \tag{21}$$

where V_2 is the speed of M_2; the angles β and α are in accordance with Fig. 1(c); δ is half of the turn angle of the trajectory of the spacecraft due to the passage by M_2, which can be expressed using two-body dynamics as (Broucke, 1988):

$$\sin\delta = \frac{1}{1 + r_p v_\infty^2/\mu}, \tag{22}$$

where v_∞ is the speed of the spacecraft at the infinity with respect to M_2 (i.e., the $X'Y'Z'$ reference system). From the two-body mechanics, v_∞ is given by:

$$v_\infty = \left(v_p^2 - \frac{2\mu}{r_p}\right)^{\frac{1}{2}}. \tag{23}$$

Now, by using Eq. 20 and Eq. 21, it is possible to calculate the speed variation obtained by the swing-by, for the patched-conics approach, as:

$$\Delta V_{PC} = V_o - V_i. \tag{24}$$

3. RESULTS

As made by Negri *et al.* (2017) for some systems of the solar system, this analysis is made using ΔVs, in km/s, measured in the inertial reference frame XYZ.

The error in ΔV is defined as:

$$\Delta V_{error} \quad = \quad \Delta V_{RP} - \Delta V_{PC}, \tag{25}$$

From Fig. 2 to Fig. 8 is shown the maximum and minimum ΔVs applied to the Galilean moons to the whole set of parameters. They are standardized as follows: the (a) subfigures represent ΔV_{RP}, the (b) represent ΔV_{PC} and the (c) represent ΔV_{error}.

Starting by the analysis of the ΔVs under the influence of r_p, as shown in Fig. 2. As observed by Negri *et al.* (2017) larger ΔVs, Fig. 2(a) and Fig. 2(b), occurs for low r_p. The errors follow this same behaviour, as shown in Fig. 2(c).

Now, studying v_p through N, as shown in Fig. 3. Analysing the ΔVs obtained by the restricted problem and by the patched-conics approach, Fig. 3(a) and Fig. 3(b), is noticeable a peak region for ΔV. For instance, in the Fig. 3(b), this peak occurs between $N = 1.2$ and $N = 1.25$. This behaviour was expected, because Eq. 23, Eq. 20 and Eq. 21 show that the Eq. 24 has quadratic terms of v_p, which makes the relation between ΔV and v_p a little more complex. The errors, Fig. 3(c), follows the same behaviour noted by Negri *et al.* (2017).

Moving now to the γ angle, Fig. 4, and starting by the patched-conics approach, Fig. 4(b) shows that the maximum and minimum ΔV_{PC}s are constant, which might bring to the conclusion that ΔV_{PC} does not depend on γ. This was not expected, since Eq. 20 and Eq. 21 show the opposite, a clear dependence on γ. However, because our analysis is based on maximums and minimums, this figure might be misleading. The Fig. 4(d) disprove this conclusion by fixing all the parameters and varying only α and γ applied to Ganymede-Jupiter and showing that ΔV_{PC} is not constant with the variation of the angle γ. The constant appearance in Fig. 4(b) is explained by the superposition of various different curves, for different sets of parameters, with equal peaks for different γ.

Figure 3(a) shows that the restricted problem does not follow the same behaviour. There is a clear advantage by using $\gamma = \pm 180°$. This is a consequence that, differently of what occurs with the patched-conics approach, which assumes that the $X'Y'Z'$ reference system is inertial, the restricted problem does not follows the same mistake. While the spacecraft performs its swing-by, M_2 keeps in its orbit around the center of mass of the system, making the velocity vector of M_2, $\vec{V}_2$, to be different at the inbound or outbound of the SOI. Figure 5 exemplifies it.

When the γ angle assumes the value of $\pm 180°$, it always acts in favour of increasing the effects of the maneuver over ΔV, whether in the gain (when $180° < \alpha < 360°$) or the loss (when $0° < \alpha < 180°$) of energy. But when γ assumes the value of $0°$, this does not happen.

