Probing a cosmological model with a $\Lambda = \Lambda_0 + 3\beta H^2$ decaying vacuum

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In this work we study the evolution of matter-density perturbations for an arbitrary $\Lambda(t)$ model and specialize our analysis to the particular phenomenological law $\Lambda = \Lambda_0 + 3\beta H^2$. We study the evolution of the cosmic star formation rate in this particular dark energy scenario and, by constraining the β parameter using both the age of the Universe and the cosmic star formation rate curve, we show that it leads to a reasonable physical model for $\beta \lesssim 0.1$.

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I. INTRODUCTION

There is plenty of observational evidence that the Universe is currently undergoing an accelerated expansion [1]. According to the Friedmann model, ordinary matter cannot bring about such cosmic acceleration; a possible way out of this unsettling picture is provided by the introduction of a fluid with negative pressure to the cosmic inventory, the so-called dark energy (DE). The simplest DE candidate is the cosmological constant Λ (CC for short), added to the right-hand side of the Einstein field equations to play the role of such a "fluid" with negative pressure. Thus, the "traditional" cold dark matter-based cosmology of the early 1990s, together with a CC (henceforth called the ΛCDM cosmology), turned out to be the standard model for describing the dynamics of the Universe, for it fits the latest observational results with a very good accuracy. However, in spite of this success, the ΛCDM model has some shortcomings (see [2] for a discussion), the most severe being the so-called *fine-tuning problem* or *the old CC problem*. This issue arises from the fact that the present-time observed value for the vacuum energy density, $\rho_\Lambda = \Lambda c^2/(8\pi G) \sim 10^{-47}$ GeV^4, is more than 100 orders of magnitude smaller than the value found by using the methods of quantum field theory ($\sim 10^{71}$ GeV^4) [3].

In the last decades, many attempts have been made to tackle these issues. In particular, models with time-dependent vacuum energy density seem to be promising, since the corresponding vacuum energy density could have a high enough value to drive inflation at the very early Universe, decaying along the expansion history to its small value observed today. This process can be implemented with the introduction of scalar fields, as in the case of *quintessence* [4], for example; another way to achieve this goal is through a phenomenological time-dependent cosmological term $\Lambda(t)$ [5–10]. There has been lately a strong interest in such a class of models, particularly on those arising from the quantum field theory methods [see [11] for a discussion and [12] for a review of $\Lambda(t)$ models arising in the context of quantum field theory in curved space-time]. In this approach, the time-dependent cosmological term implies a coupling with another cosmic component, leading to either particle production or an increase in the time-varying mass of the dark matter particles [13].

Models with varying Λ are essentially phenomenological as well as their scalar field analogs (see [14] for the canonical field description and [15] for its noncanonical counterpart), so that free parameters emerge, which must be constrained by observations. In this work we specialize to the particular $\Lambda(t)$ model given by the law $\Lambda = \Lambda_0 + 3\beta H^2$ and use the cosmic star formation rate (CSFR) to constrain the range of the β parameter.

Note that the CSFR makes the connection between the processes associated with star formation and the growth of density perturbations, of given mass, able to stand out from the Universe's expansion and collapse at a given time. Thus, CSFR is intrinsically associated with the formation of the first virialized structures (halos) in the Universe. It is therefore an observable associated with several important physical processes in the pregalactic Universe such as, for example, the chemical enrichment, reionization, early evolution of the Universe, growth of the supermassive black holes, etc. Thereby, any modification of the dark sector of the Universe, more specifically the particular type of field or fluid associated with dark energy, will produce changes in the way the CSFR evolves with redshift. Currently, observations of high-z galaxies and gamma-ray bursts has allowed estimating the CSFR up to redshift ~ 10 [16]. Although the observational uncertainties associated with the determination of the CSFR are large for $z > 3$, this is an observable that has the potential to impose constraints on the different models of dark energy in a range greater than redshift is achieved by, for example, bright high-z SNIa.

