

Bayesian network model to predict areas for sugarcane expansion in Brazilian Cerrado

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***Abstract:** The growing demand for ethanol has powered the shift of the sugarcane frontier into the Brazilian Cerrado, mainly in the states of Goiás and Mato Grosso do Sul. Therefore, this study aims to propose Bayesian Network models for identifying potential areas for sugarcane expansion in Goiás and Mato Grosso do Sul states. The models take into account constraint factors in relation to the sugarcane expansion such as topography, soil aptitude, climate conditions, and available infrastructures. Results showed that Bayesian Network models proposed in this study were able to represent the tendency of sugarcane expansion.*

1. Introduction

The intense demand for ethanol in the last years has stimulated the sugarcane expansion in new areas, which intensify the competition with other agricultural areas, increase the land prices and reduce the options for further expansions [Adami et al. 2012; Granco et al. 2015]. The saturation of traditional producing areas such as the state of São Paulo [Castro et al. 2010; Shikida 2013] along with the higher production costs for the industry has motivated the producers to search potential new areas for production of sugarcane.

The ethanol industry identified in the Brazilian Cerrado opportunities for investment, especially in the states of Goiás and Mato Grosso do Sul – the Cerrado Biome covers 97% of Goiás and 61% of Mato Grosso do Sul [BRASIL 2009]. However, even though favorable climate and soil conditions to the sugarcane cultivation [Shikida 2013] and affordable land prices, few mills were operating in the region. Until 2005, only 22 mills had been established in Goiás and Mato Grosso do Sul, which was a limiting factor for farmers to start planting sugarcane, once the crop needs to be promptly processed after harvesting [Granco et al. 2015].

To enhance the attractiveness for the sugarcane industry the government provided support through fiscal incentives, credit lines and investments in transportation infrastructure. As the result, the number of mills and areas planted to sugarcane in Goiás and Mato Grosso do Sul increased approximately three times from 2005 to 2015 [Granco et al. 2015]. Consequently, these states, which previously had an economy centered on cattle ranching and grains (soybean and corn), witnessed a strong sugarcane expansion [Shikida 2013].

The rapid sugarcane expansion and the eventual land cover changes in the Cerrado led the Brazilian government to implement the Sugarcane Agroecological Zoning to regulate the expansion and sustainable sugarcane production in Brazil, in 2009 [Manzatto et al. 2009]. To identify potential areas for sugarcane crop in the

Cerrado Biome, Ribeiro et al. (2015) carried out Boolean spatial analysis. However, Boolean analysis results only two classes (favorable and non-favorable), which do not adequately represent the spatial phenomena. Other spatial inference method for spatial data integration, such as Bayesian inference, can numerically express the potential of sugarcane areas from 0 to 1, which allows obtaining a decision surface (Moreira et al. 2000).

Within this context, this study aims to propose a Bayesian inference model for identifying potential areas for sugarcane expansion in the states of Goiás and Mato Grosso do Sul. We used a Bayesian Networks approach: the enhanced Bayesian Network for Raster Data (e-BayNeRD) method [Silva et al. 2014], which is an enhanced version of the BayNeRD algorithm [Mello et al. 2013]. The e-BayNeRD method is a probabilistic approach based on raster data observations and it is able to incorporate experts' knowledge for analysis. In the next section, we present a brief description about the theory of Bayesian Networks and the e-BayNeRD method employed in this study.

2. Bayesian Network Model

Bayesian Networks (BN) are defined in terms of two components: (i) qualitative component – a Directed Acyclic Graph (DAG), in which the nodes represent the variables in the model and the statistical dependence between pair wise variables is indicated by directed arrows that start in a parent node and end in a child node, as illustrate on Figure 1; and (ii) quantitative component – probability functions associated to each variable denoting the strengths of the links in the BN model [Aguilera et al. 2011; Landuyt et al. 2013].

Figure 1 shows an example of BN graphical model, in which *Suitable Area* variable is statistically dependent of *Soil Aptitude* variable and both are statistically dependent of *Terrain Slope* variable. Prior probability is assigned to variable without parent (e.g.: *Terrain Slope*), whereas conditional probability is assigned to descendant ones (e.g.: *Soil Aptitude* and *Suitable Area*).

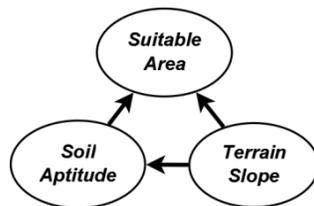


Figure 1: Example of Bayesian Network graphical model.