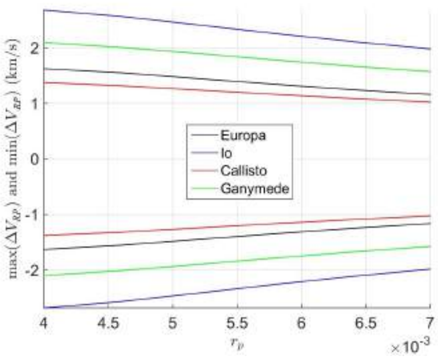
Figure 6 shows this behaviour. It represents a generic M_2 swing-by for the gain of energy as seem from the pseudo-inertial reference frame $X'Y'Z'$. But now, differently of what occurs in the patched-conics approach, this reference system is known as non-inertial and this is taken into account by changing $\vec{V}_2$ direction. One should notice that $\vec{v}_\infty$ acts in favour of $\vec{V}_2$ when $\gamma = 180°$, and the opposite when $\gamma = 0°$. The same effect is expected in the region of loss of energy, and now $\gamma = \pm 180°$ should make $\vec{v}_\infty$ acts against $\vec{V}_2$, increasing the loss of speed.

Lastly, still in γ, Fig. 4(c) is coherent with Fig. 4(a) and Fig. 4(b).

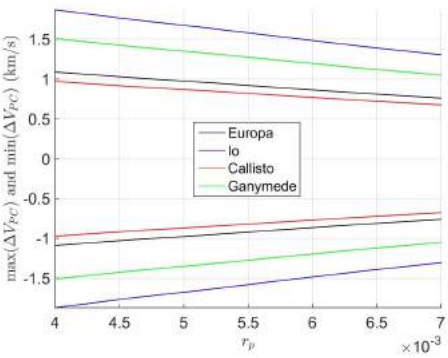
The α angle, Fig. 7, has the same behaviour noticed by Negri *et al.* (2017). Figure 7(a) and Fig. 7(b) show that the maximum and minimum values occur at the expected αs. The maximum in $\alpha = 270°$ and minimum in $\alpha = 180°$. The errors follows this line as shown in Fig. 7(c).

The last parameter, β, is presented in Fig. 8. As expected, the Fig. 8(a) and Fig. 8(b) show larger ΔVs for $\beta = 0°$. The errors, Fig. 8(c), follows this behaviour as already noted by Negri *et al.* (2017).

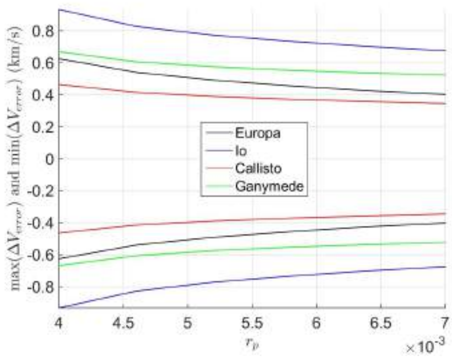
Lastly, it must be highlighted the magnitude of the ΔV_{error} found and confirmed by all Figs. 2(c) to 8(c). They show errors of about 0.9 km/s for the Io-Jupiter system, this is around 50% of the maximum value estimated by the patched-conics approach. For the Ganymede-Jupiter and Europa-Jupiter cases the error is approximately 0.6 km/s, reaching 40% of the estimated by the patched-conics approach for Ganymede and 60% for Europa. Moreover, for Callisto, the maximum error is a bit greater than 0.4 km/s, which corresponds to around 40% of the estimated by the patched-conics.



(a) ΔV_{RP}

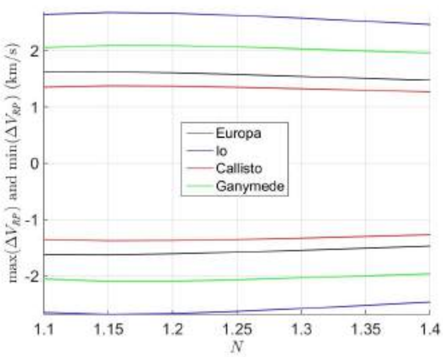


(b) ΔV_{PC}

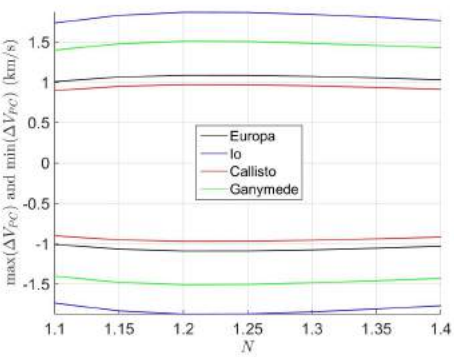


(c) ΔV_{error}

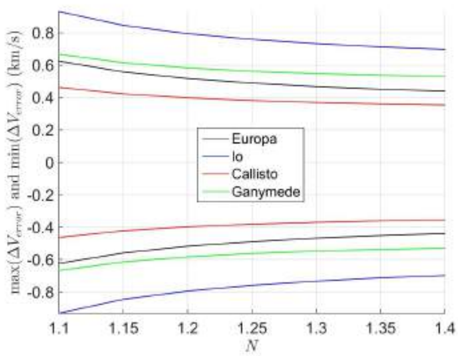
Figure 2. Maximum and minimum ΔVs as function of r_p applied to the Galilean moons.