The present paper is organized as follows: in Sec. II A, we review the basics of cosmological models with vacuum

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decay, whereas in Sec. II B, we derive an equation for the matter-density perturbations which holds for *any* $\Lambda(t)$ model, generalizing the results found in Ref. [17]. In Sec. III A, we discuss the Press-Schechter mass function to prepare the ground to derive, and in Sec. III B, the basic equations to study the time evolution of the CSFR rate. With all these results in hand, we probe the $\Lambda = \Lambda_0 + 3\beta H^2$ model, narrowing down the range of values for β in Sec. IV. In Sec. V, we make the final remarks.

II. COSMOLOGICAL MODELS WITH VACUUM DECAY

A. The background equations

Throughout this paper, we consider a flat, homogeneous, and isotropic universe described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2) \quad (1)$$

and filled with a perfect fluid with energy density ρ and pressure P described by the stress energy-momentum tensor

$$T_m^{\alpha\beta} = (\rho + P)u^\alpha u^\beta - P g^{\alpha\beta}, \quad (2)$$

where u^α is the fluid four-velocity. The quantities ρ and P are connected via the equation of state

$$P = w\rho = (\gamma - 1)\rho, \quad (3)$$

where γ is the barotropic index.

By introducing a cosmological term Λ into the Einstein field equations, one has

$$G^{\alpha\beta} - \Lambda g^{\alpha\beta} = \kappa^2 T_m^{\alpha\beta}, \quad (4)$$

where $\kappa^2 \equiv M_P^{-2} \equiv 8\pi G$, M_P being the reduced Planck mass. It is convenient to introduce the *effective* energy-momentum tensor for the two fluids through the expression

$$\bar{T}^{\alpha\beta} \equiv T_m^{\alpha\beta} + M_P^2 \Lambda g^{\alpha\beta}, \quad (5)$$

which naturally satisfies the energy and momentum conservation constraint

$$\bar{T}^{\alpha\beta}{}_{;\beta} = 0 \quad (6)$$

as a consequence of the Bianchi identities. Hence, in this description, we can interpret Λ as a second fluid, so that there is no further reason to keep this term constant with respect to time.

Next, substituting the metric (1) into (4), we get the Friedmann equations

$$\kappa^2\rho + \Lambda = 3H^2, \quad (7)$$

$$\kappa^2 P - \Lambda = -2\frac{\ddot{a}}{a} - H^2, \quad (8)$$

where $H = \dot{a}/a$ is the Hubble parameter; also, from the energy conservation constraint (6) we get the continuity equation

$$\dot{\rho} + 3H(\rho + P) = F, \quad (9)$$

where we have defined the *source term* for the particle creation process

$$F \equiv -M_P^2 \dot{\Lambda}. \quad (10)$$

Equation (9) shows that a cosmological model with varying Λ implies that the vacuum content of the model decays into particles, so that this process might lead to a nonequilibrium process; however, it is possible to find a particular configuration of the system in which equilibrium relations still hold, as pointed out in Ref. [18].

Next, we rewrite Friedmann equations (7) and (8) as

$$3\frac{\ddot{a}}{a} = -\frac{\kappa^2}{2}(3w + 1)\rho + \Lambda, \quad (11)$$

which holds for any time dependence of the cosmological term Λ; in this work, we specialize to the phenomenological model with a quadratic term in H [6,8]:

$$\Lambda(H) \equiv \Lambda_0 + 3\beta H^2, \quad (12)$$

where Λ_0 is the present-day value for the cosmological constant and β is a dimensionless constant. Substituting Eqs. (3) and (12) into (7) and (8), we get the following equation for the Hubble parameter:

$$\dot{H} = \frac{\gamma\Lambda_0}{2} - \Delta H^2, \quad (13)$$

where we have defined

$$\Delta \equiv \frac{3\gamma}{2}(1 - \beta). \quad (14)$$

It is convenient for our purposes to change the cosmic time t into the scale factor a in Eq. (13), so that

$$H' = \frac{1}{aH}\left(\frac{\gamma\Lambda_0}{2} - \Delta H^2\right), \quad (15)$$

where a prime $'$ denotes a derivative with respect to the scale factor a. The solution for the Hubble parameter is given by

$$H(a) = \left(H_0^2 - \frac{\gamma\Lambda_0}{2\Delta}\right)\left(\frac{a}{a_0}\right)^{-2\Delta} + \frac{\gamma\Lambda_0}{2\Delta}. \quad (16)$$

