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STUDY OF PERTURBATION INTEGRALS APPLIED TO THE DYNAMICS OF SPACECRAFTS AROUND GALILEAN MOONS

A. F. B. A. Prado,[*] and J. Cardoso dos Santos,[†] and J. P. S. Carvalho,[‡] and R. Vilhena de Moraes[§]

Several scientific missions have been proposed for a deeper exploration of planetary systems. These missions require a lot of specialized techniques in order to reach a better understanding of the dynamics involved in their planning. In order to help to get important features about each system under study, the present study proposes to use different definitions of integrals of the perturbing forces received by spacecrafts to help to find the best orbits for them. These computations present important information about level of perturbation acting in the spacecraft due to the effects of each force considered in the system and they also help to understand the evolution of these perturbations. The system of Galilean moons is explored to test these techniques.

INTRODUCTION

The investigation of the orbital motion of celestial bodies under the influence of disturbed (non-Keplerian) gravitational fields is a classical topic in celestial mechanics. Specially in the context of the space exploration, there are many topics to be taken into account when a science mission is designed, just like to better understand the types of orbits that require a smaller number of orbital maneuvers. It is also important to understand the types of orbits that have lower changes in the orbital energy and that are less disturbed by the perturbation forces considered. In recent years, this type of research has become important for the design of space missions that intend to visit minor bodies in the Solar System. The Galilean satellites of Jupiter are objects that have been studied since they were discovered by Galileo Galilei, in 1609. The information about these bodies obtained by means of data collected by some missions that explored the Jovian system (Pioneer 10 and 11, Voyager 1 and 2, Galileu and others) brought issues that have to be answered by future missions, like: to obtain information about inner liquid oceans; to characterize the ocean layers and the distribution of mass; to detect putative subsurface water reservoirs; to study physical properties of the icy crusts and the intensive volcanic activity; to reach a better understand of the Laplacian resonance and its implications. These questions and others are object of several studies and can be better answered by sending a specific mission having these issues as the one of the main goals.

[*]President of the Board of the Graduate School at INPE in Brazil, National Institute for Space Research (INPE), São José dos Campos, Brazil. AIAA Associate Fellow. email: antonio.prado@inpe.br

[†]Graduate Researcher, São Paulo State University (UNESP), Guaratinguetá, Brazil and Visiting Scholar, University of Colorado at Boulder (CCAR-UCB), USA. email: josuesantosunesp@gmail.com

[‡]Researcher, Center for Energy and Sustainability Science and Technology, Federal University of the Reconcavo of Bahia (UFRB), Feira de Santana, Brazil. email: jeanfeg@gmail.com

[§]Senior Visiting Researcher, Institute of Science and Technology, Federal University of São Paulo (UNIFESP), São José dos Campos, Brazil. email:rodolpho.vilhena@gmail.com

In the context of the astrodynamics, several studies have been explored the dynamics of orbits for applications in such missions that intend to visit other celestial bodies, and the Galilean moons were included (References 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11). In this scenario, space missions having such goals will require orbits having lower costs of operation and fuel consumption. Therefore, the search for these orbits is a fundamental issue to be reached, in order to save costs for the space agencies. In this context, the present work proposes to search for orbits that suffer smaller perturbations, to use them in scientific missions. This comprehension is essential for the success of this type of mission. All the results are obtained by performing numerical simulations and they can be used in applications in the fields of astrodynamics and aerospace engineering. We compare three types of indexes based in integrals. It is assumed that the orbital elements are constant during one orbital period in all the methods used. This assumption needs to be investigated better in the future.

The system of planetary moons considered in our applications is the system of Galilean moons of Jupiter: Io, Europa, Ganymede and Callisto. An analytical model for each perturbation, here called Perturbation Integral, is developed considering the restricted three-body problem and the effects of the irregular gravity field of the moon (J_2 and C_{22} terms). In the present study, the central body is assumed to be a planetary moon that is being orbited by a spacecraft. The disturbances due to an external planet that in the present context is assumed to come from Jupiter.

THE INTEGRAL APPROACHES

The concept of *"Perturbation Integrals"* as an useful tool to analyze the dynamics of a spacecraft's orbit was introduced recently (References 12, 13). Since the first papers, this technique have been applied to several studies in the literature, (References 14, 15, 16, 17, 18, 19, 20, 21, 22). In these applications, the the main effect of the forces involved in the system were to change the velocity of the spacecraft, according to the physical law of impulse.

For the present work, three different *Perturbation Integral* are studied, in order to be presented as useful tools to analyze the orbital dynamics of different celestial bodies, specially in the context of applications in the design of science missions. Each *Perturbation Integral* will be denoted by PI_1, PI_2 and PI_3. For the present work, these applications could be done for missions being planned to visit the system of Galilean moons of Jupiter.

The Integral Approaches: PI_1

The first *Perturbation Integral* considered in the present work is called PI_1 and it is defined as (Reference 12):

$$PI_1 = \frac{1}{T}\int_0^T |\vec{a}|dt \tag{1}$$

where $\vec{a}$ is the disturbing acceleration, t is the time and T is the period of the orbit.

This approach is useful to map orbits with respect to oscillations during the orbital period, when compared to a Keplerian reference orbit. This approach measures the integral of the norm of the perturbations over the time and it is useful to observe which orbits are more or less disturbed during the orbital period of the spacecraft.

The Integral Approaches: PI_2

The second approach for the *Perturbation Integral* considered here is called PI_2 and it is defined as:

$$PI_2 = \frac{1}{T}\int_0^T \vec{a}.\vec{v}dt \tag{2}$$

where $\vec{a}$ is the disturbing acceleration suffered by the spacecraft, $\vec{v}$ is the velocity of the spacecraft, t is the time and T is the period of the orbit. This approach is useful to measure the energy transferred to the spacecraft after one revolution.

The Integral Approaches: PI_3

The third approach considered for *PI* is defined as:

$$PI_3 = \frac{1}{T}\left[\left(\int_0^T a_x dt\right)^2 + \left(\int_0^T a_y dt\right)^2 + \left(\int_0^T a_z dt\right)^2\right]^{\frac{1}{2}} \tag{3}$$

where $\vec{a} = (a_x, a_y, a_z)$ is the disturbing acceleration suffered by the spacecraft, t is the time and T is the period of the orbit.

This approach is useful to map orbits with respect to the net result due to the perturbations after one revolution, not considering the oscillations during the orbital period. The idea of using the integral of the components appeared in Reference 22. The present paper performs a normalization using the orbital period of the spacecraft to make the results comparable with the previous definitions of *"Perturbation Integrals"*.

