Rendezvous Maneuvers with Final Approach by *R-bar* to Target Satellite in Leo Orbit

William Reis Silva*
Ijar Milagre da Fonseca
Space Mechanics and Control Division
INPE - Brazilian Institute for Space Research
São José dos Campos - SP, Brazil
*reis.william@gmail.com,
ijar@dem.inpe.br

Maria Cecília Zanardi
Group of Orbital Dynamics and Planetology
UNESP – São Paulo State University
Guaratinguetá – SP, Brazil
cecilia@feg.unesp.br

***Abstract*—This work aims to realize a study of rendezvous maneuvers between satellite atmospheric reentry (SARA), regarded as a chaser vehicle, and a target vehicle in permanent LEO orbit. A theoretical study of the modeling of the dynamic equations of relative motion, proposed by Hill-Clohessy-Wiltshire, and a study of control system for such rendezvous maneuver is performed, considering the dynamics of the actuators negligible. The technical control of multivariable Linear Quadratic Regulator (LQR) is used as a method to design the control system making use of the computational tool MATLAB. In simulations of rendezvous maneuvers, it was used as a strategy, approaches between the chaser and target vehicles by R-bar and are analyzed the temporal evolution of the position, velocity and the force needs to the control with the intention evaluate the response velocity of such control system. With the graphical visualization of 3D maneuvers, it is observed that the LQR control technique applied to the problem plant was adequate and with satisfactory results. Stresses those, to design, understand and manipulate controllers for increasingly accurate orbital rendezvous maneuvers can make the difference between success or failure of a project that certainly involves high budget.**

Keywords—Orbital Dynamics; Rendezvous; Hill-Clohessy-Wiltshire Equation; LQR Controller.

I. INTRODUCTION

Recently, due to the high risk and cost of sending astronauts into space, a new trend is emerging and being implemented to carry out rendezvous and docking operations. These operations are migrating to run in a fully autonomous, that is, without human interference in place. However, to develop systems with this level of self-shading, one should take into account the need for innovative technologies, accurate and robust.

The process of rendezvous, refers to the orbital maneuvers responsible for aligning the flight of two spacecraft, synchronizing their orbital elements and bringing them together in the same orbital plane. At the end of this operation, both vehicles will be paired.

The chaser vehicle should come to engage with the target withlinear and angular relative velocity zero or close to zero. To make this possible, the vehicle chaser must reduce your velocity of approach and also synchronize your attitude with the vehicle target [1].

In this work the equations of relative motion between the chaser vehicle and the target vehicle are described by the Hill-Clohessy-Wiltshire equations, where is considered the target vehicle into a circular orbit. The control method used for rendezvous maneuvers is the Linear Quadratic Regulator (LQR).

II. THE HILL-CLOHESSY-WILTSHIRE EQUATIONS

The methodology used to calculate the guiding of chaser vehicle during rendezvous maneuvers are introduced by Clohessy-Wiltshire [2], where is regarded as a reference axis system composed as follows: Its origin is centered on the center of mass of the target vehicle for all other coordinates are represented in relation to this vehicle. Thus, when the chaser vehicle to find the origin of the reference system (while a certain offset so there are no collisions) it will have effected the maneuver coupling.

This coordinate system, in red, is shown in Figure 1 below.

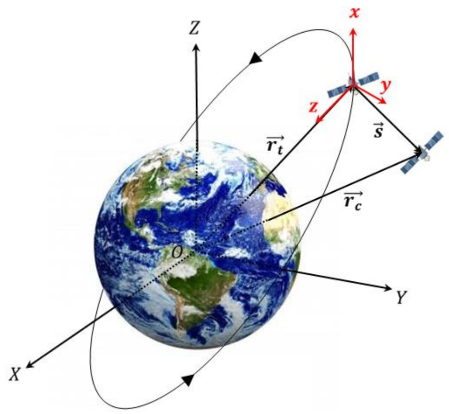

Fig. 1. Chaser vehicle and target vehicle orbits. Reference system centered on the target vehicle.

Besides its center is contained in the target satellite and its reference plane being the plane of the local horizon of this vehicle, its axes have the following guidance:

- **X-Axis**: in the same direction and orientation the orbital velocity vector (*V-bar*).
- **Y-Axis**: normal to the orbit, with the opposite direction of the orbital angular momentum vector (*H-bar*).
- **Z-Axis**: complete de system, oriented in the radial direction, perpendicular to the plane of the horizon, nadir direction (*R-bar*).