In the equation above, all the quantities with a "0" subscript are evaluated in present time. It is convenient to factor out the present-time Hubble constant H_0 by defining the expansion factor

$$E(a) \equiv \frac{H(a)}{H_0}; \quad (17)$$

then, specializing to the case of a dust- (with $\gamma = 1$) and vacuum-dominated universe, Eqs. (7), (12), and (14) yield

$$\Omega_{m,0} + \Omega_\Lambda = \frac{2}{3}\Delta, \tag{18}$$

where

$$\Omega_\Lambda \equiv \frac{\Lambda_0}{3H_0^2}, \qquad \Omega_m \equiv \frac{8\pi G\rho_{m,0}}{3H_0^2}; \tag{19}$$

hence, it follows from expression (16) that

$$E(a) = \sqrt{\frac{3}{2\Delta}\Omega_{m,0}\left(\frac{a}{a_0}\right)^{-2\Delta} + \frac{3}{2\Delta}\Omega_\Lambda}. \tag{20}$$

For a universe dominated by dust and the cosmological constant, that is, $\Delta = 3/2$, Eq. (20) reduces to the well-known formula for the ΛCDM model

$$E(a) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda}, \tag{21}$$

as expected.

B. The equation for matter-density perturbations

Following Ref. [19], the hydrodynamical equations that describe the dynamics of the perfect fluid are given, respectively, by the Euler, continuity, and Poisson equations:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla\Phi + \frac{F}{\rho_m}(\mathbf{V} - \mathbf{u}), \tag{22}$$

$$\frac{\partial}{\partial t}\rho_m + \nabla\cdot(\rho_m \mathbf{u}) = F, \tag{23}$$

$$\nabla^2\Phi = 4\pi G\rho - \Lambda, \tag{24}$$

where $\mathbf{u}$ and $\mathbf{V}$ are, respectively, the velocity of a fluid volume element and of the created particles, ρ_m is the fluid mass density, Φ is the Newtonian gravitational potential, and F is the source term responsible for the matter creation due to the vacuum decay, given in Eq. (10).

We introduce next a comoving coordinate related to the proper coordinate $\mathbf{r}$ as

$$\mathbf{x} \equiv \frac{\mathbf{r}}{a} \tag{25}$$

and expand the velocity $\mathbf{u}$ and the matter density ρ_m to first order:

$$\mathbf{u} = aH\mathbf{x} + \mathbf{v}(\mathbf{x}, t), \tag{26}$$

$$\rho_m = \bar{\rho}_m(t)[1 + \delta_m(\mathbf{x}, t)], \tag{27}$$

where δ_m is the matter-density contrast; hence, Eqs. (22)–(24) become

$$\frac{\partial}{\partial t}\mathbf{v} + H\mathbf{v} + \ddot{a}\mathbf{x} = -\frac{1}{a}\nabla\Phi, \tag{28}$$

$$\nabla\cdot\mathbf{v} = -a\left(\frac{\partial}{\partial t}\delta_m + Q\delta_m\right), \tag{29}$$

$$\nabla^2\Phi = 4\pi G a^2\bar{\rho}_m(1+\delta_m) - \Lambda^2 a^2, \tag{30}$$

where we have used Eq. (9) to zeroth order and defined

$$Q(t) \equiv \frac{F}{\rho_0}. \tag{31}$$

Next, by expanding Φ as

$$\Phi(\mathbf{x}, t) = \phi(\mathbf{x}, t) + \frac{2\pi}{3}G\bar{\rho}_m a^2 x^2 - \frac{1}{6}\Lambda a^2 x^2 \tag{32}$$

and using the background equation (11) with $w = 0$, expressions (28) and (30) turn into

$$\frac{\partial \mathbf{v}}{\partial t} + H\mathbf{v} = -\frac{1}{a}\nabla\phi, \tag{33}$$

$$\nabla^2\phi = 4\pi G a^2\bar{\rho}_m\delta_m. \tag{34}$$

Taking the divergence of (33) and using (29) and (34), we find

$$\ddot{\delta}_m + (2H + Q)\dot{\delta}_m - (4\pi G\bar{\rho}_m - 2HQ - \dot{Q})\delta_m = 0. \tag{35}$$