(a) ΔV_{RP}

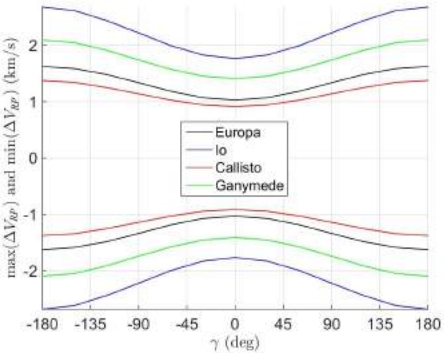


(b) ΔV_{PC}

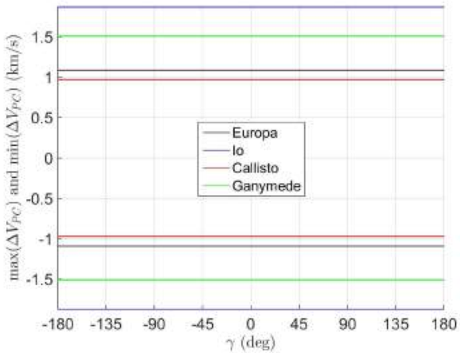


(c) ΔV_{error}

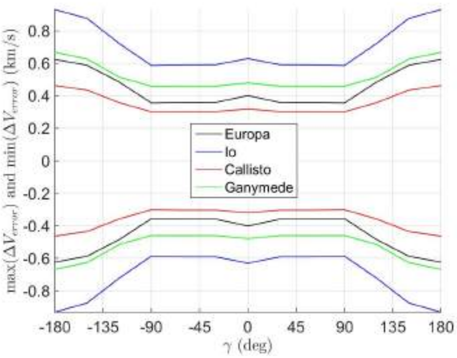
Figure 3. Maximum and minimum ΔVs as function of N applied to the Galilean moons.



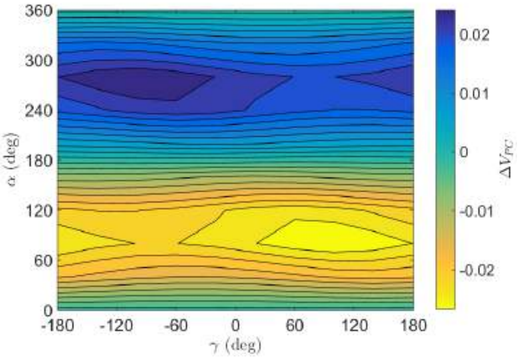
(a) ΔV_{RP}



(b) ΔV_{PC}



(c) ΔV_{error}



(d) ΔV_{PC} for $\beta = 80°$, $N = 1.2$, $r_p = 0.004$ and $\mu = 7.8 \times 10^{-5}$ (Ganymede-Jupiter) as function of α and γ.

Figure 4. Maximum and minimum ΔVs as function of γ applied to the Galilean moons.

