Searching for modified gravity in the astrophysical gravitational wave background: Application to ground-based interferometers

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We investigate how the propagation of an astrophysical gravitational wave background (AGWB) is modified over cosmological volumes when considering theories beyond general relativity of the type Horndeski gravity. We first deduce an amplitude correction on the AGWB induced for the presence of a possible running in the Planck mass. Then, we apply the spectral noise density from some ground-based interferometers, namely, the Advanced LIGO (aLIGO), Einstein Telescope (ET) and Cosmic Explore (CE), to evaluate the signal-to-noise ratio as a function of the amplitude of the running of the Planck mass for two different scenarios. We find that for observation time period $\gtrsim 5$ yrs and $\gtrsim 1$ yr, we can have a significant signal of the AGWB in the band [1–100] Hz from the ET and CE sensitivity, respectively. Finally, using Fisher information, we find some forecast bounds, and we deduce $\lesssim 27\%$ and $\lesssim 18\%$ correction at 1σ confidence level on the amplitude of the running of the Planck mass from ET and CE, respectively. It is clear that a detection of a AGWB in the future can open a new window to probe the nature of gravity with good accuracy.

I. INTRODUCTION

One century after the formulation of Einstein's general relativity (GR), the gravitational waves (GWs), being one of the main theoretical predictions of GR, were recently observed in LIGO/VIRGO [1]. It was possible to observe the Universe, and discover several objects and physical phenomena in the past 100 years through various observations mainly via electromagnetic signal. Now, discovery of the GWs has opened a new observational window to investigate the Universe under a new spectrum of possibilities. Also, through possible joint observation in GWs and electromagnetic signal, called the multimessenger search, just like the recent GW170817 and GRB 170817A events [2,3], and other events should be detected soon.

The GWs are emitted mainly by individual binary systems, like for instance, from binary black holes (BBH), binary neutron stars (BNS) and binary black hole-neutron stars (BBH-NS). It is expected that the superposition of the signal from these sources over cosmological volumes can form an astrophysical gravitational wave background (AGWB) [4–12]. The AGWB is characterized by the energy density parameter $\Omega_{GW}(f)$, which represents the present-day fractional energy as a function of frequency f. The AGWB signal strongly depends on the type of sources that produce them, and we expect that signal to exist in the most diverse frequencies [13–26].

Until the present moment, the AGWB has not been detected, and only some upper limits on the stochastic gravitational-wave background signal have been obtained. The LIGO/VIRGO collaboration reported an astrophysical background with amplitude $< 4.8 \times 10^{-8}$ [27] and $\Omega_{GW} = 1.8^{+2.7}_{-1.3} \times 10^{-9}$ at 25 Hz in [28]. With the improvements in instrumental sensitivity in the coming years, as well as from the prospects of the future detectors like the Einstein Telescope (ET) [29], Cosmic Explorer (CE) [30] and LISA [31,32], it is expected to achieve enough sensitivity to detect the AGWB.

There are theoretical and observational reasons to believe that GR should be modified when gravitational fields are strong and/or on large scales. From an observational point of view, the physical mechanism responsible for accelerating the Universe at late times is still an open question, and new degrees of freedom of the gravitational origin are alternatives to explain such an accelerated stage (see [33–35] for review). Theories beyond GR can serve as alternatives to explain the current tension in the Hubble constant that persists in the framework of the ΛCDM model [36–39]. Also, modified gravity models are motivated to drive the accelerating expansion of the Universe at early times (inflation). See [40] and references therein for motivation of modified gravity scenarios under the regime of strong gravitational field.

All the first GW observations to date have revealed to be consistent with GR theory [41,42], and imposed strong bounds on modified gravity/dark energy scenarios in the

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local Universe [43–45]. On the other hand, going beyond general relativity means changing the properties of GWs in different ways, such as corrections in amplitude, phase, extra polarization states, etc. It is expected that on large scales and cosmological distances, the GR theory needs to be corrected and, in return, on such scales, it will change the GW behavior/properties [46–52]. Current detectors are not sensitive enough to probe the Universe at cosmological distances, not more than $z \lesssim 1$. But, certainly, some promising projects like ET, CE and LISA will be able to detect GW events with great accuracy at cosmological distances and provide information for powerful cosmological tests [53–70].

The goal of this article is to extend the standard calculation of the gravitational wave background from compact binary coalescences, including correction on GR theory, which modifies the propagation of an AGWB. Where we consider a sum on the contributions from the binary neutron stars + black hole-neutron + black holes, incorporating corrections on the propagation of the AGWB signal over cosmic time in the presence of possible changes in GR theory. We find that the gravitational coupling, quantified in terms of the running of the Planck mass α_M, can induce amplitude corrections on $\Omega_{GW}(f)$ propagation. Then, we analyze the signal-to-noise ratio as a function of α_M for two scenarios, assuming the possibilities $\alpha_M < 0$ and $\alpha_M > 0$, from the perspective of the LIGO, ET and CE sensitivity. We note that from ET and CE, we can have a significant signal, and then we perform a forecast analysis on the free parameter that determines the α_M amplitude.

The paper is structured as follows. In Sec. II, we present the theoretical framework for the AGWB propagation in modified gravity. In Sec. III, we present our main results. Lastly, in Sec. IV, we outline our final considerations and perspectives.

II. MODIFIED GRAVITATIONAL WAVE BACKGROUND FROM COMPACT BINARY COALESCENCES

In this section, we summarize the formalism used to calculate the GW energy spectrum $\Omega_{GW}(f)$ as presented in [4,13]. Let us check the theories beyond the GR inducing corrections on the $\Omega_{GW}(f)$ propagation, and when evaluated at the present moment, i.e, $z = 0$, we can compare the spectrum in possible GWs experiments/observatories.