EQUATIONS OF MOTION

The geometry of the problem considered in the present study considers the classical restricted three body problem presented in Figure 1. The equations of motion considered are introduced as follows:

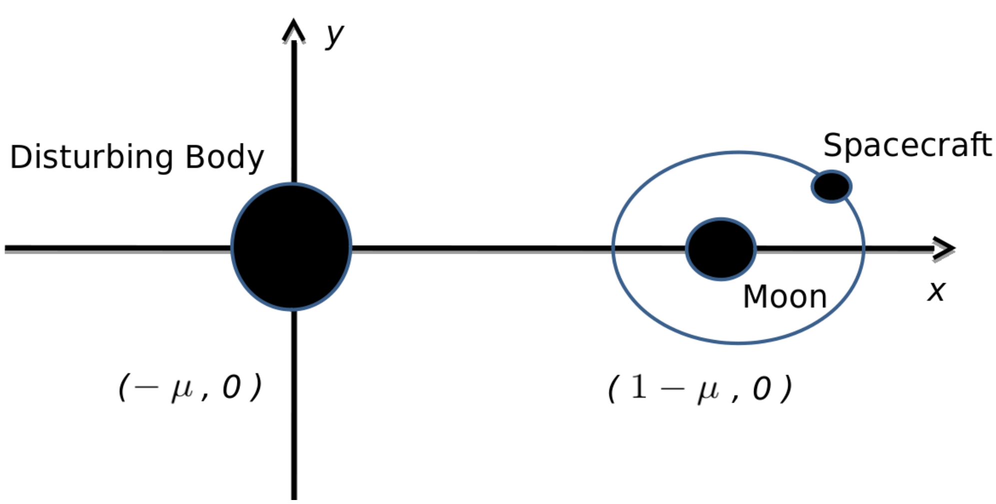

Figure 1. Geometry of the Restricted Three Body Problem.

$$a_x = 2v_y + x - \frac{\mu(x-1+\mu)}{{d_{ms}}^3} - \frac{(1-\mu)(x+\mu)}{{d_{js}}^3} \tag{4}$$

$$a_y = -2v_x + y - \frac{\mu y}{{d_{ms}}^3} - \frac{(1-\mu)y}{{d_{js}}^3} \tag{5}$$

$$a_z = -\frac{\mu z}{{d_{ms}}^3} - \frac{(1-\mu)z}{{d_{js}}^3} \tag{6}$$

where a_x, a_y and a_z are the components of the disturbing acceleration $\vec{a}$, v_x, v_y and v_z are the components of the orbital velocity $\vec{v}$ and x, y and z are the components of the position vector. The values d_{ms} and d_{js} are the distances moon-spacecraft and disturbing body (Jupiter)-spacecraft, respectively. By means of classical relations from celestial mechanics, it is possible to relate all these values to a specific set of orbital elements that describe the characteristic of each orbit: semimajor axis a, eccentricity e, inclination i, argument of the pericenter ω, longitude of the ascending node Ω. There is also the mean anomaly M, that describes the motion of the spacecraft in a specific orbit.

As a next step in the development of the gravitational model for the orbits, the influence of some gravity coefficients in the disturbing potential due to the non-sphericity of the central body, assumed to be the planetary moon, were considered to improve the analysis. This points particularly important for orbits close to the moon.

To obtain this improvement for the physical model, it is important to compute the disturbing accelerations due to some gravity terms related to the non-uniform gravity field of the planetary moons. The process to include these perturbations can be made by including the forces by unity of mass related to the potentials associated to each gravity coefficient desired in the analysis. In the present work, the gravity coefficients considered are J_2 and C_{22}. This is compatible with a model that assumes that the central body has an ellipsoidal shape. All of them will help to obtain a model that can take into account some anomalies that occur when the true gravitational potential is compared to the potential of a perfect spherical body. The potential related to J_2 and C_{22} can be written as (References 23, 24):

$$R = -\frac{\mu J_2}{2}\left(\frac{3z^2}{{d_{ms}}^5} - \frac{1}{{d_{ms}}^3}\right) + \frac{3\mu C_{22}}{{d_{ms}}^5}\left(x^2 - y^2\right) \tag{7}$$

where the gravity coefficients J_2 and C_{22} used to study each Galilean moon are presented in Table 2. The dynamics of these orbits are explored by numerical simulations.

NUMERICAL SIMULATIONS

A study considering spacecrafts orbiting each Galilean moon under the influence of the central body's non-uniform potential and the Jupiter's disturbing potential is performed in order to measure the perturbation levels of the orbits. Here, it is proposed the use of three integrals as tools in the context of space mission analysis. The Galilean moons Io, Europa, Ganymede and Callisto were chosen due to their higher potential to receive scientific missions in the next decades.

The data presented in Table 1 are used as input in the numerical simulations of the equations of motion, where m is the mass of the bodies of the system and the orbital elements a_p, e_p, ω_p, ℓ_p, Ω_p, i_p, M_p are the orbital elements of the disturbing body (planet) refereed to the central body (planetary moon). The results obtained (Figures 2 - 19) present several minimum values for the

Table 1. **Physical and orbital data for the Galilean system of moons. Source: http://ssd.jpl.nasa.gov/**

Body	m ($\times 10^{22}$ kg)	a_p (km)	e_p	ω_p (deg)	ℓ_p (deg)	Ω_p (deg)	i_p (deg)	M_p (deg)
Io	8.9319	421800	0.0041	84.129	342.021	43.977	0.036	342.021
Europa	4.7998	671100	0.0094	88.970	171.016	219.106	0.466	171.016
Ganymede	14.819	1070400	0.0013	192.417	317.540	63.552	0.177	317.540
Callisto	10.759	1882700	0.0074	52.643	181.408	298.848	0.192	181.408
Jupiter	189860.0							

Table 2. Gravity coefficients for the Galilean moons

Planetary Moon	J_2 $\times 10^{-6}$	C_{22} $\times 10^{-6}$	Reference
Io	1845.9000	553.70000	27
Europa	435.50000	130.65000	6
Ganymede	61.436994	63.943452	28
Callisto	15.456880	16.808453	28

integrals (perturbations and energy) that can be considered for the design of the orbits. These techniques can be used to find less perturbed orbits for spacecrafts orbiting a specific central body. It can also be useful to search for frozen orbits. It could be done by analyzing the regions where there is the presence of smaller perturbations over the spacecraft's orbital elements e, ω and i. Among theoretical and practical interests of this study are the better comprehension of the spacecraft dynamics under the influence of gravitational disturbing potentials, as well the applications to space missions by means of the search of orbital configurations that reduces the cost of station-keeping maneuvers.

Testing the model for an Europa orbiter in circular orbit

A preliminary study was performed considering the system Europa-spacecraft-Jupiter. Nowadays, Europa is among the most desired targets for future exploration missions and there are some missions under study to visit this planetary moon (References 25, 26). The Europa's non-sphericity and the Jupiter disturbing potential were considered in the simulations. Europa was chosen because it is a body that is receiving a special attention in the last years and some missions are being designed to study its surface and structure. Therefore, some results for an Europa orbiter were firstly obtained.

After the previous simulations considering the effects of Jupiter (disturber body), it is performed a more detailed analysis of the perturbation integrals by considering the gravity coefficients (i.e. harmonic terms) due to the non-sphericity of the planetary moon. The terms considered for the Galilean moons were: J_2 (related to the flattening at the poles) and C_{22} (due to the elliptical shapes of the central body's equator). The data considered for these simulations are present in Table 2. The values for the gravity coefficients are presented without a multiplicative factor of 10^{-6}. To test the integrals concept, several simulations are made. First of all the moon Europa is considered. In the simulation In the simulation presented in Figure 2 it is considered a scenario where the semimajor-axis goes from 2000 km to 10000 km (expressed in canonical units).