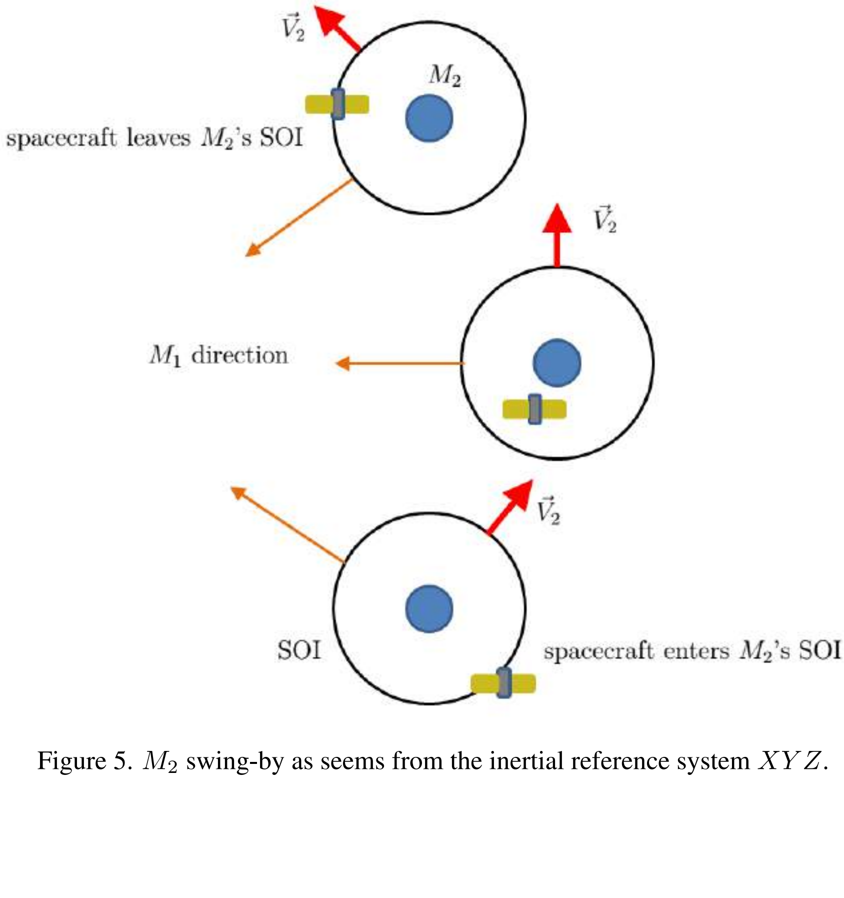


Figure 5. M_2 swing-by as seems from the inertial reference system XYZ.

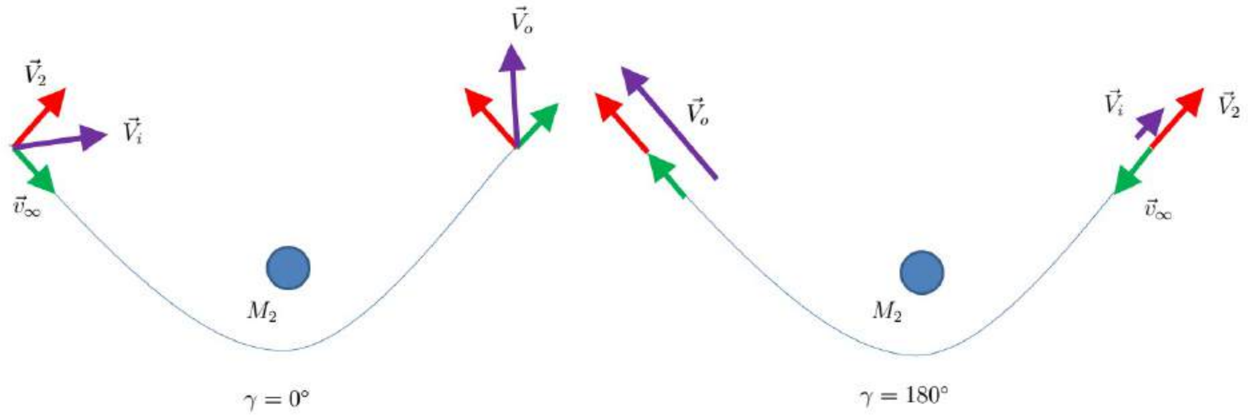
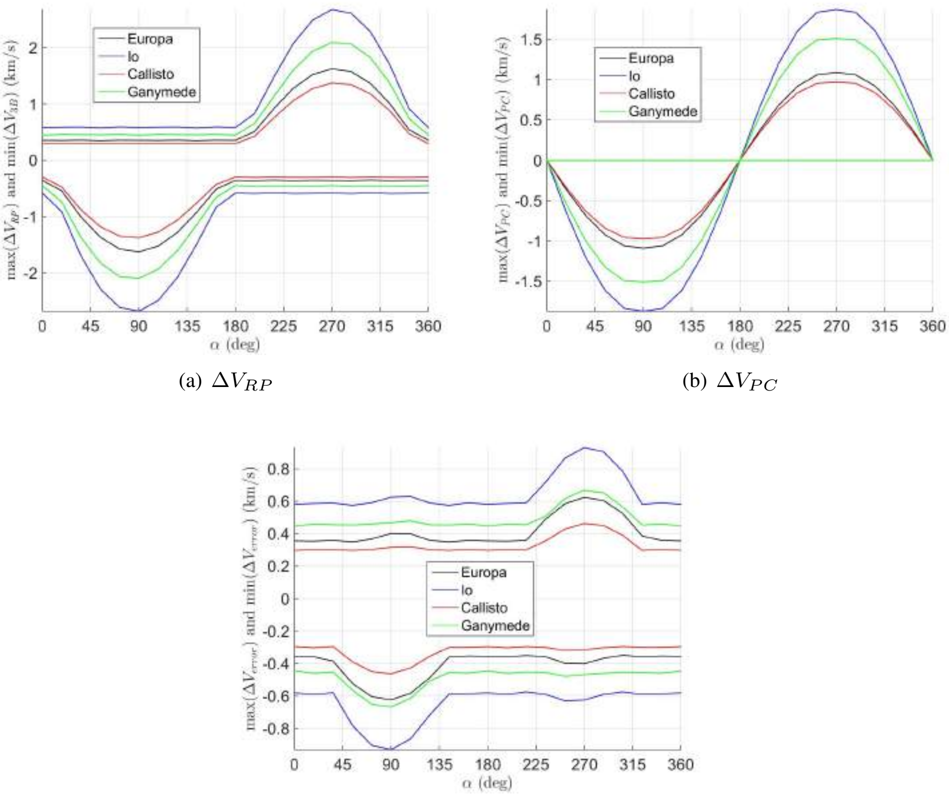