The GWs spectrum can be computed by

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \int_0^{z_{max}} \frac{N(z)}{1+z} \left(\frac{dE_{GW}}{d \ln f_r} \right) dz, \quad (1)$$

where $N(z)$ is the spatial number density of GW events at z. The factor $(1 + z)$ accounts for redshifting of GW energy since emission, and $f_r = f(1 + z)$ is the GW frequency in the source frame. The function $dE_{GW}/d \ln f_r$ quantifies the single source energy spectrum. It is convenient to replace $N(z)$ with the differential GWs event rate

$$\frac{dN}{dz} = N(z)4\pi r^2 = \frac{R(z)}{1+z} \frac{dV}{dz}, \quad (2)$$

where $dV/dz = 4\pi c r^2 / H(z)$ is the comoving volume element, and r the comoving distance. Here, $R(z) = r_0 e(z)$ is the rate density measured in cosmic time local to the event [71,72], where the parameter r_0 is the local rate density, used to estimate detection rates for different detectors. The amount $e(z)$ is a dimensionless factor which models the source rate evolution over cosmic time. The factor $(1 + z)$ in the above equation converts $R(z)$ to an earth time based quantity. Then, Eq. (1) can be written as

$$\Omega_{GW}(f) = \frac{f r_0}{\rho_c H_0} \int_0^{z_{max}} \frac{e(z)}{(1+z)E(z)} \left(\frac{dE_{GW}}{d \ln f} \right) dz. \quad (3)$$

The GW energy spectrum for an inspiralling circular binary of component masses m_1 and m_2 is given by [73]

$$\frac{dE_{GW}}{d \ln f_r} = \frac{(\pi G)^{2/3} M_c^{5/3}}{3} f_r^{-1/3}, \quad (4)$$

where $M_c = M\eta^{5/3}$ is the chirp mass, $M = m_1 + m_2$ being the total mass and $\eta = m_1 m_2 / M^2$ the symmetric mass ratio. Inserting this into Eq. (3), we have

$$\Omega_{GW}(f) = A \times J \times f^{2/3}, \quad (5)$$

where we have defined the quantity

$$A = \frac{8 r_0}{9 c^2 H_0^3} (\pi G M_c)^{5/3} \quad (6)$$

and

$$J = \int_0^{z_{max}} \frac{e(z)(1+z)^{-4/3}}{\sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}}. \quad (7)$$

To determine the applicable frequency range of the above relation, one has f_{min} well below 1 Hz, and f_{max} given by the frequency at the last stable orbit during inspiral, $f_{max} = 1/(63/2^2 \pi M_z)$, with $M_z = (1+z)M$.

We are interested in checking how alternative scenarios to GR can change $\Omega_{GW}(f)$. In principle, we have two major possibilities to look into this point, which can globally affect the AGWB.

(i) A common feature in almost all the gravity theories beyond the GR, at the level of the Universe on large scales, is that the new degrees of freedom in each theory modify the gravitational force/interaction on cosmological scales, mainly motivated to explain the late-time acceleration of the Universe (dark-energy-dominated era). This case is generally featured by an

effective time variable gravitational constant, which will affect the propagation of the GWs along the cosmic expansion. See [74] for a review.

(ii) By changing gravity, we also change the generation mechanism of the gravitational radiation emitted by the binary systems. Such methodology can be quantified through the parametrized post-Einsteinian framework [75–79]. In this case, changes in GR will modify the waveform, but keep propagation corrections on GWs unchanged. The AGWB is recently studied in the parametrized post-Einsteinian context in [80].

In general, at local level, the GW information from isolated or binary systems in the strong space-time curvature regime can provide several tests to GR [40]. But, we are interested in the AGWB, which is a global source. Thus, these two points above should be the main sources of corrections in this sense. In this work, let us focus on case (i), where the presence of some scalar field can significantly modify the gravitational force and the effective gravitational couplings vary in time at the cosmological scales, so that it is possible to see the variation of the gravitational couplings as a function of the cosmic time. As the AGWB is evaluated over large cosmic time intervals, corrections in this sense can become an interesting source of the information about gravity. It is important to note that this framework is quantifying directly dark energy effects and its fingerprint on the AGWB propagation over cosmological time. Here, these dark energy effects are not considered/significant on the compact objects structure.

As this proposal, let us formulate these corrections on the AGWB in the context of the Horndeski gravity [81–83], which is the most general scalar-tensor theory with second-order equations in $D = 4$. In the Appendix, we briefly review this gravity scenario. Following [48,49], the effective Newton constant can be written as

$$\frac{G_{\mathrm{gw}}}{G_N} = \frac{M_*^2(0)}{M_*^2(z)}, \tag{8}$$

where G_N is the Newton gravitational constant, and we define G_{gw} as the gravitational coupling for GWs, where

$$M_*^2 = 2G_4 \quad \text{and} \quad \alpha_M = \frac{1}{H}\frac{d\log M_*^2}{dt}, \tag{9}$$

with M_*^2 being the effective Planck mass and α_M the running of the Planck mass.

Interpreting the gravitational constant in Eq. (5) as the gravitational coupling for GWs, we can write the spectrum as

$$\Omega_{GW}^{\mathrm{MG}}(f) = \tilde{A}^{-10/3}\Omega_{GW}^{\mathrm{GR}}(f), \tag{10}$$

with

$$\tilde{A} = \exp\left[-\frac{1}{2}\int_0^z \frac{\alpha_M(z')}{1+z'}\,dz'\right], \tag{11}$$

where the indices GR and MG refer to the spectrum predicted in general relativity and modified gravity, respectively. Note that this correction comes due to the energy spectrum from the inspiralling binary systems, and can be interpreted as an amplitude correction, a cumulative effect throughout the propagation of the AGWB through the cosmic evolution.

Now, in order to move on, it is usual to choose phenomenological functional forms for the functions α_M (see, e.g., [84,85]). In the present work, we will adopt two parametrizations.