Knowing that, $\vec{r_t}$ and $\vec{r_c}$ are the distances from the center of mass of the Earth to the vehicles, target and chaser respectively and $\vec{s}$ the relative distance between the vehicles.

$$\vec{s} = \vec{r_c} - \vec{r_t} \tag{1}$$

The general equation of motion of the body under the influence of a central force is given by Newton's law of gravitation [3].

$$m_c\ddot{\vec{r_c}} = -\mu \frac{m_c}{{r_c}^3}\vec{r_c} + \vec{F}_{cng} \tag{2}$$

$$m_t\ddot{\vec{r_t}} = -\mu \frac{m_t}{{r_t}^3}\vec{r_t} + \vec{F}_{tng} \tag{3}$$

being, m_c and m_t the masses of chaser and target vehicles, respectively; $\vec{F}_{cng}$ and $\vec{F}_{tng}$ are non-gravitational forces on the chaser and target vehicle, respectively. These forces include external disturbances and the performance of control.

Substituting (2) and (3) in the second derivative of (1), considering only the control is performed only by the propulsion system in this chaser vehicle and disregarding the influence of external disturbance on the system ($\vec{F}_{tng} = \vec{0}$ and $\vec{F}_{cng} = \vec{u_c}$), then:

$$\ddot{\vec{s}} = -\frac{\mu}{{r_c}^3}\vec{r_c} + \frac{\mu}{{r_t}^3}\vec{r_t} + \frac{\vec{u_c}}{m_c} \tag{4}$$

Linearizing the gravitational force on the chaser vehicle around the position of the target vehicle by means of a Taylor series expansion up to the first order, then [4].

$$\ddot{\vec{s}} = -\frac{\mu}{{r_c}^3}M\vec{s} + \frac{\vec{u_c}}{m_c} \tag{5}$$

where

$$M = \begin{bmatrix} 1 - 3\frac{{r_x}^2}{{r_t}^2} & -3\frac{r_x r_y}{{r_t}^2} & -3\frac{r_x r_z}{{r_t}^2} \\ -3\frac{r_x r_y}{{r_t}^2} & 1 - 3\frac{{r_y}^2}{{r_t}^2} & -3\frac{r_y r_z}{{r_t}^2} \\ -3\frac{r_x r_z}{{r_t}^2} & -3\frac{r_z r_y}{{r_t}^2} & 1 - 3\frac{{r_z}^2}{{r_t}^2} \end{bmatrix} \tag{6}$$

The objective is to represent the motion of the chaser vehicle in the referential of target vehicle, but this is a non-inertial reference. Thus this relationship is given by [5]:

$$\ddot{\vec{s}} = \ddot{\vec{s^*}} + \vec{\omega} \times (\vec{\omega} \times \vec{s^*}) + 2\vec{\omega} \times \dot{\vec{s^*}} + \dot{\vec{\omega}} \times \vec{s^*} \tag{7}$$

Substituting (7) in (5), found:

$$\ddot{\vec{s^*}} + \vec{\omega} \times (\vec{\omega} \times \vec{s^*}) + 2\vec{\omega} \times \dot{\vec{s^*}} + \dot{\vec{\omega}} \times \vec{s^*} = -\frac{\mu}{{r_c}^3}M\vec{s} + \frac{\vec{u_c}}{m_c} \tag{8}$$

Knowing that, $\vec{s^*} = [x \quad y \quad z]^T$ and representing $\vec{\omega}$ and $\vec{r_t}$ in *VHR-bar* with:

$$\vec{\omega} = \begin{bmatrix} 0 \\ -\omega \\ 0 \end{bmatrix} \tag{9}$$

$$\vec{r_t} = \begin{bmatrix} 0 \\ 0 \\ r_t \end{bmatrix} \tag{10}$$

Substituting (9) and (10) in the (8) and evolving calculations, we obtain the equations of motion of the problem [6].

$$\ddot{x} = 2\omega\dot{z} + \omega^2 x + \dot{\omega} z - \frac{\mu}{{r_t}^3}x + \frac{1}{m_c}u_x \tag{11}$$

$$\ddot{y} = -\frac{\mu}{{r_t}^3}y + \frac{1}{m_c}u_y \tag{12}$$

$$\ddot{z} = -2\omega\dot{x} + \omega^2 z - \dot{\omega} x + 2\frac{\mu}{{r_t}^3}z + \frac{1}{m_c}u_z \tag{13}$$