The first tests consider circular orbits. Figure 2 shows PI_1 as a function of the semimajor axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering only the disturbance due to the irregular shape of Europa. Some expected results are confirmed and quantified. The effects decrease with the semi-major axis of the orbit, because the effects of the irregular shape of Europa is stronger when the spacecraft is closer to the surface of the body. The effects of the inclination have an oscillatory behavior, with the curve showing a symmetry with respect to the polar orbit. The physical reasons are explained in details in (Reference 29).

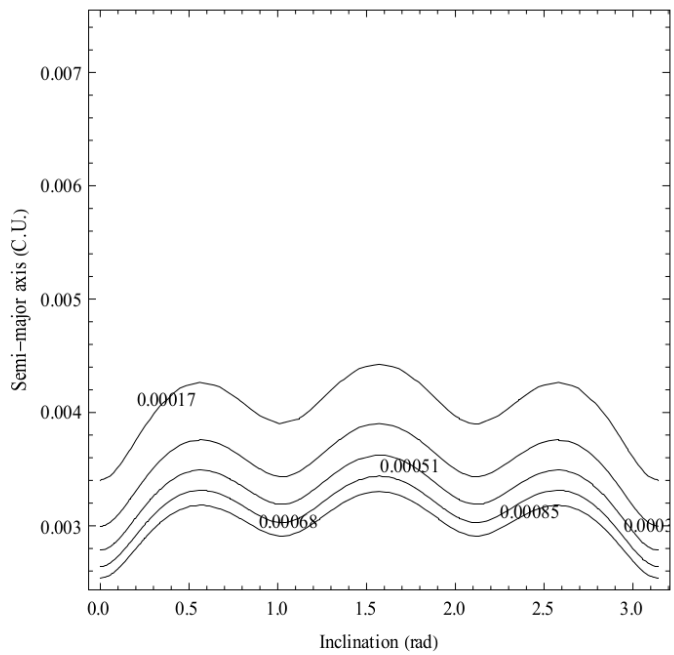

Figure 2. PI_1 as a function of the semi-major axis (in canonical units) and inclination (in radians) of circular orbits for the spacecraft considering only the effects of the irregular shape of Europa.

The next simulation show the effects of Jupiter perturbing the orbit of the spacecraft. Figure 3 shows PI_1 as a function of the semi-major axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering the perturbations of the third-body coming from Jupiter and the terms due to the rotating frame. The study is made in the rotating frame because the equations of motion are simple in this frame. There is also an oscillation with respect to the inclination, with a minimum perturbation located in the plane perpendicular to the orbital plane of Jupiter around Europa.

The next simulation show the total effects of Jupiter, the terms due to the rotating frame and the irregular shape of Europa perturbing the orbit of the spacecraft. Figure 4 shows PI_1 as a function of the semi-major axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering both perturbations. It is clear that the perturbation due to the irregular shape os Europa is much smaller in the orbit of the spacecraft. This is due to the smaller mass of the moon. The results considering all the perturbations are very similar to the results considering only Jupiter and the terms coming from the rotation of the frame, just showing a small increase in the magnitude. The extreme values are located around the same locations. The importance of the effects of the irregular shape of the moon would be larger if its mass were not so small. It means that, in general, there is a combination of effects coming from all the perturbations.

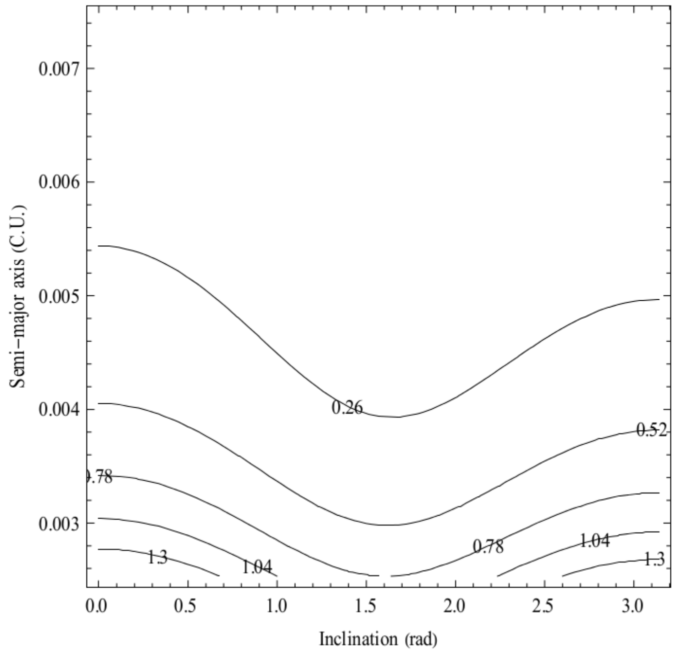

Figure 3. PI_1 **as a function of the semi-major axis (in canonical units) and inclination (in radians) of circular orbits for the spacecraft considering only the third-body effects of Jupiter and the terms coming from the rotating frame.**

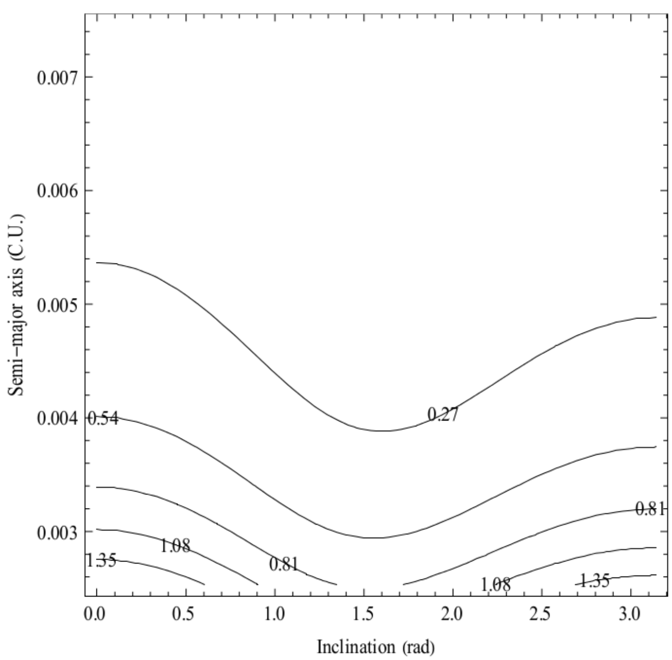

Figure 4. PI_1 **as a function of the semi-major axis (in canonical units) and inclination (in radians) of circular orbits for the spacecraft considering the third-body effects of Jupiter, the terms coming from the rotating frame and the irregular shape of Europa.**

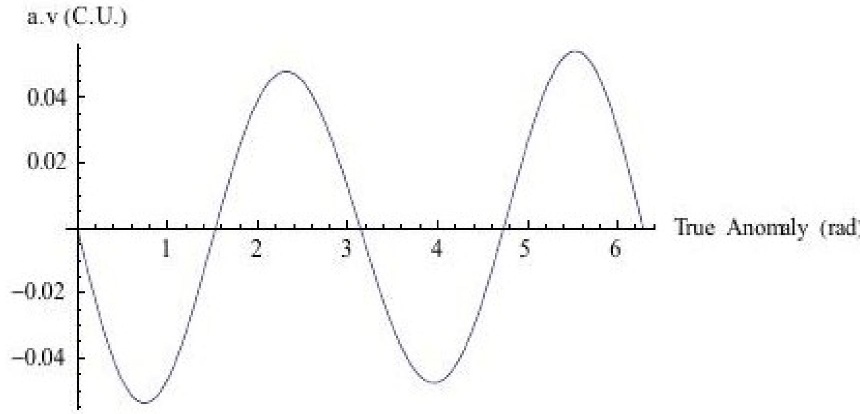

Figure 5. PI_2 **for an Europa orbiter**

It was performed a study where we obtained several results for the perturbation integrals PI_2. These results were always zero, showing that energy is not given or removed to/from the system by all perturbations. These figures are omitted here. It it is possible to perform an analysis considering the perturbation of Jupiter and the terms due to the rotating frame over the spacecraft's orbital motion, which is shown in Figure 5. It is useful to illustrate these zero-results for PI_2. Note that the curve has equals values in the positive and negative sides, so leaving a zero result for the integral over one orbital period. The longitude of the ascending node and the argument of periapsis have little effects. After an analysis of all the tests performed considering different cases for PI_2, similar results were obtained, so they are omitted too.