Scenario I: The sub-Horndeski gravity called by *No Slip Gravity*, proposed in [86]. The main characteristics of this model read as the speed of gravitational wave propagation equal to the speed of light, and equality between the effective gravitational coupling strengths to matter and light, but yet different from Newton's constant, capable of generating an effective G_{gw}. In this scenario we have

$$\alpha_M = \frac{c_M(a/a_t)^\tau}{[(a/a_t)^\tau + 1]^2}, \tag{12}$$

where a is the cosmic scale factor. The main parameter here is c_M, featuring the amplitude of the running of the Planck mass. The stability condition requires $c_M > 0$ and $0 < \tau < 3/2$. In what follows, through all the simulations carried out in this work, we will adopt $a_t = 0.5$ and $\tau = 1$. See [86] for more details.

Scenario II: To quantify $\alpha_M < 0$ effects, let us consider $\alpha_M = c_M a^n$. Following [85], the stability conditions for $\alpha_M < 0$ can be summarized as stable for $c_M < 0$ and $n > 3/2$. Throughout our results below, we will assume $n = 1$.

Once α_M is defined, we can evaluate correction on the spectrum due to the modified propagation and compare the theoretical spectrum with sensitivity curves planned for GW observations. To the author's knowledge, this methodology is new and never investigated in the literature before. In what follows, we discuss our main results.

III. RESULTS AND DISCUSSIONS

First, we need to define some input properties of an AGWB, before performing numerical simulations. We list here the main ones.

(1) We define $e(z) = \rho_*(z)/\rho_*(0)$, where $\rho_*(z)$ is the cosmic star formation rate density (in units of $M_\odot \mathrm{yr}^{-1} \mathrm{Mpc}^{-3}$). We consider $\rho_*(z)$ derived from the observations in [87].

(2) With respect to information about compact binary coalescence populations, we consider that $\Omega_{\mathrm{GW}}(f)$ as the sum due to contributions from stellar mass BBH, BNS and BBH-NS, i.e., $\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW}}^{\mathrm{BBH}}(f) + \Omega_{\mathrm{GW}}^{\mathrm{BNS}}(f) + \Omega_{\mathrm{GW}}^{\mathrm{BBH-NS}}(f)$. These are the
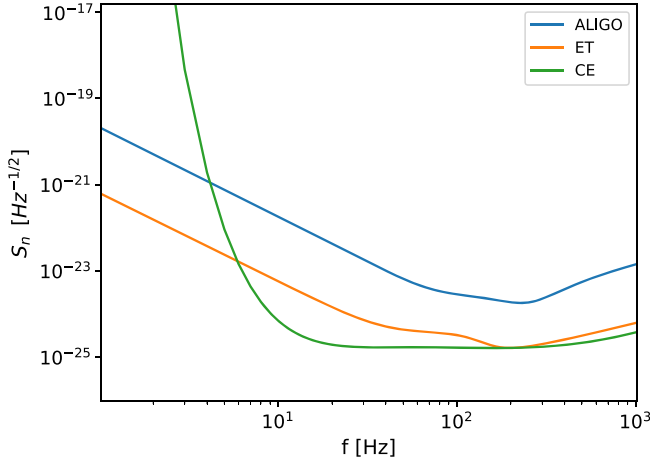
FIG. 1. Detector spectral noise density for the Advanced LIGO (aLIGO), Einstein Telescope (ET) and Cosmic Explore (CE).

most promising GW sources for ground-based interferometers. In this sense, we use r_0 values corresponding to the realistic estimates, $r_0 = 1, 0.03, 0.05$ Mpc^{-3} Myr^{-1}, for BNS, BBH-NS, BBH, respectively. Also, we replaced $M_c^{5/3}$ with $\langle M_c^{5/3} \rangle$ to account for a distribution of system masses with an average over individual energy spectra as presented in [13].

From this input information, we can search our template applied to ground-based interferometers. We will consider the sensitivity as predicted by the detector spectral noise density for the Advanced LIGO (aLIGO) [89], Einstein Telescope (ET) [29] and Cosmic Explore (CE) [30]. Figure 1 shows S_n for these detectors. The signal-to-noise ratio (SNR) to a SGWB in terms of the above quantities is

$$\text{SNR} = \sqrt{T \int_{f_{\min}}^{f_{\max}} \left(\frac{S_h}{S_n}\right)^2}, \qquad (13)$$
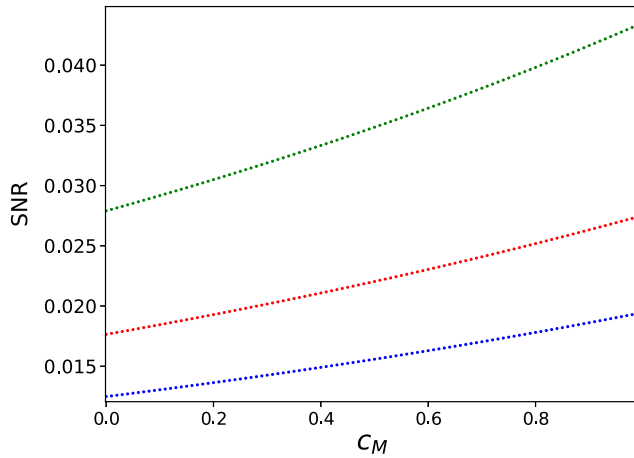
where $f_{\min}$, $f_{\max}$ denote, respectively, the minimal and maximal frequencies accessible at the detector and/or some range of the interest for research. The SNR increases as the square root of the observation time T. In the above equation, we define [88],

$$S_h(f) = \frac{3H_0^2}{2\pi^2} f^{-3} \Omega_{\text{GW}}(f). \qquad (14)$$

Note that assuming an isotropic GW background, a factor of $1/5$ should be included to account for the average detector response over all source locations in the sky. In what follows, in all results, we consider the input: $z_{\max} = 2$, $f_{\min} = 1$ Hz and $f_{\max} = 100$ Hz. These values are justified because up to $z = 2$, it comprises the range for the majority compact binary coalescence populations from stellar mass, which in return presents greater amplitude in the range $f \sim [1, 100]$. We use $H_0 = 70$ km/s/Mpc and $\Omega_m = 0.31$ to fix the background expansion. Evaluation for values greater than this range does not change the main results considerably.