Next we change the cosmic time variable t into the scale factor, so that Eq. (35) becomes

$$\delta_m'' + \left[\frac{3}{a} + \frac{E'}{E} - \frac{a^3\Lambda'}{3H_0^2\Omega_{m,0}}\right]\delta_m' - \left[\frac{3}{2}\frac{\Omega_{m,0}}{a^5E^2} + \frac{a^3}{3H_0^2\Omega_{m,0}} \times\left(6\frac{\Lambda'}{a} + \frac{E'}{E}\Lambda' + \Lambda''\right)\right]\delta_m = 0. \tag{36}$$

It is important to stress that Eq. (36) is quite general, holding for *any* cosmological model with $\Lambda(t)$, thus generalizing the approach developed in [17]. In particular, it reduces to the ΛCDM matter-density contrast when $\Lambda' = 0$:

$$\delta_m'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta_m' - \frac{3}{2}\frac{\Omega_{m,0}}{a^5E^2}\delta_m = 0 \tag{37}$$

[see Eq. (19) in Ref. [17]].

III. THE HIERARCHICAL STRUCTURE FORMATION SCENARIO

A. The halo mass function

Press and Schechter (hereafter PS) heuristically derived a mass function for bounded virialized objects in 1974 [20]. The basic idea of the PS approach is to define halos as concentrations of mass that have already left the linear regime by crossing the threshold δ_c for nonlinear collapse. Thus, given a power spectrum and a window function, it should then be relatively straightforward to calculate the halo mass function as a function of the mass and redshift. In particular, the scale differential mass function $f(\sigma, z)$ [21], defined as a fraction of the total mass per $\ln\sigma^{-1}$ that belongs to halos, is

$$f(\sigma, z) \equiv \frac{d\rho/\rho_B}{d\ln\sigma^{-1}} = \frac{M}{\rho_B(z)} \frac{dn(M, z)}{d\ln[\sigma^{-1}(M, z)]}, \quad (38)$$

where $n(M, z)$ is the number density of halos with mass M, $\rho_B(z)$ is the background density at redshift z, and $\sigma(M, z)$ is the variance of the linear density field. As pointed out in [21], this definition of the mass function has the advantage that it does not explicitly depend on redshift, power spectrum, or cosmology; all of these are contained in $\sigma(M, z)$.

In order to calculate $\sigma(M, z)$, the power spectrum $P(k)$ is smoothed with a spherical top-hat filter function of radius R, which on average encloses a mass M ($R = [3M/4\pi\rho_B(z)]^{1/3}$). As usual, $P(k)$ is assumed to have a power-law dependence on scale, that is, $P(k) \propto k^n$ (with $n \approx 1$). Thus,

$$\sigma^2(M, z) = \frac{D^2(z)}{2\pi^2} \int_0^\infty k^2 P(k) W^2(k, M) dk, \quad (39)$$

where $W(k, M)$ is the Fourier transform of the real-space top-hat window function of radius R. Thus,

$$W(k, M) = \frac{3}{(kR)^3}[\sin(kR) - kR\cos(kR)], \quad (40)$$

and the redshift dependence enters only through the linear growth factor $D(z)$. That is, $\sigma(M, z) = \sigma(M, 0)D(z)$.

On the other hand, the linear growth function is defined as $D(z) \equiv \delta_m(z)/\delta_m(z = 0)$, and it is obtained as a solution from Eq. (36) or (37) (see [17,22] for details).