The prior knowledge of an event is updated taking into account new evidence through the Bayes' theorem:

$$P(A = a | B = b) = \frac{P(B = b | A = a)P(A = a)}{P(B = b)},$$

in which $P(A = a)$ is the prior probability of the event A; $P(B = b | A = a)$ is the likelihood function and $P(A = a | B = b)$ is the posterior probability; and $P(B = b)$ is a normalizing constant [Neapolitan 2004]. Upper-case letters denote the variables and the same but lower-case letters denote the state or value of the variable. This ability to compute posterior probabilities given new evidence is called inference.

Bayesian Networks Model has been used in many applications employing Bayesian inference to develop a plausible reasoning and to describe the occurrence probability of a phenomenon. Aguilera et al. (2011) argue that the BN have mainly been used as a technique for inference in Environmental Sciences. Dlamini (2010) presented a BN approach to estimate the probability of wildfire occurrence based on satellite-detected wildfires data and a set of social, physical, environmental and climatic factors. McCloskey et al. (2011) proposed a BN to identify suitable sites for the economic development and landscape conservation, while Gonzalez-Redin et al. (2016) used BN approach to find areas for a sustainable timber production and biodiversity conservation. Mello et al. (2013) developed a BN model for raster data analysis to study soybean mapping based on remote sensing variables.

However, Aguilera et al. (2011) reported that few studies have used methods based on BN in agriculture applications. Therefore, we expected that this study contribute to show the potentiality of BN models, which are used to infer potential areas for sugarcane expansion in this study.

2.1. e-BayNeRD algorithm

To identify the appropriate areas for sugarcane expansion, we used e-BayNeRD method [Silva et al. 2014], which is briefly described below. Figure 2 shows the e-BayNeRD's workflow.

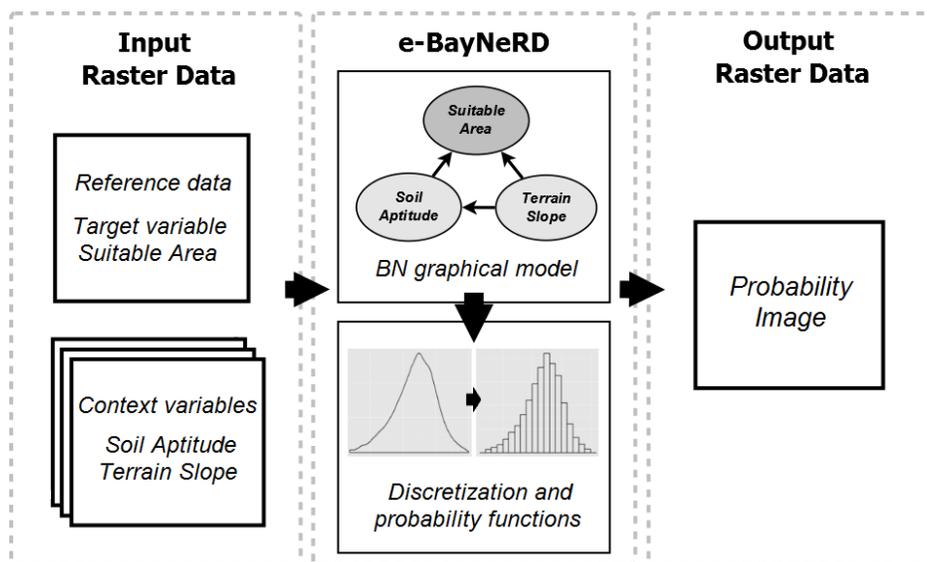


Figure 2. e-BayNeRD's Workflow.

The method deals with raster data in GeoTiff format, in which each GeoTiff corresponds to a variable (node) in the BN model. The variable that represents the studied phenomenon is called *target variable* and its GeoTiff must contain the reference data for training. To illustrate, suppose that *Suitable Area* is the target variable in the BN model exemplified in Figure 1. The others variables in the model are called *context variables*, and they can have some relation with the target one and they can have some interrelation. After entering all the raster data, the user designs the BN graphical model (i.e., the DAG) by defining the relations among all variables.

Once the BN graphical model is defined, user needs to convert continuous context variables into categorical ones. In the discretization phase, the range of observed values for each variable is divided into intervals according to the lower and upper limits chosen by the user. Each interval is one category; therefore, a variable will have n categories if user discretizes it into n intervals. Figure 3 illustrates the *Terrain Slope* variable discretized in three categories: $[0, 8)$, $[8, 13)$, $[13, \max]$. Values into the bars indicate the probability to find a pixel within the defined interval limits. The limits of the intervals for each variable should be appropriately chosen to describe as best as possible the variable according to the phenomenon studied.