For the special case in which the target vehicle is in a circular orbit r_t and ω are constant, thus, according to Bate et. al. [7].

$$\omega^2 = \frac{\mu}{{r_t}^3} \tag{14}$$

Thus, using (14) in (11), (12) and (13) the equations of motion are reduced in an easy way known as Hill-Clohessy-Wiltshire Equations.

$$\ddot{x} = 2\omega\dot{z} + \frac{1}{m_c}u_x \tag{15}$$

$$\ddot{y} = -\omega^2 y + \frac{1}{m_c}u_y \tag{16}$$

$$\ddot{z} = -2\omega\dot{x} + 3\omega^2 z + \frac{1}{m_c}u_z \tag{17}$$

These are the equations of motion used in this work, since the great majority of the rendezvous maneuver is performed with the target vehicle on a circular orbit.

It is important to note that the (15) and (17) are coupled to each other, so it will be convenient to separate the movements: outside the orbital plane *H-bar* (component y) and in the orbital plane *RV-bar* (component xz).

Thus, the strategy to control the system separates into two subsystems, one SISO (*H-bar*) and other MIMO (*RV-bar*).

III. REPRESENTATION IN SPACE OF STATES

A. SISO model for the motion on H-bar

Here the subsystem is presented with two states containing one input and one output.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega^2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_c}\end{bmatrix}u_y \qquad (18)$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad (19)$$

where x_1 and x_2 represent the position y and the velocity $\dot{y}$, respectively.

B. MIMO model for the motion on RV-bar

Here the subsystem is presented with four states containing two input and two output.

$$\begin{bmatrix}\dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 2\omega\\ 0 & 3\omega^2 & -2\omega & 0\end{bmatrix}\begin{bmatrix}x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ \frac{1}{m_c} & 0\\ 0 & \frac{1}{m_c}\end{bmatrix}\begin{bmatrix}u_y\\ u_y\end{bmatrix} \qquad (20)$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} \qquad (21)$$

where $x_3, x_4,\ x_5$ and x_6 represent the position, x,z the velocity $\dot{x}$ and $\dot{z}$, respectively

IV. DATA USED

In this section are presented the physical and orbital characteristics of chaser satellite, such data are used in the simulations in this work.

A. Orbital platform recoverable: SARA

The Atmospheric Reentry Satellite (SARA) it is an Orbital Platform Recoverable for scientific applications of approximately $350kg$ of the mass, currently under development at the Aeronautics and Space Institute (IAE), of the CTA, in São José dos Campos, SP [8].

This satellite will be launched to a low Earth orbit (LEO) with $300km$ of the altitude in equatorial orbit and about after 10 days in orbit the spacecraft will perform a maneuver controlled atmospheric reentry. After reentry, the vehicle should be recovered for subsequent reuse your module reentrable [9].

The SARA is being designed for the purpose of scientific applications and will comprise a vehicle with a payload of $55kg$ containing small scientific and technological experiments. Some of these experiments may need to be serviced or serviced in orbit to be successful throughout the duration of the mission.

Thus, SARA should incorporate couplings to be used by another thread orbital permanent (target vehicle). Consequently, the pursuer must perform a series of rendezvous and docking operations in order to complete the mission objectives.

The configurations of SARA are shown in Table 1 and 2 below.

TABLE I. PHYSICAL SETTING OF ATMOSPHERIC REENTRY SATELLITE (SARA).

Physical characteristics	Values
Mass (m_c)	350 kg

TABLE II. ORBITAL SETTING OF ATMOSPHERIC REENTRY SATELLITE (SARA).

Orbital parameters	Values
Altitude (h)	300 km
Eccentricity (e)	≅ 0
Inclination (I)	0°
Angular velocity (ω)	$1{,}16 \times 10^{-3}$ rad/s

V. CONTROL METHOD

The control technique used in this paper to design the optimal controller for the rendezvous maneuver is the Linear Quadratic Regulator (LQR). This technique will be used with the aid of computational tool *MATLAB®*.

A. Linear Quadratic Regulator

In the case of Linear Quadratic Regulator (LQR), we assume that all states are available for feedback [10]. Consider the system dynamics given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (22)$$
$$y(t) = Cx(t)$$

where A a matrix $n \times n$, B a matrix $n \times m$, C a matrix $k \times n$, $x(t)$ the state variables $n \times 1$ and $u(t)$ the control signal $m \times 1$.