Thus, the function $f(\sigma, z)$ is, in Eq. (39), the σ-weighted distribution separating collapsed objects (that is, $\delta > \delta_c$, with $\delta_c \approx 1.69$) from uncollapsed regions. Here, we consider the function $f(\sigma, z)$ given by [23]

$$f_{\rm ST}(\sigma) = 0.3222\sqrt{\frac{2a}{\pi}}\frac{\delta_c}{\sigma}\exp\left(-\frac{a\delta_c^2}{2\sigma^2}\right)\left[1 + \left(\frac{\sigma^2}{a\delta_c^2}\right)^p\right], \quad (41)$$

where $a = 0.707$ and $p = 0.3$. In particular, Eq. (41) is known as the Sheth-Tormen mass function.

Simulations [21] show that the mass function of dark matter halos in the mass range from galaxies to clusters is reasonably well described by Eq. (41).

Once a halo is formed, baryonic matter falls towards its center. Considering that the density of baryons is proportional to the density of dark matter or, in other words, considering that the baryon distribution traces the dark matter, it is possible to write a equation describing the fraction of baryons that are in structures as

$$f_b(z) = \frac{\int_{M_{\rm min}}^{M_{\rm max}} f_{\rm ST}(\sigma) M dM}{\int_0^\infty f_{\rm ST}(\sigma) M dM}, \quad (42)$$

where we have used $M_{\rm min} = 10^6 M_\odot$ and $M_{\rm min} = 10^{18} M_\odot$ (see [24] for details).

From the above equation, we can obtain the baryon accretion rate $a_{\rm b}(t)$, which accounts for the increase in the fraction of baryons in structures. It is given by

$$a_b(t) = \Omega_b \rho_c \left(\frac{dt}{dz}\right)^{-1} \left|\frac{df_b(z)}{dz}\right|, \quad (43)$$

where $\rho_c = 3H_0^2/8\pi G$ is the critical density of the Universe.

The age of the Universe that appears in (43) is related to the redshift by

$$\left|\frac{dt}{dz}\right| = \frac{9.78h^{-1}\ \text{Gyr}}{(1+z)E(z)}. \quad (44)$$

B. The cosmic star formation rate density

Since galaxies form in dark matter halos and their evolution is influenced by the baryonic accretion rate [see Eq. (43)], it is reasonable to assume that the physical properties of galaxies should correlate to those of the host halos. In this way, the CSFR density, which is an integral constraint averaged over the volume of the Universe observable at a given redshift, could be obtained by a similar procedure as that used to study stellar populations during the past 40 years, since the pioneering model developed by [25].

The key point is to consider halos as reservoirs of neutral gas that is converted into stars. In this way, the equation governing the total gas mass (ρ_g) in the halos is

$$\dot{\rho}_g = -\frac{d^2 M_\star}{dV dt} + \frac{d^2 M_{ej}}{dV dt} + a_b(t). \quad (45)$$

The first term on the right-hand side of Eq. (45) represents the stars which are formed by the gas contained in the halos. Using a Schmidt law [26], we can write for the star formation rate

$$\frac{d^2 M_\star}{dV dt} = \Psi(t) = k\rho_g(t), \quad (46)$$

where k is the inverse of the time scale for star formation. That is, $k = 1/\tau_s$.

The second term on the right-hand side of Eq. (45) considers the mass ejected from stars through winds and supernovae. Therefore, this term represents the gas which is returned to the "interstellar medium of the system." Thus, we can write (see, e.g., [25])

$$\frac{d^2 M_{ej}}{dV dt} = \int_{m(t)}^{120 M_\odot} (m - m_r)\Phi(m)\Psi(t - \tau_m) dm, \quad (47)$$

where the limit $m(t)$ corresponds to the stellar mass whose lifetime is equal to t. In the integrand, m_r is the mass of the remnant, which depends on the progenitor mass (see [24,25] for details), and the star formation rate is taken at the retarded time $(t - \tau_m)$, where τ_m is the lifetime of a star of mass m.

Since the stars that are formed within the halos have masses up to $\sim 120 M_\odot$, we can use for τ_m the metallicity-independent fit of Ref. [27]. Thus,

$$\log_{10}(\tau_m) = 10.0 - 3.6\log_{10}\left(\frac{M}{M_\odot}\right) + \left[\log_{10}\left(\frac{M}{M_\odot}\right)\right]^2, \tag{48}$$

where τ_m is the stellar lifetime given in years.