Figure 2 shows the SNR as a function of the parameter c_M for both scenarios, and for different observation time periods within the sensibility of the aLIGO. In all cases, the GR theory corresponds to $c_M = 0$. As expected, we have a signal and SNR very low, making its detectability difficult, for a wide range of intervals in c_M.

We can note a pattern around the features of the theoretical framework. For $\alpha_M > 0$, the SNR tends to increase, when c_M also increases. For $\alpha_M < 0$, we notice the opposite, where c_M decreases, we have SNR also decreasing. Once the α_M function quantifies a general property for all gravitational theories, we can conjecture that this should happen for any model beyond the GR in general, which can be written in terms of the α_M.

Figures 3 and 4 show the influence of the c_M on the SNR from the spectral noise density for the ET and CE,
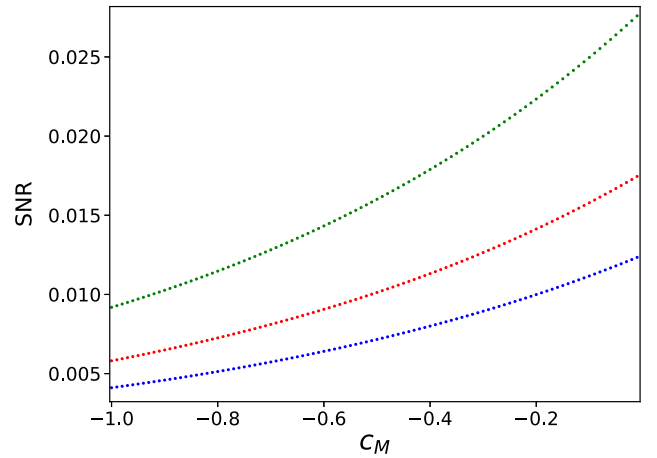

FIG. 2. The signal-to-noise ratio (SNR) as a function of the parameter c_M (amplitude of the running of the Planck mass), assuming Advanced LIGO noise power spectral density sensibility for different observation times, 1, 3, and 5 yr in blue, red and green, respectively. Left panel: Theoretical framework given by scenario I. Right panel: Scenario II.
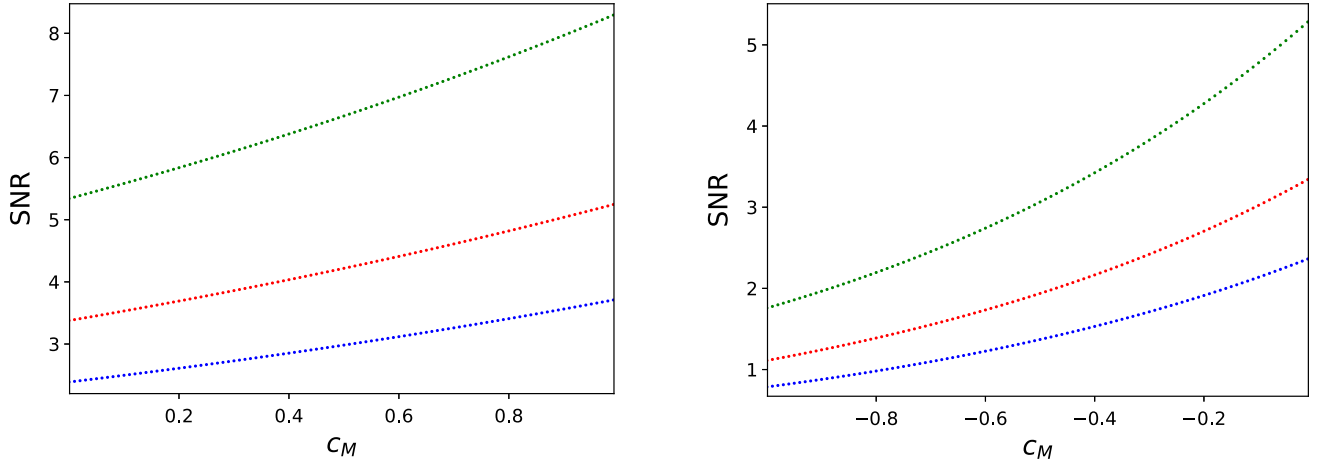
FIG. 3.   The same as in Fig. 2, but assuming Einstein Telescope noise power spectral density sensibility.

respectively. Here, we also take different observation time periods. For ET, we find that only for $\gtrsim 5$ yrs, we can note significant SNR values, which we can talk about for a possible observation of the signal for a AGWB. Note that GR always has SNR value smaller (larger), with respect to $c_M > 0 (< 0)$, respectively. Within CE sensitivity, we can detect a strong signal, with high SNR values for both scenarios, even for 1 yr of operation. This is because CE can be more sensitive than ET by up to 2 orders of magnitude in the band [1–100] Hz.

It is interesting to note that the residual foreground in the range [1–100] Hz, should be considered in future ground-based stochastic searches for AGWB signal. In principle, this signal can be detected in GR as well as in modified gravity models, as shown in Figs. 3 and 4.