To solve this problem of regulating the output signal must optimize the cost functional defined as:

$$J = \int_{t_0}^{\infty}(x'Qx + u'Ru)dt \qquad (23)$$

where, Q and R are matrices semi-positive definite, symmetrical real and generally diagonal.

If Q is greater than R, the system response improves, but the energy exerted by the control u possibility will increase. If R is larger than the Q, the system response worsens because the energy exerted by the control u is more penalized.

Using the Pontriagin principle [11], can be written the Hamiltonian system, such as:

$$H = \frac{1}{2}(x'Qx + u'Ru) + \lambda'(Ax + Bu) \qquad (24)$$

The minimum principle establishes that the optimal control and trajectories of the states must meet the following three conditions: the state equation, co-state equation and stationary equation [11].

Evolving calculations is the following equations:

$$\dot{x} = Ax + Bu \tag{25}$$

$$-\dot{\lambda} = Qx + A'\lambda \tag{26}$$

$$u^* = -R^{-1}B'\lambda \tag{27}$$

where u^* is the optimal control signal.

The boundary value problem need not necessarily be solved in this way, it is considered that.

$$\lambda = P(t)x \tag{28}$$

The formulation presented for the solution of the LQR problem is known as infinite time which results in a state feedback controller with linear time-variant, given by:

$$u(t) = -K(t)x(t) \tag{29}$$

where

$$K(t) = R^{-1}B'P(t) \tag{30}$$

In the situation where the process to be controlled resides in an infinite time interval provides a simple solution to find the LQR controller [11]. If the pair (A, B) is controllable, the pair (A, C) is observable and the matrices A, B, R and Q s are constants, when t_f ends to infinity, it follows that $P(t)$ and $K(t)$ become constant matrices P and$K$.

Substituting (29) in (22) has control of dynamic closed-loop.

$$\dot{x}(t) = (A - BK)x(t) \tag{31}$$

$$y(t) = Cx(t)$$

VI. RESULTS OBTAINED

In this section, we present the results obtained for the LQR controller with the plant simulations using *MATLAB®* tool. The system is divided into two subsystems, one MIMO representing movement in *RV*-bar and the other SISO representing the movement in *H-bar*.

In the simulations it was found that the target vehicle was in a circular orbit LEO permanent and the vehicle tracker should maneuvers to rendezvous approaching the target vehicle by *R-bar*.

A. LQR controller for SISO system

The states of this space are: the position (y) and the velocity $(\dot{y})$.

The control signal (u_y) is a force applied to the system in y and $\dot{y}$. The system output is the end position.

To validate the controller was applied to a situation fictitious satellite SARA, chaser vehicle, and this should make a rendezvous maneuver to the target vehicle.

The Table 3, shows the initial position and velocity on *H-bar* of chaser vehicle.

TABLE III. INITIAL POSITION AND VELOCITY IN H-BAR OF SARA RELATIVE TO TARGET VEHICLE

Parameters	Values
y_0	$5{,}175$ m
$\dot{y}_0$	$3{,}1$ m/s

Figure 2, below, shows the temporal evolution of the position, velocity and the control force for the LQR controller.

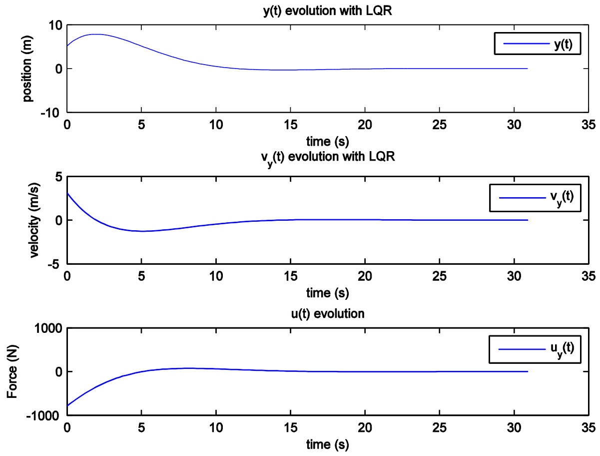

Fig. 2. Behavior of the position, velocity and control force exerted on H-bar.

The chaser vehicle reaching a peak in y of $7{,}82m$ in $1{,}97s$, reaching the desired position ($y = 0m$) and stabilizing at 21,22s. The peak force exerted by the control is at the beginning of the process, the module achieves $780N$ on y-axis.

B. LQR controller for MIMO system

The states of this space are: the position (x, z) and the velocity $(\dot{x}, \dot{z})$.

The control signal (u_x, u_z) are a force applied to the system in $x, z, \dot{x}, \dot{z}$. The system output is the end position.