In Eq. (47), the term $\Phi(m)$ represents the initial mass function (IMF), which gives the distribution function of stellar masses. Thus,

$$\Phi(m) = Am^{-(1+x)}, \tag{49}$$

where x is the slope of the IMF and A is a normalization factor determined by

$$\int_{0.1M_\odot}^{120M_\odot} m\Phi(m)dm = 1, \tag{50}$$

where $0.1M_\odot$ corresponds to the minimum stellar mass capable of nuclear fusion which represents the stellar or brown dwarf mass limit.

Numerical integration of (45) produces the function $\rho_g(t)$ at each time t (or redshift z). Once we have obtained $\rho_g(t)$, we return to Eq. (46) in order to obtain the CSFR. Just by replacing $\Psi(t)$ by $\dot{\rho}_\star(t)$, we have the CSFR as given by

$$\dot{\rho}_\star = k\rho_g. \tag{51}$$

IV. TESTING THE $\Lambda = \Lambda_0 + 3\beta H^2$ MODEL

Once we have established the basic ideas underlying vacuum-decaying cosmological models and the DE contribution to the CSFR, we now proceed to test the $\Lambda = \Lambda_0 + 3\beta H^2$ model using the CSFR. The main equation to be solved is the one associated with the time evolution of the matter-density contrast (36); the derivatives of Λ can be read from (12) and (17):

$$\frac{\Lambda'}{3H_0^2} = 2\beta E'E, \tag{52}$$

$$\frac{\Lambda''}{3H_0^2} = 2\beta(E''E + E'^2), \tag{53}$$

so that Eq. (36) becomes

$$\delta_m'' + \left[\frac{3}{a} + \frac{E'}{E} - \frac{2\beta a^3}{\Omega_{m,0}}\right]\delta_m' - \left[\frac{3}{2}\frac{\Omega_{m,0}}{a^5E^2} + \frac{2\beta a^3}{\Omega_{m,0}} \times \left(6\frac{E'E}{a} + 2E'^2 + E''E\right)\right]\delta_m = 0. \tag{54}$$

The expansion function for this model is given by Eq. (20), and taking its derivatives we get the other terms appearing in (54):

$$\frac{E'}{E} = \frac{1}{a}\left(\frac{3}{2}\frac{\Omega_\Lambda}{E^2} - \Delta\right), \tag{55}$$

$$EE'' = \frac{1}{a}\left[\frac{3}{2}\Omega_\Lambda\left(-\frac{1}{a} - \frac{E'}{E}\right) - \Delta\left(E'E - \frac{E^2}{a}\right)\right]. \tag{56}$$

From the mathematical formalism developed above, we are able to obtain the CSFR in a self-consistent way. That is, taking into account Eq. (56) in the hierarchical structure formation scenario, described in the previous section, one can obtain a consistent formalism to analyze cosmological models with decaying vacuum from the point of view of both the structure of the Universe and star formation at high redshifts.

In Fig. 1, we present our results for the CSFR as a function of the redshift. In particular, HP stands for the observational data as those derived by Ref. [28]. We have fixed the cosmological parameters for the following values: $\Omega_d = 0.721$, $\Omega_m = 0.279$, $\Omega_b = 0.046$, and Hubble constant $H_0 = 100h$ km s^{-1} Mpc^{-1} with $h = 0.700$. For the variance of the overdensity field smoothed on a scale of size $8h^{-1}$ Mpc, we consider $\sigma_8 = 0.821$. These values are consistent with nine-year Wilkinson Microwave Anisotropy Probe observations [29].

For other model parameters associated with the hierarchical scenario of structure formation, one uses $x = 1.35$ (IMF), and $\tau_s = 2.0 \times 10^9$ Gyr (we refer the reader to Refs. [22,24,30], who have analyzed the influence of these parameters on the CSFR).