For a high enough SNR, we can use the Fisher matrix analysis to provide upper bounds for the free parameters of the models. We refer the reader to [90–92] for a discussion on the Fisher analysis in GWs signal. Thus, let us limit ourselves to apply the Fisher information only to ET and CE. The root-mean-squared error on any parameter is determined by

$$\Delta\theta^i = \sqrt{\Sigma^{ii}}, \tag{15}$$

where Σ^{ij} is the covariance matrix, i.e, the inverse of the Fisher matrix, $\Sigma^{ij} = \Gamma_{ij}^{-1}$. The Fisher matrix is given by

$$\Gamma_{ij} = \left(\frac{\partial\tilde{h}}{\partial\theta^i}\bigg|\frac{\partial\tilde{h}}{\partial\theta^j}\right), \tag{16}$$

where we define the inner product as

$$(\tilde{h}_1|\tilde{h}_2) \equiv 2\int_{f_{\mathrm{low}}}^{f_{\mathrm{upper}}} \frac{\tilde{h}_1\tilde{h}_2^* + \tilde{h}_1^*\tilde{h}_2}{S_n(f)}\, df, \tag{17}$$

where the "star" stands for complex conjugation, and $S_n(f)$ is the detector spectral noise density. The characteristic amplitude of a AGWB signal is given by $h = fS_h(f)$ [88].

Analyzing Eq. (10), we can see that even within GR theory prediction, possible different values of the parameter r_0 can induce corrections in amplitude. Thus, when considering a parameter estimation in modified gravity context, r_0 estimation can play an important role, since this
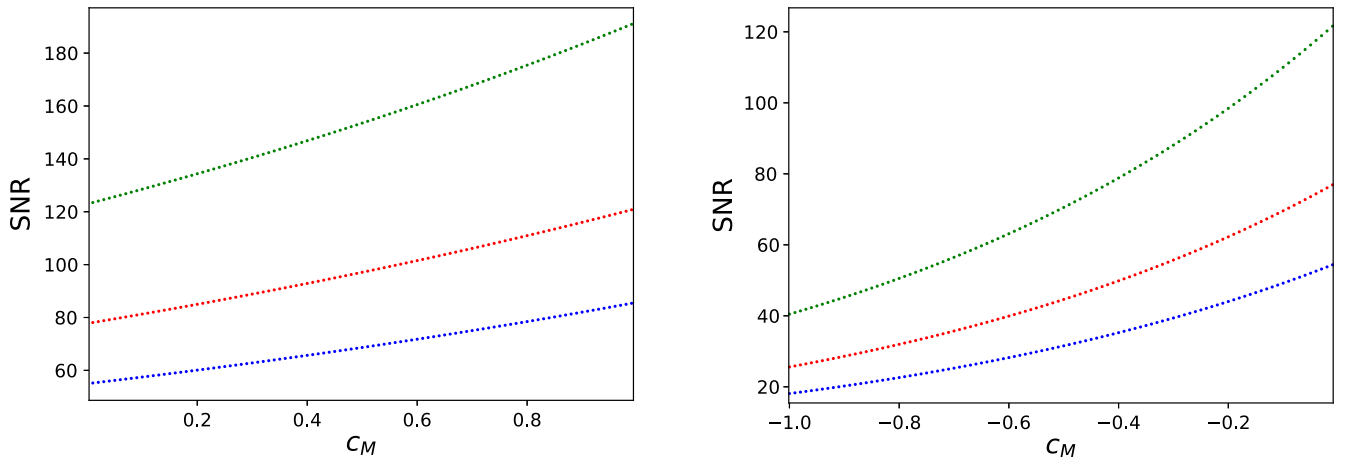

FIG. 4.   The same as in Figs. 2 and 3, but assuming Cosmic Explore noise power spectral density sensibility.
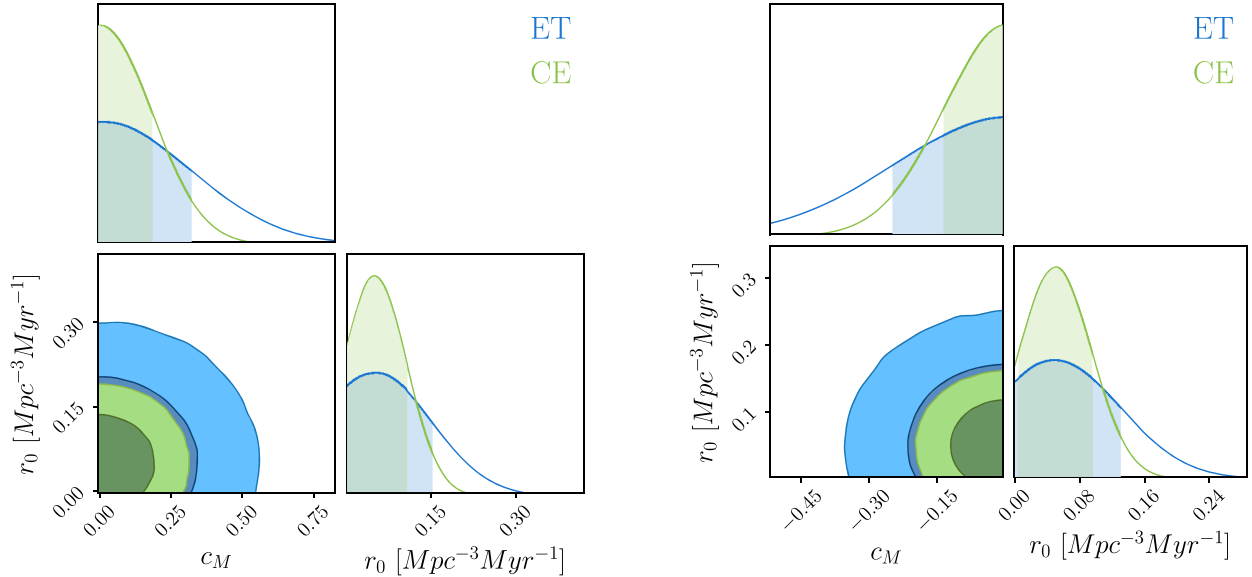
FIG. 5. Two-dimensional marginalized distributions of the free parameters c_M (amplitude of the running of the Planck mass) and r_0 (local BBH rate density) forecasted from the ET and CE sensitivity. Left panel: Theoretical framework given by scenario I. Right panel: Scenario II.