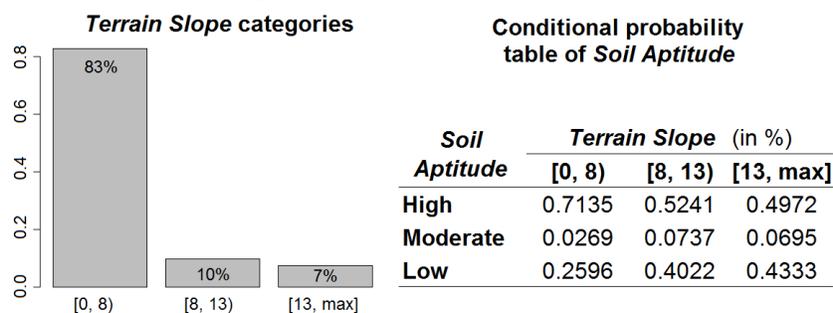


Figure 3. Example of the discretization phase and computed conditional probability table.

In the e-BayNeRD algorithm, probability functions are computed based on pixel counting according to the dependence relations among variables and their categories. Prior probability is assigned to those variables without parents (e.g.: *Terrain Slope* variable), whereas conditional probability is assigned to descendant ones. Figure 3 illustrates the conditional probability table associate to the *Soil Aptitude* variable, in which *High*, *Moderate* and *Low* are categories for *Soil Aptitude*. Values in table are conditional probabilities $P(\text{Soil Aptitude} | \text{Terrain Slope})$, which indicate the probability of finding a pixel equal to some *Soil Aptitude* category given that this pixel is equal to some *Terrain Slope* category.

After compute the probability functions associated with each variable, e-BayNeRD is able to calculate the probability of target presence given the values observed in the context variables. When the probability is computed for each pixel in the study area, the output, called Probability Image, is formed.

3. Study area

The study area covers Goiás and Mato Grosso do Sul states in the Brazilian Cerrado biome excluding some regions of no interest for sugarcane expansion, as showed in Figure 4. Some areas excluded in study are: urban areas, water bodies, areas under environmental protection laws (i.e. conservation units, indigenous lands, the Upper Paraguay River basin, Pantanal Biome), and areas where it is not allowed commercial sugarcane production such as agrarian settlements or quilombo communities.

Figure 4 shows also cultivated sugarcane areas in Goiás and Mato Grosso do Sul for 2008/2009 crop year (dark green color). These regions were excluded from the study area because they were also excluded in the Sugarcane Agroecological Zoning, which was used as a reference data for training and assessing the model for infer potential areas for sugarcane expansion proposed in this work.

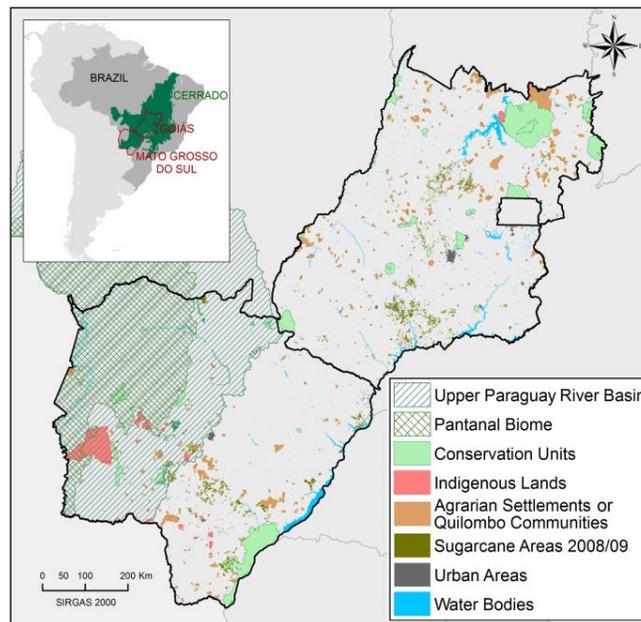


Figure 4. States of Goiás and Mato Grosso do Sul and masked out areas from the study.

4. Selected variables to build the BN model

This section presents the variables chosen to compose the BN model. The *target variable* represents appropriate areas for sugarcane expansion and the *context variables* are constraining factors that exhibit any kind of relation with the sugarcane expansion.

4.1. Target variable

Most of the suitable areas for sugarcane expansion in Brazil are located in Goiás and Mato Grosso do Sul states [Manzatto et al. 2009]. Figure 5 shows the *target variable Suitable Area* for sugarcane expansion according to the Sugarcane Agroecological Zoning (Source: <http://geo.cnpma.embrapa.br/>). Suitable and non-suitable areas are denoted by green and yellow colors, respectively. About 70% of the pixels in each class were randomly selected to compose the reference data for training. The remaining 30% were used for accuracy assessment.