In the present study, it is enough to fix the same input parameters for both cases: cosmological constant and vacuum decay. We are interested in verifying whether these two different cosmological models can produce a difference in the evolution of CSFR.

As can be seen from Fig. 1, the process of baryonic matter infall from the halos is more efficient, for the same set of parameters, if $\beta \neq 0$ (decaying-vacuum cosmology). Note that $\beta = 0.10$ produces 3 times more stars at redshift $\sim$5 than the cosmological constant cosmology ($\beta = 0$).

In Table I, we present two important characteristics of the models: the redshift where the CSFR peaks and the age of the Universe. Thus, the β model increases the redshift

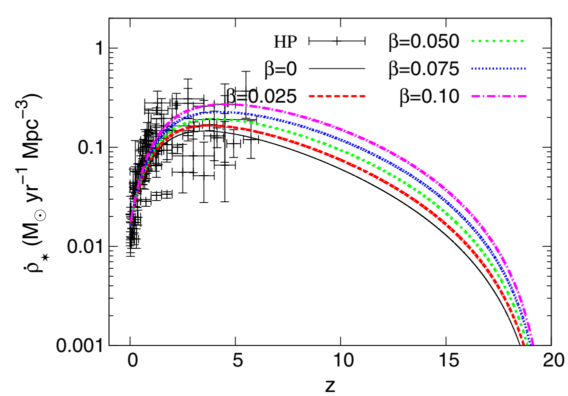

FIG. 1 (color online). The CSFR derived in this work compared to the observational points (HP) taken from Ref. [28]. The case $\beta = 0$ corresponds to cosmological constant model.

TABLE I. The results for the CSFR as a function of the β parameter. In column 2 is presented the redshift (z_p) where the CSFR peaks, and finally, in column 3, we have the age of the Universe (t_u).

β	z_p	t_u (Gyr)
0	3.55	13.70
0.025	3.78	13.95
0.050	4.02	14.22
0.075	4.28	14.50
0.10	4.54	14.79

where the CSFR peaks when compared to the cosmological constant.

Moreover, the cosmological models with $\beta \neq 0$ are older than in the case of the cosmological constant. In particular, we can use the value inferred for the CSFR at $z = 0$, which is $\dot{\rho}_\star \approx 0.016 M_\odot$ yr^{-1} Mpc^{-1}, as a constraint for the maximum value of the β parameter.

Indeed, for $\beta > 0.15$ the CSFR at $z = 0$ falls well below this observational reference value. In this way, a cosmological model with a $\Lambda = \Lambda_0 + 3\beta H^2$ decaying vacuum can produce a reasonable physical model only if $\beta \lesssim 0.1$.

V. CONCLUSIONS

In this paper, we generalize the evolution equation for the matter-density contrast found in Ref. [17] to the case of DE scenarios with an arbitrary time-varying cosmological term Λ. We have studied the CSFR density for the particular vacuum-decaying model $\Lambda(t) = \Lambda_0 + 3\beta H^2$ for a spatially flat Friedmann-Robertson-Walker geometry and find that the amplitude of the CSFR depends on the specific value of the β parameter. We verify that in the case $\beta \neq 0$ the star formation is more efficient and the CSFR peaks at high redshifts. As a result, the CSFR can become 3 times higher (if $\beta \approx 0.1$) than the cosmological constant model at $z \sim 5$.

However, using the best estimate for the CSFR at $z = 0$, which is $\dot{\rho}_\star \approx 0.016 M_\odot$ yr^{-1} Mpc^{-1}, produces an important constraint on the vacuum decay scenario. That is, models with $\beta > 0.15$ have $\dot{\rho}_\star(z = 0)$ so far below this observational limit. Thus, models with $\beta \gtrsim 0.10$ can be ruled out.

In general, a variety of cosmologically relevant observations has been used to constrain the vacuum decay models. They are the baryonic acoustic oscillations, cosmic microwave background shift parameter, and SNIa distance moduli [31]. However, another relevant observable could be constructed to study the Universe at least up to redshift $z \sim 6$–7. This new observable is the CSFR, which could help to understand the physical character of the dark energy.

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