The study now focus in the index PI_3, which is an index that allows the compensations among positive and negative values of the perturbations, similar to what was done in Reference 22. The simulations show that there is no dependence with respect to the inclination for this index and that it decreases with the altitude. The figure is omitted here because it is trivial. The integrals for the perturbations due to the irregular shape of the moon is zero, due to the symmetry given by the circular orbits. It means that the perturbation from Jupiter and the terms due to the rotating frame are the only ones actuating in the long range for the circular orbits.

Testing an eccentric orbit for the Europa orbiter

After the first simulations considering circular orbits, now we test elliptical orbits. The next figure show the results of the simulations using an orbit with $e = 0.01$ for the spacecraft. Figure 6 shows PI_1 as a function of the semi-major axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering only the disturbance due to the irregular shape of Europa for the elliptical orbit. It is clear that this figure is very similar to Figure 2. The effect of the eccentricity is just to increase the magnitude of the perturbation. It happens because a larger eccentricity makes the spacecraft to get closer to the moon, where it suffers stronger effects of the irregular shape of the body. There is also the effect of loosing the simmetry that a circular orbit has.

The study considering the effects of Jupiter and the terms due to the rotating frame and the combined effects of all perturbations showed results that are very similar to the case of circular orbits, so they are not repeated here.

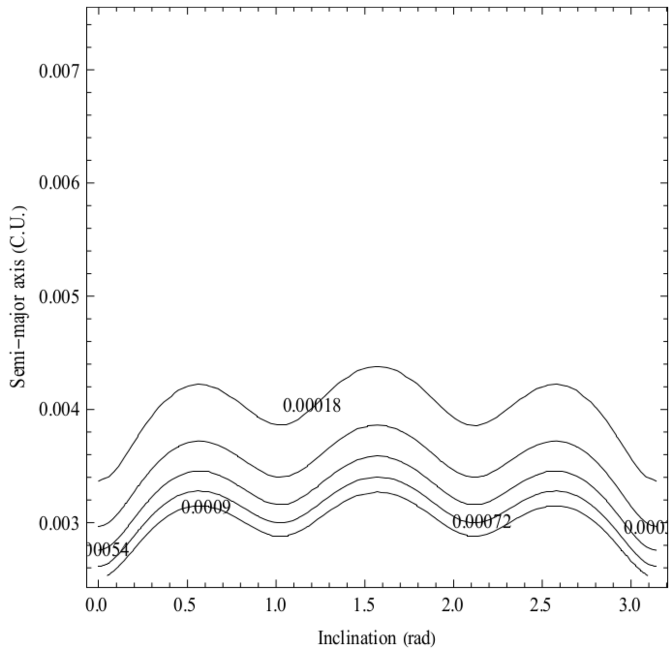

Figure 6. ***PI_1*** **as a function of the semi-major axis (in canonical units) and inclination (in radians) for eccentric orbits for spacecraft considering the effects of the irregular shape of Europa.**

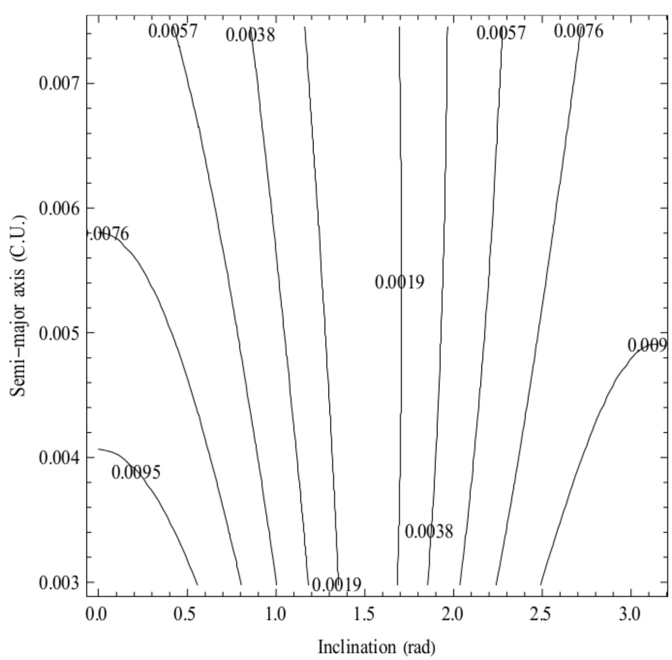

Figure 7. ***PI_3*** **as a function of the semi-major axis (in canonical units) and inclination (in radians) for eccentric orbits for spacecraft considering the third-body effects of Jupiter, the irregular shape of Europa and the terms due to the rotating frame.**

The next step is to perform a similar study for PI_3. Figure 7 shows this quantity as a function of the semi-major axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering all the perturbations. It is noted a symmetry around the inclination of 90 degrees. The orbits located in the orbital plane of Jupiter are more perturbed, as expected. They are also more dependent on the semi-major axis of the orbit.

Next, Figure 8 shows PI_3 as a function of the semi-major axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering the effects of the irregular shape of Europa. It is noted the expected decrease of the perturbation with the semi-major axis. This index is also much smaller than the corresponding index due to the perturbation of Jupiter and the terms due to the rotation of the frame, as occurred for PI_1.

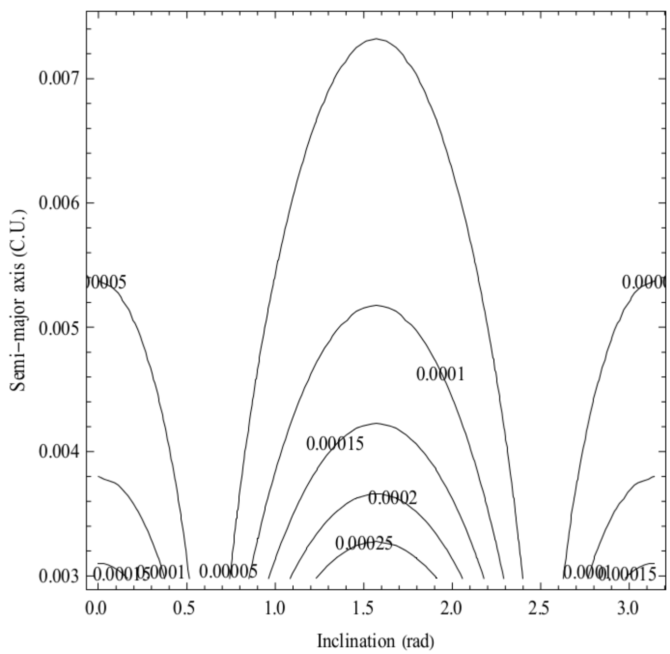

Figure 8. PI_3 as a function of the semi-major axis (in canonical units) and inclination (in radians) for eccentric orbits for spacecraft considering the effects of the irregular shape of Europa.