parameter can generate effects that can falsify possible real deviations due to α_M contribution. We are assuming that the merger rate follows the star formation rate. So, greater uncertainty comes from r_0 for BBH. As shown in [93], the BH redshift distribution can lead to uncertainties, which in principle, should affect parameter estimation analysis. Thus, when analyzing forecasts on c_M, also let us take r_0 for BBH as a baseline parameter. We keep r_0 to be 1 and 0.03 $Mpc^{-3}\,Myr^{-1}$, for BNS and BBH-NS, respectively, once that these systems can be well modeled from the star formation rate. Therefore, we can interpret the results below leading to an optimistic scenario.

Figure 5 shows the marginalized distributions for the parameter c_M and r_0 from the ET and CE sensitivity for both scenarios. For the ET forecast, we find $c_M < 0.27$ and $c_M > -0.20$, at 1σ confidence level (C.L.) in the framework of scenarios I and II, respectively. For the CE forecast, we find $c_M < 0.18$ and $c_M > -012$ at 1σ C.L., for scenarios I and II, respectively.

We can note that these bounds are of the accuracy matching the current measures [94–96]. Thus, we can expect that future constraints using real data from a possible AGWB in this band will impose strong limits on possible deviations from general relativity.

IV. FINAL REMARKS

The detectability of an isotropic and stochastic AGWB in the near future could open new ways to investigate fundamental physics, once many different astrophysical sources and physical properties contribute to the AGWB. The observational landscape is growing, and it covers a large range of frequencies where the AGWB is present.

With regard to gravity, two main means can modify the AGWB properties: (i) Generation mechanism of the signal by the sources; (ii) Modified propagation of the signal over cosmological volumes. In this work, we have investigated the latter in a parametric scenario given in terms of the Horndeski gravity. We find that the gravitational-wave detector of third-generation, like ET and CE, can detect the AGWB with significant SNR in our simple approximation, in particular from the perspectives of the CE experiment, in both, GR theory and modified gravity. Within the sensitivity of these instruments, a forecast analysis shows that the corrections on the amplitude of the running of the Planck mass can be bounded with the same precision as current measures. Therefore, an AGWB signal can put significant bounds in modified gravity models, and certainly some tight constraints in combination with other datasets.

Other interesting astrophysical sources can contribute to the AGWB in frequency well below the 1 Hz like supermassive black hole, primordial black hole, binaries white dwarfs, r-mode instability of neutron stars, as well as several phenomenological physical aspects in the early Universe. It may be interesting to investigate possible corrections on this type of signal, which is in LISA band frequency, as well as to consider a network of interferometric detectors like LISA + ET and/or LISA + CE, and explore the parameter space of some models to probe gravity bounds.

APPENDIX: THE HORNDESKI GRAVITY

In this Appendix, we briefly review the functions in Horndeski gravity, used in the main text of the paper. The Horndeski theories of gravity [81–83] are the most general Lorentz invariant scalar-tensor theories with second-order equations of motion. The Horndeski action reads

$$S = \int d^4x\sqrt{-g}\left[\sum_{i=2}^{5}\frac{1}{8\pi G}\mathcal{L}_i + \mathcal{L}_m\right], \qquad (A1)$$

$$\mathcal{L}_2 = G_2(\phi, X), \qquad (A2)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \qquad (A3)$$

$$\mathcal{L}_4 = -G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}], \qquad (A4)$$

$$\mathcal{L}_5 = -G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}[(\Box\phi)^3 + \qquad (A5)$$

$$2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi_{;\sigma}^{;\nu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi], \qquad (A6)$$

where the functions G_i (i runs over 2, 3, 4, 5) depend on ϕ and $X = -1/2\nabla^\nu\phi\nabla_\nu\phi$, with $G_{iX} = \partial G_i/\partial X$. For $G_2 = \Lambda$,

$G_4 = M_p^2/2$ and $G_3 = G_5 = 0$, we recover GR with a cosmological constant. The running of the Planck mass, α_M, is given by [97]

$$\alpha_M = \frac{1}{HM_*^2}\frac{dM_*^2}{dt}, \qquad (A7)$$

where

$$M_*^2 = 2(G_4 - 2XG_{4X} + XG_{5\phi} - \dot{\phi}HXG_{5X}) \qquad (A8)$$

is the effective Planck mass.

Another relevant quantity for the GW context is the tensor speed excess, α_T, which can be written as [97]

$$\alpha_T = \frac{2X(2G_{4X} - 2G_{5\phi} - (\ddot{\phi} - \dot{\phi}H)G_{5X})}{M_*^2}. \qquad (A9)$$