As previously done for the SISO system, to validate the controller was applied to a situation fictitious satellite SARA, chaser vehicle, and this should make a rendezvous maneuver to the target vehicle by an approach in *R-bar*.

The Table 4, shows the initial position and velocity on *RV-bar* of chaser vehicle.

TABLE IV. INITIAL POSITION AND VELOCITY IN RV-BAR OF SARA RELATIVE TO TARGET VEHICLE

Parameters	Values
x_0	0,175 m
z_0	15,305 m
$\dot{x}_0$	3,27 m/s
$\dot{z}_0$	−3,05 m/s

Figure 3, below, shows the temporal evolution of the position, velocity and the control force for the LQR controller.

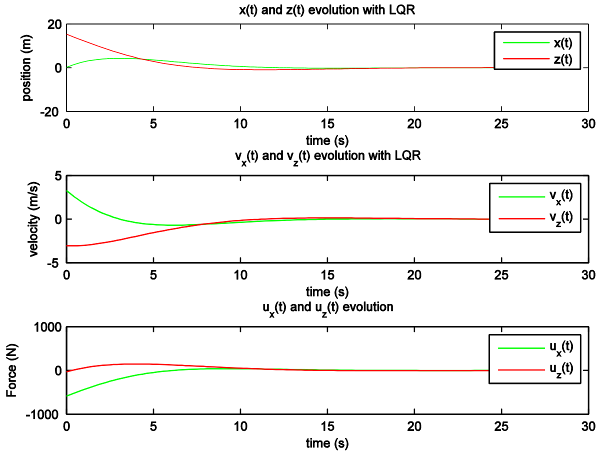

Fig. 3. Behavior of the position, velocity and control force exerted on RV-bar.

The chaser vehicle reaching a peak in x of $4{,}28m$ in $3{,}11s$ and a peak in z of $-0{,}97m$ in$11{,}42s$, reaching the desired position ($x = z = 0m$) and stabilizing at $18{,}68s$. The peak force exerted by the control is at the beginning of the process, the module achieves $583{,}6N$ on x-axis and $30{,}69N$ on z-axis.

The Figures (4) and (5) represent the motion of the chaser relative to the target *VR-bar* and *H-bar* versus $t(s)$, respectively.

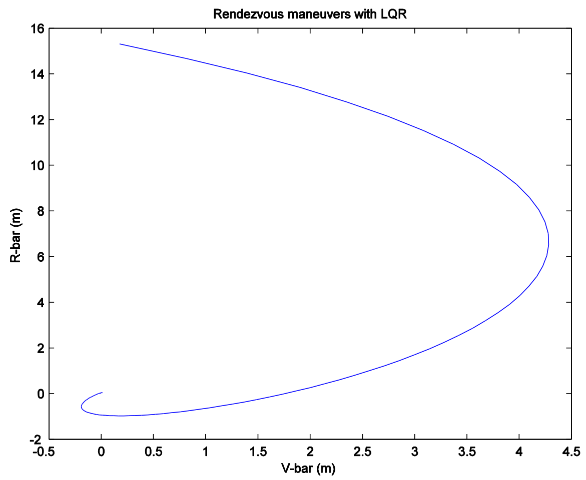

Fig. 4. Motion in *RV-bar*, R-bar approach with LQR control.

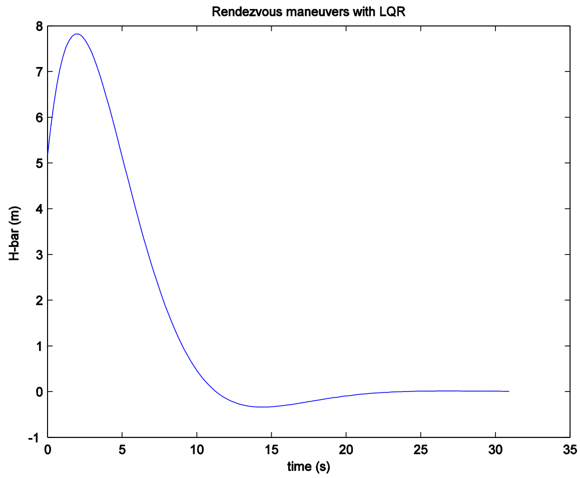

Fig. 5. Motion in *H-bar* versus t(s), R-bar approach with LQR control.

To facilitate the visualization, the Figure 6 represents the chaser vehicle motion relative to target vehicle on *VHR-bar*.