To complete this study, Figure 9 shows PI_3 as a function of the semi-major axis (in canonical units) and inclination (in radians) of the orbit of the spacecraft considering the effects of the third-body perturbation from Jupiter and the terms coming from the rotating frame. The results are very similar to the equivalent ones considering both perturbations, also due to the fact that the disturbing potential of Jupiter is much larger than the irregular shape of the moon, due to its large mass.

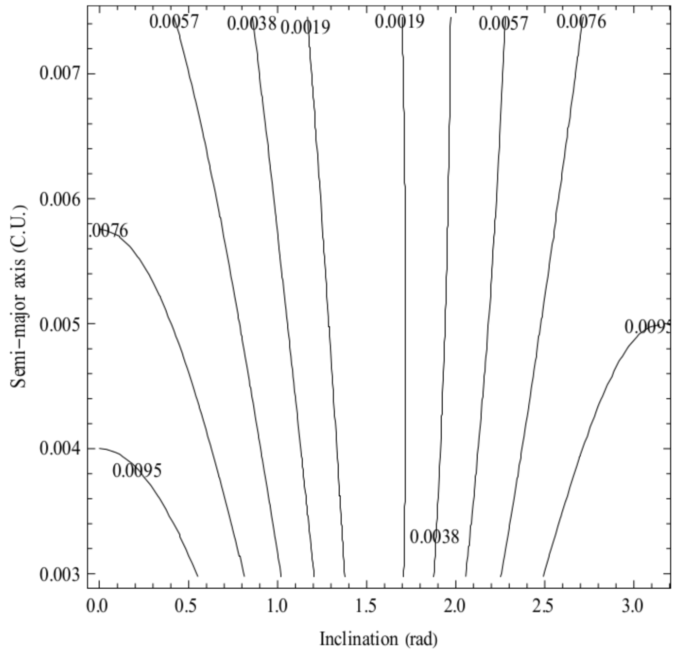

Figure 9. ***PI_3*** **as a function of the semi-major axis (in canonical units) and inclination (in radians) for eccentric orbits for spacecraft considering the effects of the third-body perturbation from Jupiter and the effects of the rotating frame.**

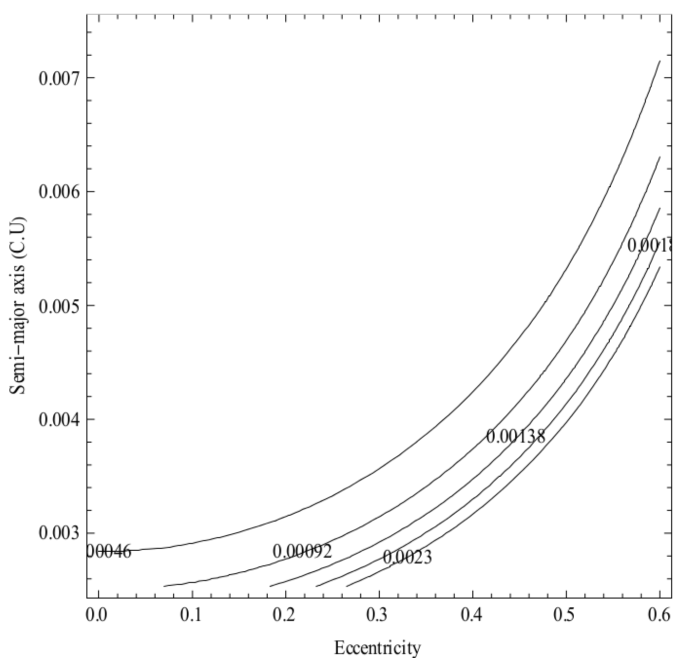

Figure 10. ***PI_1*** **as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Europa.**

The next study considers an equatorial orbit and investigate the effects of the semi-major axis (in canonical units) and the eccentricity. Figure 10 shows PI_1 as a function of the semi-major axis (in canonical units) and eccentricity of the orbit of the spacecraft considering the effects of the irregular shape of Europa. It is clear the expected behavior of increasing effects with the eccentricity, because the spacecraft gets closer to the third-body and the moon. It also looses the symmetry, which increases the effects of the terms coming from the rotating frame.

Figure 11 shows PI_1 as a function of the semi-major axis (in canonical units) and eccentricity of the orbit of the spacecraft considering the effects of all the perturbations. The same effects are observed. The study considering only the effects of Jupiter are very similar, just a little bit smaller in magnitude, so the figure is ommitted here.

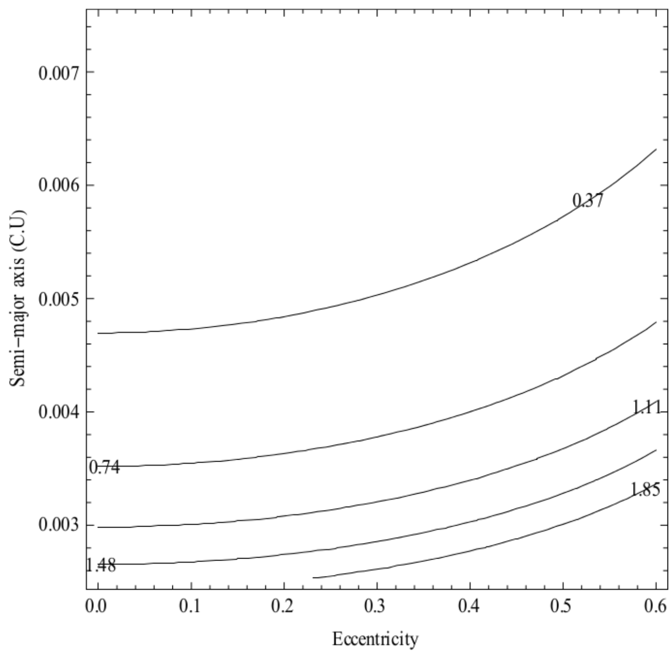

Figure 11. PI_1 as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Europa, the third-body perturbation from Jupiter and the effects of rotating frame.

Figure 12 shows PI_3 as a function of the semi-major axis (in canonical units) and eccentricity of the orbit of the spacecraft considering the effects of the irregular shape of Europa and the third-body perturbation from Jupiter.

Figure 13 shows equivalent results considering only the effects of the irregular shape of Europa. The same type of conclusions are obtained. The figure considering only the effects of Jupiter is also omitted, because it is very similar to the one considering both effects.