The functions α_M and α_T depend on the parameters of the theory and on the cosmological dynamics of the scalar field. On the other hand, the event GW170817 showed that the speed of GW, c_T, is very close to that of light for $z < 0.01$, that is, $|c_T/c - 1| < 10^{-15}$. Thus, we have $\alpha_{T0} \simeq 0$, leading to consider $G_{4X} = G_{5\phi} = G_{5X} \simeq 0$. Thus, under that consideration, from Eq. (A8), we can write

$$M_*^2 = 2G_4. \qquad (A10)$$

[1] The LIGO Scientific and the Virgo Collaborations, Phys. Rev. Lett. **116**, 061102 (2016).
[2] The LIGO Scientific and the Virgo Collaborations, Phys. Rev. Lett. **119**, 161101 (2017).
[3] The LIGO Scientific and the Virgo Collaborations, Astrophys. J. Lett. **848**, L13 (2017).
[4] E. S. Phinney, arXiv:astro-ph/0108028.
[5] N. Christensen, Rep. Prog. Phys. **82**, 016903 (2019).
[6] T. Regimbau and V. Mandic, Classical Quantum Gravity **25**, 184018 (2008).
[7] D. Meacher, E. Thrane, and T. Regimbau, Phys. Rev. D **89**, 084063 (2014).
[8] X. J. Zhu, E. Howell, T. Regimbau, D. Blair, and Z. H. Zhu, Astrophys. J. **86**, 739 (2011).
[9] P. A. Rosado, Phys. Rev. D **84**, 084004 (2011).
[10] S. Marassi, R. Schneider, G. Corvino, V. Ferrari, and S. P. Zwart, Phys. Rev. D **84**, 124037 (2011).
[11] C. Wu, V. Mandic, and T. Regimbau, Phys. Rev. D **85**, 104024 (2012).
[12] J. D. Romano and N. J. Cornish, Living Rev. Relativity **20**, 2 (2017).

[13] X. J. Zhu, E. J. Howell, D. G. Blair, and Z. H. Zhu, Mon. Not. R. Astron. Soc. **431**, 882 (2013).
[14] G. Cusin, I. Dvorkin, C. Pitrou, and J. P. Uzan, Phys. Rev. D **100**, 063004 (2019).
[15] I. Dvorkin, J. P. Uzan, E. Vangioni, and J. Silk, Phys. Rev. D **94**, 103011 (2016).
[16] K. Nakazato, Y. Niino, and N. Sago, Astrophys. J. **832**, 146 (2016).
[17] E. F. D. Evangelista and J. C. N. Araujo, Braz. J. Phys. **44**, 824 (2014).
[18] Ch. Filloux, J. A. de Freitas Pacheco, F. Durier, and J. C. N. de Araujo, Int. J. Mod. Phys. D **20**, 2399 (2011).
[19] T. Regimbau, Res. Astron. Astrophys. **11**, 369 (2011).
[20] M. Isi and L. C. Stein, Phys. Rev. D **98**, 104025 (2018).
[21] C. M. F. Mingarelli, S. R. Taylor, B. S. Sathyaprakash, and W. M. Farr, arXiv:1911.09745.
[22] A. Sharma and J. Harms, arXiv:2006.16116.
[23] G. C. Herrera, O. Contigiani, and V. Vardanyan, arXiv:1910.08353.
[24] A. C. Jenkins, M. Sakellariadou, T. Regimbau, and E. Slezak, Phys. Rev. D **98**, 063501 (2018).

[25] G. Cusin, I. Dvorkin, C. Pitrou, and J. P. Uzan, arXiv: 1904.07757.

[26] T. Namikawa, arXiv:2007.04359.

[27] The LIGO Scientific and the Virgo Collaborations, Phys. Rev. D **100,** 061101 (2019).

[28] The LIGO Scientific and the Virgo Collaborations, Phys. Rev. Lett. **120,** 091101 (2018).

[29] M. Maggiore *et al.*, J. Cosmol. Astropart. Phys. 03 (2020) 050.

[30] D. Reitze *et al.*, Bull. Am. Astron. Soc. **51,** 035 (2019).

[31] P. Amaro-Seoane *et al.*, arXiv:1702.00786.

[32] C. Caprini, D. G. Figueroa, R. Flauger, G. Nardini, M. Peloso, M. Pieroni, A. Ricciardone, and G. Tasinato, J. Cosmol. Astropart. Phys. 11 (2019) 017.

[33] D. Huterer and D. L. Shafer, Rep. Prog. Phys. **81,** 016901 (2018).

[34] S. Capozziello, R. D'Agostino, and O. Luongo, Int. J. Mod. Phys. D **28,** 1930016 (2019).

[35] M. Ishak, Living Rev. Relativity **22,** 1 (2019).

[36] M. Ballardini, M. Braglia, F. Finelli, D. Paoletti, and A. A. Starobinsky, arXiv:2004.14349.

[37] R. C. Nunes, J. Cosmol. Astropart. Phys. 05 (2018) 052.

[38] M. Raveri, Phys. Rev. D **101,** 083524 (2020).

[39] R. D'Agostino and R. C. Nunes, Phys. Rev. D **101,** 103505 (2020).

[40] E. Berti *et al.*, Classical Quantum Gravity **32,** 243001 (2015).

[41] The LIGO Scientific and the Virgo Collaborations, Phys. Rev. D **100,** 104036 (2019).

[42] C. J. Haster, arXiv:2005.05472.

[43] J. M. Ezquiaga and M. Zumalacárregui, Phys. Rev. Lett. **119,** 251304 (2017).

[44] P. Creminelli and F. Vernizzi, Phys. Rev. Lett. **119,** 251302 (2017).

[45] T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller, and I. Sawicki, Phys. Rev. Lett. **119,** 251301 (2017).

[46] L. Amendola, M. Kunz, I. D. Saltas, and I. Sawicki, Phys. Rev. Lett. **120,** 131101 (2018).

[47] A. Nishizawa, Phys. Rev. D **97,** 104037 (2018).

[48] E. Belgacem, Y. Dirian, S. Foffa, and M. Maggiore, Phys. Rev. D **98,** 023510 (2018).

[49] A. Nishizawa and S. Arai, Phys. Rev. D **99,** 104038 (2019).

[50] I. D. Saltas, I. Sawicki, L. Amendola, and M. Kunz, Phys. Rev. Lett. **113,** 191101 (2014).

[51] C. Deffayet and K. Menou, Astrophys. J. **668,** L143 (2007).

[52] L. Lombriser and A. Taylor, J. Cosmol. Astropart. Phys. 03 (2016) 031.