Figure 6. M_2 swing-by as seem from the pseudo-inertial reference frame $X'Y'Z'$, and recognizing it as a non-inertial reference frame.

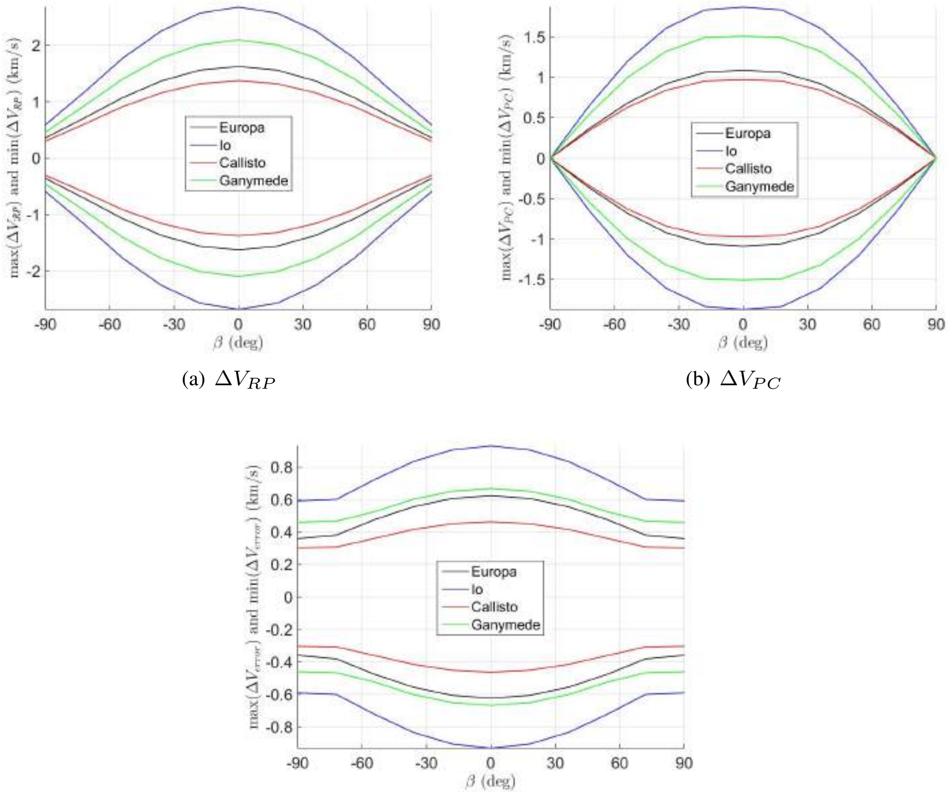


(a) ΔV_{RP}

(b) ΔV_{PC}

(c) ΔV_{error}

Figure 7. Maximum and minimum ΔVs as function of α applied to the Galilean moons.



(a) ΔV_{RP} (b) ΔV_{PC}

(c) ΔV_{error}

Figure 8. Maximum and minimum ΔVs as function of β applied to the Galilean moons.

4. CONCLUSIONS

This paper highlights the importance of taking a more realistic approach to design swing-bys applied to the Galilean moons, by showing that errors in ΔVs might arise up to 60% of the value estimated by the patched-conics approach.

Also, through the analysis of the γ angle, the importance of the non-instantaneity of the maneuver, which is a consequence of considering the pseudo-inertial reference system as inertial, is highlighted.

The other results are in accordance with previous studies, as the ones made by Negri *et al.* (2017), Prado (2007) and Broucke (1988).

5. ACKNOWLEDGEMENTS

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