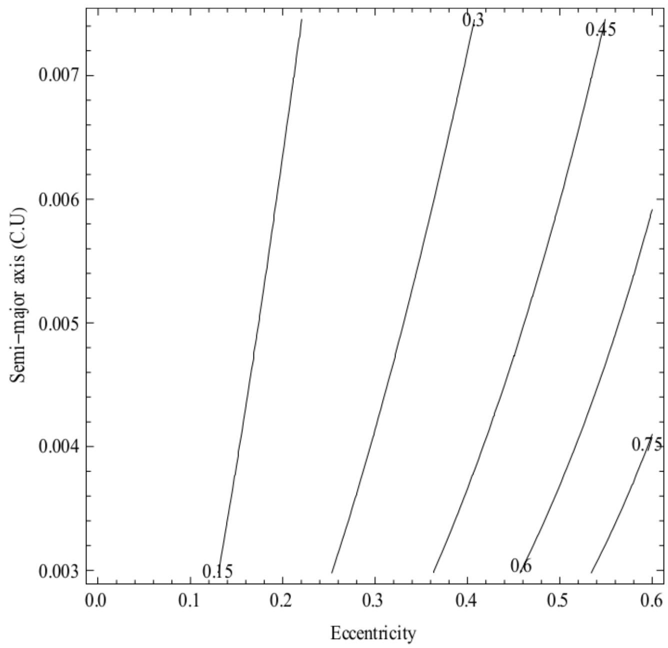

Figure 12. ***PI_3* as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Europa, the third-body perturbation from Jupiter and the effects of the rotating frame.**

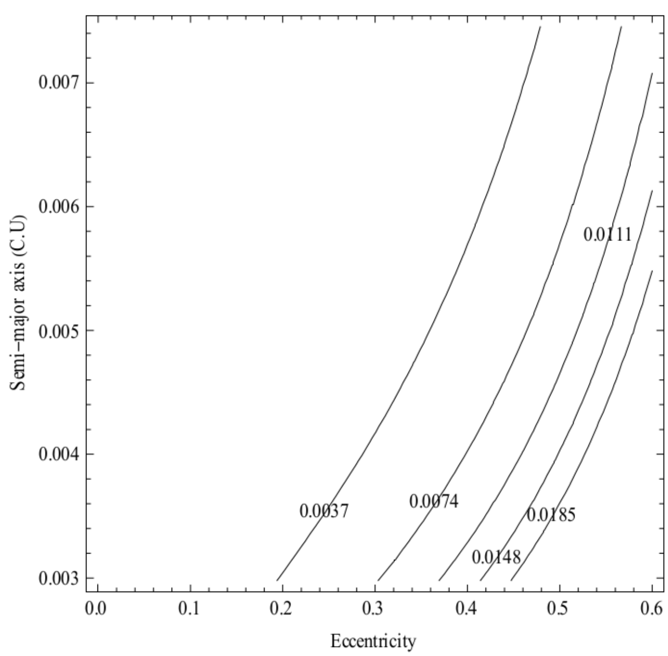

Figure 13. ***PI_3* as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Europa.**

Disturbances over spacecrafts orbiting other Galilean moons

After the study of the system of Europa, several simulations were performed to study the systems of the other Galilean moons: Io, Ganymede and Callisto. The results obtained for others moons (Figures 14 to 19) present very similar characteristics as those obtained for Europa orbiter.

Figures 14 and 15 shows the results for an Io orbiter under the influence of the irregular shape of Io and the same effects plus the attraction of Jupiter and the rotation of the frame, respectively.

Similar behavior happens when it is analyzed the case of orbits for Ganymede and Callisto orbiters. Figures 16 and 17 are related to the study considering the moon Ganymede as the central body of the system. Analogously, Figures 18 and 19 are presenting results obtained by considering the case of a system having the moon Callisto as the orbited body.

Among the reasons for these similar behaviors are the similarity between the magnitude of the masses and gravity coefficients of these planetary moons and their scale of distance to Jupiter.

The results for all the cases considering PI_2 and PI_3 are very similar and these figures for Europa will not be presented here. All the analyzes can be applied in the search for less disturbed orbits, analysis of orbital energy, study of the fuel comsumption, analysis of station-keeping maneuvers and so forth.

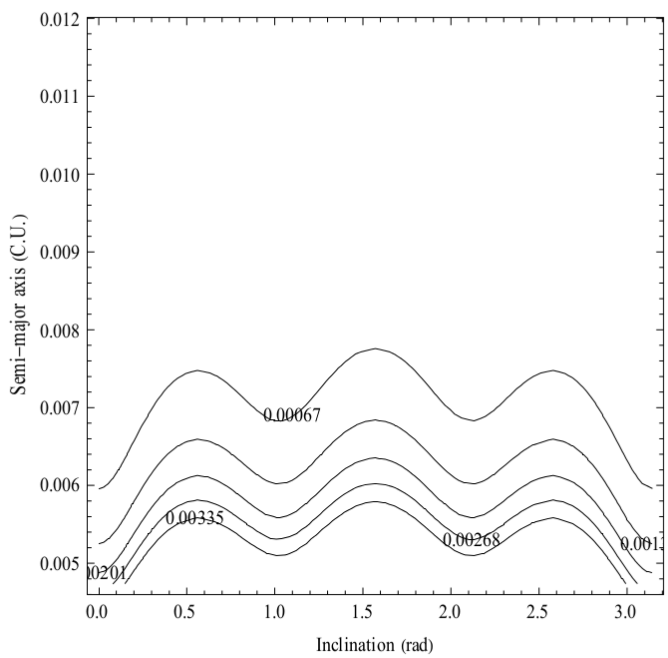

Figure 14. $\boldsymbol{PI_1}$ **as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Io.**

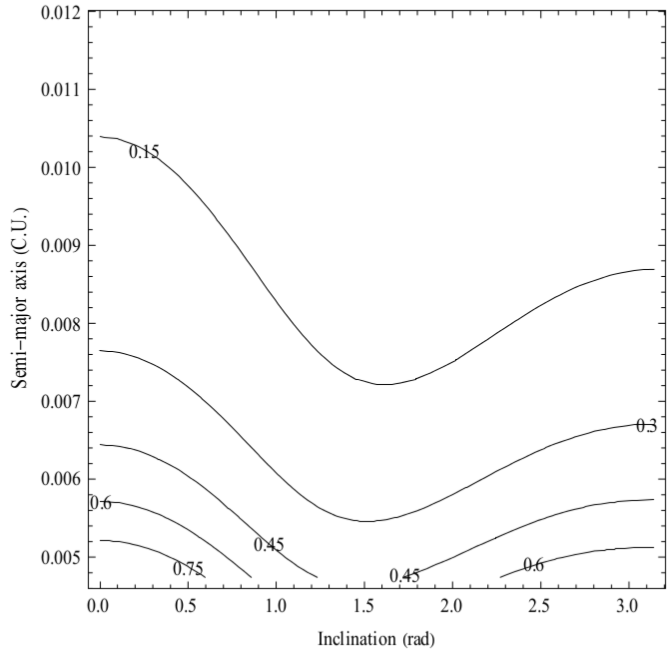

Figure 15. PI_1 **as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Io, the third-body perturbation from Jupiter and the effects of the rotating frame.**