[53] S. Mastrogiovanni, D. Steer, and M. Barsuglia, arXiv: 2004.01632.

[54] R. D'Agostino and R. C. Nunes, Phys. Rev. D **100,** 044041 (2019).

[55] R. C. Nunes, M. E. S. Alves, and J. C. N. de Araujo, Phys. Rev. D **100,** 064012 (2019).

[56] X. Gao and X. Y. Hong, Phys. Rev. D **101,** 064057 (2020).

[57] G. Calcagni, S. Kuroyanagi, S. Marsat, M. Sakellariadou, N. Tamanini, and G. Tasinato, J. Cosmol. Astropart. Phys. 10 (2019) 012.

[58] E. Belgacem, Y. Dirian, S. Foffa, E. J. Howell, M. Maggiore, and T. Regimbau, J. Cosmol. Astropart. Phys. 08 (2019) 015.

[59] S. J. Jin, D. Z. He, Y. Xu, J. F. Zhang, and X. Zhang, J. Cosmol. Astropart. Phys. 03 (2020) 051.

[60] A. Bonilla, R. D'Agostino, R. C. Nunes, and J. C. N. de Araujo, J. Cosmol. Astropart. Phys. 03 (2020) 015.

[61] W. J. Wolf and M. Lagos, Phys. Rev. Lett. **124,** 061101 (2020).

[62] M. Lagos and H. Zhu, J. Cosmol. Astropart. Phys. 06 (2020) 061.

[63] P. T. H. Pang, R. K. L. Lo, I. C. F. Wong, T. G. F. Li, and C. Van Den Broeck, Phys. Rev. D **101,** 104055 (2020).

[64] X. Liu, V. F. He, T. M. Mikulski, D. Palenova, C. E. Williams, J. Creighton, and J. D. Tasson, Phys. Rev. D **102,** 024028 (2020).

[65] M. Du, W. Yang, L. Xu, S. Pan, and D. F. Mota, Phys. Rev. D **100,** 043535 (2019).

[66] W. Yang, S. Vagnozzi, E. Di Valentino, R. C. Nunes, S. Pan, and D. F. Mota, J. Cosmol. Astropart. Phys. 07 (2019) 037.

[67] E. Belgacem *et al.*, J. Cosmol. Astropart. Phys. 07 (2019) 024.

[68] E. Belgacem, Y. Dirian, A. Finke, S. Foffa, and M. Maggiore, J. Cosmol. Astropart. Phys. 11 (2019) 022.

[69] E. Belgacem, Y. Dirian, A. Finke, S. Foffa, and M. Maggiore, J. Cosmol. Astropart. Phys. 04 (2020) 010.

[70] A. Maselli, S. Marassi, and M. Branchesi, Astron. Astrophys. **635,** A120 (2020).

[71] E. Howell, D. Coward, R. Burman, D. Blair, and J. Gilmore, Mon. Not. R. Astron. Soc. **351,** 1237 (2004).

[72] D. Coward, R. Burman, and D. Blair, Mon. Not. R. Astron. Soc. **324,** 1015 (2001).

[73] *Gravitational Radiation in 300 Years of Gravitation*, edited by K. Thorne, S. Hawking, and W. Israel (Cambridge University Press, Cambridge, England, 1987), p. 330.

[74] J. M. Ezquiaga and M. Zumalacárregui, Front. Astron. Space Sci. **5,** 44 (2018).

[75] N. Yunes and F. Pretorius, Phys. Rev. D **80,** 122003 (2009).

[76] N. Cornish, L. Sampson, N. Yunes, and F. Pretorius, Phys. Rev. D **84,** 062003 (2011).

[77] Z. Carson and K. Yagi, Phys. Rev. D **101,** 044047 (2020).

[78] A. Saffer and K. Yagi, Phys. Rev. D **102,** 024001 (2020).

[79] A. Maselli, S. Marassi, V. Ferrari, K. Kokkotas, and R. Schneider, Phys. Rev. Lett. **117,** 091102 (2016).

[80] A. Saffer and K. Yagi, Phys. Rev. D **102,** 024001 (2020).

[81] G. W. Horndeski, Int. J. Theor. Phys. **10,** 363 (1974).

[82] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, Phys. Rev. D **84,** 064039 (2011).

[83] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Prog. Theor. Phys. **126,** 511 (2011).

[84] J. Kennedy, L. Lombriser, and Andy Taylor, Phys. Rev. D **98,** 044051 (2018).

[85] M. Denissenya and E. V. Linder, J. Cosmol. Astropart. Phys. 11 (2018) 010.

[86] E. V. Linder, J. Cosmol. Astropart. Phys. 03 (2018) 005.

[87] B. E. Robertson and R. S. Ellis, Astrophys. J. **744,** 95 (2012).

[88] M. Maggiore, Phys. Rep. **331,** 283 (2000).

[89] https://dcc.ligo.org/LIGO-T1800044/public.

[90] K. G. Arun, B. R. Iyer, B. S. Sathyaprakash, and P. A. Sundararajan, Phys. Rev. D **71,** 084008 (2005).

[91] H. S. Cho and C. H. Lee, arXiv:1310.3050.

[92] E. Berti, A. Buonanno, and C. M. Will, Phys. Rev. D **71,** 084025 (2005).

[93] M. Fishbach, D. E. Holz, and W. M. Farr, Astrophys. J. Lett. **863,** L41 (2018).

[94] M. Brush, E. V. Linder, and M. Zumalacarregui, J. Cosmol. Astropart. Phys. 01 (2019) 029.

[95] C. D. Kreisch and E. Komatsu, J. Cosmol. Astropart. Phys. 12 (2018) 030.

[96] J. Noller, Phys. Rev. D **101,** 063524 (2020).

[97] E. Bellini and I. Sawicki, J. Cosmol. Astropart. Phys. 07 (2014) 050.