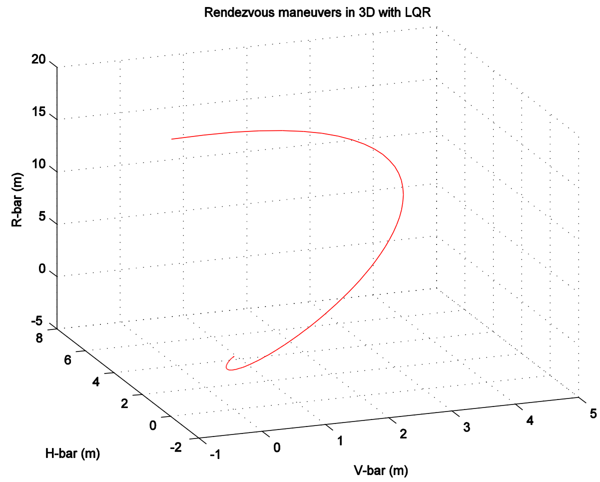

Fig. 6. Motion in *VHR-bar* with LQR control, *R-bar* approach.

VII. CONCLUSION

In this paper we present a simulation analysis of rendezvous maneuvers between the Atmospheric Reentry Satellite (SARA) in a target vehicle in a LEO orbit with the LQR controller.

The LQR control technique applied in the plant of the problem, which was separated into two subsystems being one SISO, to represent motion in H-bar and other MIMO, to represent motion in RV-bar, showed satisfactory results possible design specifications.

For the rendezvous maneuver, simulated a situation in which a stalker as a vehicle approaching from the *R-bar* target vehicle. Recalling that the *V-bar* approach is considered the safest and generally of lower energy consumption since, when the spacecraft approaches the target the "front", that is, in the direction of the velocity vector, she performs "jumps" successive toward the target through his orbital survey.

So when its orbit increases, its orbital velocity decreases, thus increasing its velocities relative to the target (in lower orbit, therefore faster). This causes the tracker approaches the target successively at low relative velocities, also requiring low fuel consumption.

The LQR control technique is easy to apply; both in SIMO systems as in MIMO systems, but one should remember that she carries some limitations. This technique does not take into account dynamic noise and uncertainties in the measurements of the states, moreover it is necessary to know all this states plan to implement it, and need to load the chaser satellite with sensors to measure position and velocity with respect to the target vehicle.

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REFERENCES

[1] J. Gentina, "Desenvolvimento e Simulação da LógicaEmbarcada paraManobras de Rendezvous e Docking da Plataforma Orbital Recuperável SARA". Dissertação de Mestrado, Instituto Nacional dePesquisas Espaciais (INPE), São José dos Campos, 2009.

[2] W. H. Clohessy and R. S.Wiltshire, "Terminal Guidance System for Satellite Rendezvous". Journal of the Aerospace Sciences, v. 27, n. 9, p. 653-659. 1960.

[3] I. Newton, "Mathematical Principles of Natural Philosophy". 1713.

[4] W. Fehse, "Automated Rendezvous and Docking of Spacecraft". New York, USA: Cambridge University Press. p. 517. 2003.

[5] N. A. Lemos, "Mecânica Analítica". São Paulo, Brasil: Editora Livraria da Física, p. 386, 2.ed. 2007.

[6] G. A. Junior, "Rendezvous with a Non-cooperating Target". Thesis (PH.D). Universitat Bremen, Bremen. 2011.

[7] R. R Bate; D. D. Mueller and J. E. White, "Fundamentals of Astrodynamics". New York,USA: Dover. p. 455. 1971.

[8] P. M. Junior, "Design Aspects of the Recoverable Orbital Platform SARA". Chilean Congress of Mechanical Engineering, Unesp, Concepcion, p. 8. 2008.

[9] N. Seito, I. M. da Fonseca and P. M. Junior, "Sara Orbital Autonomous Rendezvous and Docking". AScientific and Technological Challenge. Brazilian Conference on Dynamics, Control and their Applications, Unesp, São José do Rio Preto. 2007.

[10] E. R. Pinheiro, "Aplicaçao do Controlador misto H_2/H_∞no Controle da Atitude de um Microssatélite na Presença de Incertezas Estruturadas e não Estruturadas", p. 60 p. Proposta de Dissertação de Mestrado, Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos. 2012.

[11] D. E. Kirk, "Optimal Control Theory: An Introduction". New York, USA: Dover Publications. 3.ed. 2004.