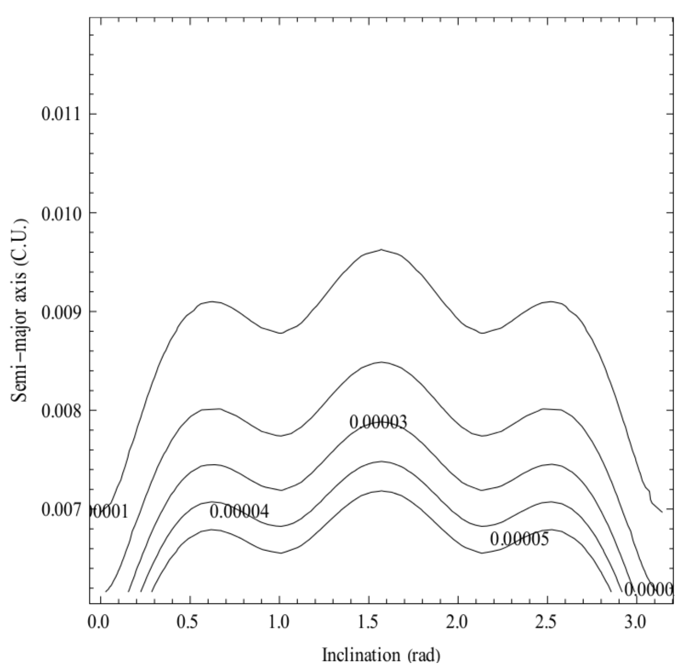

Figure 16. PI_1 **as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Ganymede.**

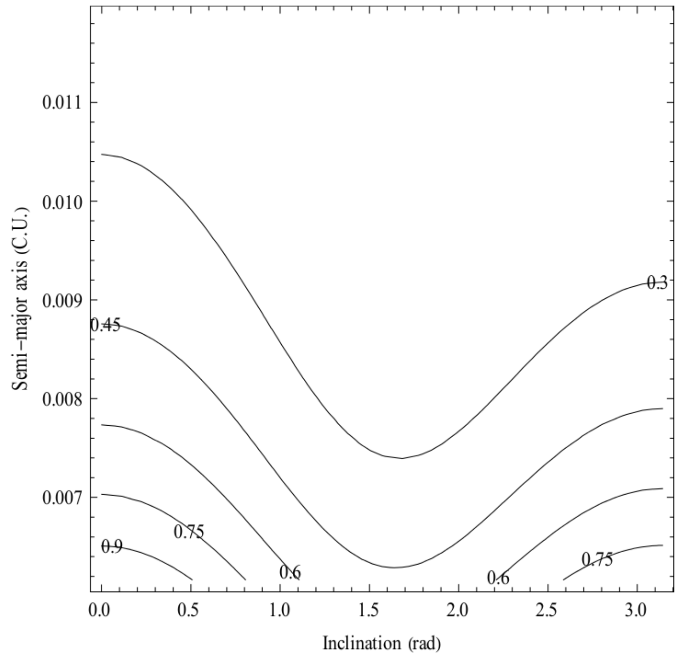

Figure 17. $\boldsymbol{PI_1}$ **as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Ganymede, the third-body perturbation from Jupiter and effects of the rotating frame.**

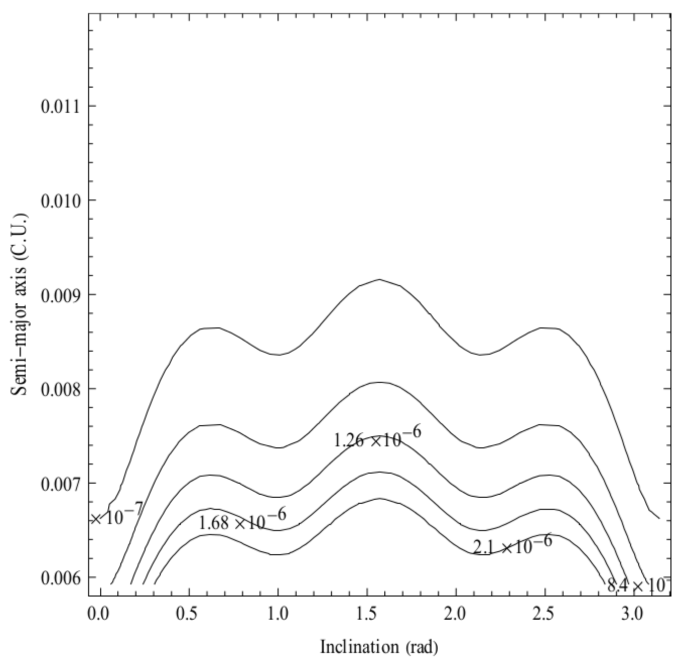

Figure 18. $\boldsymbol{PI_1}$ **as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Callisto.**

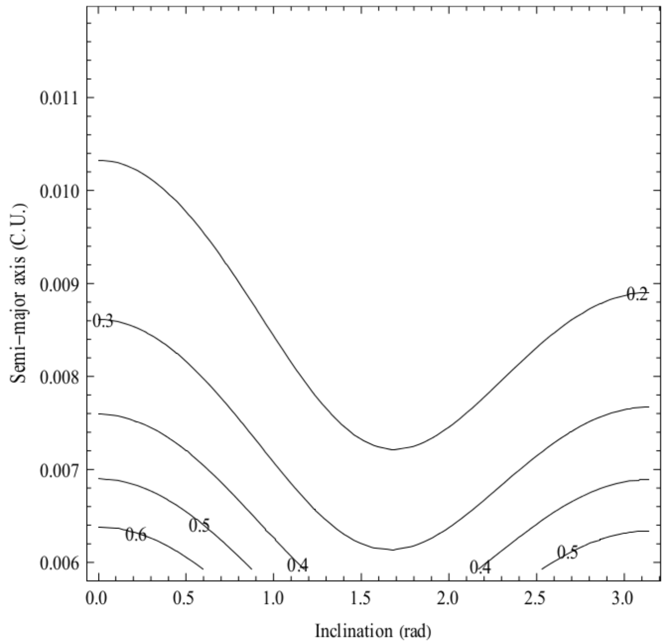

Figure 19. PI_1 as a function of the semi-major axis (in canonical units) and eccentricity for eccentric orbits for spacecraft considering the effects of the irregular shape of Callisto, the third-body perturbation from Jupiter and the effects of the rotating frame.

CONCLUSIONS

The study of the Perturbation Integrals is useful in the context of searching initial conditions for spacecrafts orbiting different celestial bodies and under different perturbations. The computations presented in this work offer information about the level of perturbation acting in the spacecraft due to the effects of each perturbation considered. The role of each perturbation can be obtained. The three approaches PI_1, PI_2 and PI_3 were presented as techniques for applications in astrodynamics and they are based on the idea of perturbations over the spacecraft's orbital motion. These approaches were applied to study the orbital dynamics of spacecrafts orbiting each one of the Galilean moons of Jupiter and they offered valuable physical information for each system. The results for all the Galilean moons presented similar behavior due to the close scales of physical quantities like mass, distance to Jupiter and values for the gravity coefficients considered. By using these techniques, it is possible to get features of the dynamics involveld in each problem under study. The results also show the relative effects of each force, quantifying how stronger is the disturbance coming from one force compared to other. The locations of less disturbed orbits are also performed. In the continuation of this work, more studies have to be developed to explore these techniques and they can be useful for planning future space missions that intend to visit these and others celestial bodies.

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REFERENCES

[1] G. E. O. Giacaglia, J. P. Murphy, and T. L. Felsentreger, "A Semi-analytic Theory for the Motion of a Lunar Satellite," *Celestial Mechanics*, Vol. 3, Mar. 1970, pp. 3–66, 10.1007/BF01230432.

[2] D. J. Scheeres, M. D. Guman, and B. F. Villac, "Stability Analysis of Planetary Satellite Orbiters: Application to the Europa Orbiter," *Journal of Guidance Control Dynamics*, Vol. 24, July 2001, pp. 778–787, 10.2514/2.4778.

[3] A. F. B. A. Prado, "Third-Body Perturbation in Orbits Around Natural Satellites," *Journal of Guidance Control Dynamics*, Vol. 26, Jan. 2003, pp. 33–40, 10.2514/2.5042.

[4] M. Lara and J. F. San Juan, "Dynamic Behavior of an Orbiter around Europa," *Journal of Guidance Control Dynamics*, Vol. 28, Mar. 2005, pp. 291–297, 10.2514/1.5686.

[5] B. d. Saedeleer, "Analytical theory of a lunar artificial satellite with third body perturbations," *Celestial Mechanics and Dynamical Astronomy*, Vol. 95, May 2006, pp. 407–423, 10.1007/s10569-006-9029-6.

[6] M. Lara and R. Russell, "Computation of a Science Orbit About Europa," *Journal of Guidance Control Dynamics*, Vol. 30, Jan. 2007, pp. 259–263, 10.2514/1.22493.

[7] M. E. Paskowitz and D. J. Scheeres, "Design of Science Orbits About Planetary Satellites: Application to Europa," *Journal of Guidance Control Dynamics*, Vol. 29, Sept. 2006, pp. 1147–1158, 10.2514/1.19464.

[8] R. P. Russell and A. T. Brinckerhoff, "Circulating Eccentric Orbits Around Planetary Moons," *Journal of Guidance Control Dynamics*, Vol. 32, Mar. 2009, pp. 424–436, 10.2514/1.38593.

[9] J. P. S. Carvalho, R. Vilhena De Moraes, and A. F. B. A. Prado, "Some orbital characteristics of lunar artificial satellites," *Celestial Mechanics and Dynamical Astronomy*, Vol. 108, Dec. 2010, pp. 371–388, 10.1007/s10569-010-9310-6.

[10] J. P. S. Carvalho, A. Elipe, R. Vilhena De Moraes, and A. F. B. A. Prado, "Low-altitude, near-polar and near-circular orbits around Europa," *Advances in Space Research*, Vol. 49, Mar. 2012, pp. 994–1006, 10.1016/j.asr.2011.11.036.

[11] J. P. S. Carvalho, R. Vilhena De Moraes, and A. F. B. A. Prado, "Dynamics of Artificial Satellites around Europa," *Mathematical Problems in Engineering*, Vol. 2013, 2013, 10.1155/2013/182079.

[12] A. F. B. A. Prado, "Searching for Orbits with Minimum Fuel Consumption for Station-Keeping Maneuvers: An Application to Lunisolar Perturbations," *Mathematical Problems in Engineering*, Vol. 2013, 2013, 10.1155/2013/415015.

[13] A. F. B. A. Prado, "Mapping orbits around the asteroid 2001SN_{263}," *Advances in Space Research*, Vol. 53, Mar. 2014, pp. 877–889, 10.1016/j.asr.2013.12.034.

[14] L. D. Gonçalves, E. M. Rocco, R. Vilhena De Moraes, and A. F. B. A. Prado, "Estudo dos efeitos do potencial lunar em trajetrias de veculos espaciais," *Proceeding Series of the Brazilian Society of Applied and Computational Mathematics*, Vol. 2013, Oct. 2013, 10.5540/03.2013.001.01.0095.

[15] J. Cardoso Dos Santos, J. P. S. Carvalho, and R. Vilhena De Moraes, "Stable low-altitude polar orbits around Europa," *Proceeding Series of the Brazilian Society of Applied and Computational Mathematics*, Vol. 2013, Oct. 2013, 10.5540/03.2013.001.01.0189.

[16] J. P. S. Carvalho, R. Vilhena De Moraes, A. F. B. A. Prado, and J. Cardoso Dos Santos, "Searching for orbits aound Europa that requires lower fuel consumption for stationkeeping," *Proceeding Series of the Brazilian Society of Applied and Computational Mathematics*, Vol. 2013, Oct. 2013, 10.5540/03.2013.001.01.0009.

[17] T. C. Oliveira and A. F. B. A. Prado, "Mapping orbits with low station keeping costs for constellations of satellites based on the integral over the time of the perturbing forces," *Acta Astronautica*, Vol. 104, Nov. 2014, pp. 350–361, 10.1016/j.actaastro.2014.06.035.

[18] D. M. Sanchez, A. F. B. A. Prado, and T. Yokoyama, "On the effects of each term of the geopotential perturbation along the time I: Quasi-circular orbits," *Advances in Space Research*, Vol. 54, Sept. 2014, pp. 1008–1018, 10.1016/j.asr.2014.06.003.

[19] J. P. S. Carvalho, R. Vilhena De Moraes, and A. F. B. A. Prado, "Searching Less Perturbed Circular Orbits for a Spacecraft Travelling around Europa," *Mathematical Problems in Engineering*, Vol. 2014, 2014, 10.1155/2014/529716.

[20] J. Cardoso Dos Santos, J. P. S. Carvalho, A. F. B. A. Prado, and R. Vilhena De Moraes, "Searching for less perturbed elliptical orbits around Europa," *Journal of Physics Conference Series*, Vol. 641, Oct. 2015, p. 012011, 10.1088/1742-6596/641/1/012011.

[21] J. Cardoso Dos Santos, J. P. S. Carvalho, R. Vilhena De Moraes, and A. F. B. A. Prado, "Dynamics of orbits for space missions taking into account a disturbing body in an elliptical-inclined orbit: Applications to planetary moons," *International Astronautical Congress, Jerusalem, Israel*, 2015, IAC paper.

[22] M. Lara, "Equivalent delta-v per orbit of gravitational perturbations," *Journal of Guidance Control Dynamics*, 2016, p. 16. in press.

[23] D. Brouwer and G. M. Clemence, *Methods of Celestial Mechanics*. Academic Press Inc., New Yor, 1961.

[24] M. E. Paskowitz and D. J. Scheeres, "Control of Science Orbits About Planetary Satellites," *Journal of Guidance Control Dynamics*, Vol. 32, 2009, pp. 223–231, 10.2514/1.36220.

[25] ESA, "JUICE definition study report (Red Book). Access: http://sci.esa.int/juice/54994-juice-definition-tudy-report/,"

[26] NASA, "Mission to Europa. Access: http://www.nasa.gov/europa/,"

[27] J. D. Anderson, R. A. Jacobson, E. L. Lau, W. B. Moore, and G. Schubert, "Io's gravity field and interior structure," , Vol. 106, Dec. 2001, pp. 32963–32970, 10.1029/2000JE001367.

[28] J. Aiello, "Numerical investigation of mapping orbits about Jupiter's icy moons," *2005 AAS/AIAA Astrodynamics Specialist Conference, Lake Tahoe, CA, August 7-11, 2005 - http://hdl.handle.net/2014/37832*, Aug. 2005.

[29] F. C. Faria Venditti and A. F. B. A. Prado, "A Method to Evaluate the Perturbation of Non-spherical Bodies," *Advances in the Astronautical Sciences*, Vol. 153, 2015, pp. 1143–1163.