4.2. Context variables

Considering that Goiás and Mato Grosso do Sul states have topography adequate for sugarcane expansion [Shikida 2013], we selected *Terrain Slope* variable as a context variable in the model. *Terrain Slope* variable was settled as parent of *Suitable Area* variable in the BN model. (*Terrain Slope* variable source: <http://srtm.csi.cgiar.org/>).

Soil in our study area presents favorable conditions for sugarcane cultivation although there are some poor-nutrient areas as stated by [Shikida 2013]. As soil conditions influence sugarcane cultivation, we set *Soil Aptitude* variable as parent of *Suitable Area* variable. Considering that terrain slope is an important factor to determine the aptitude to some crop cultivation, we set *Soil Aptitude* variable as descendant of *Terrain Slope* variable. (*Soil Aptitude* variable source: <http://mapas.mma.gov.br/i3geo/datadownload.htm>).

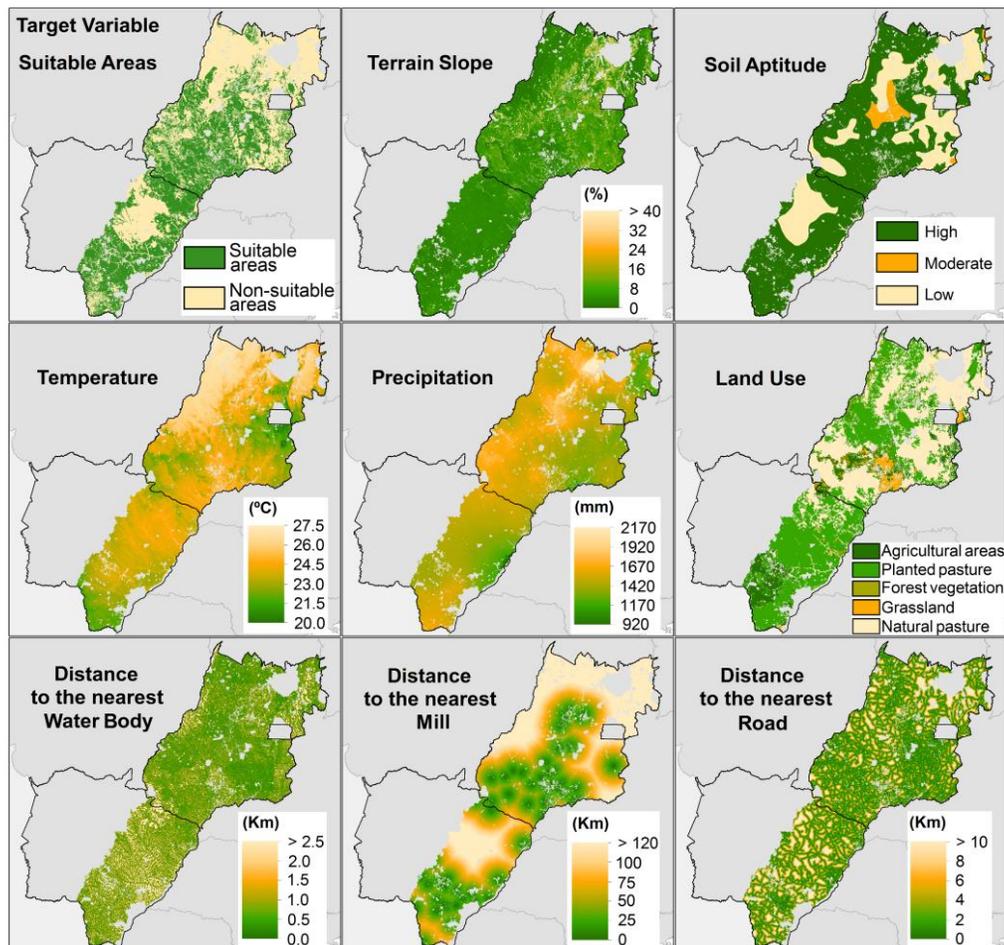


Figure 5. GeoTiff data of target and context variables.

Furthermore, it is important to consider climate risks to cultivate sugarcane [Brunini et al. 2008]. In this sense, the climate data *annual mean Temperature* and *annual Precipitation* were also selected as context variables. As these variables indicate suitable climate conditions regions for sugarcane cultivation, we set them as parent of *Suitable Area* variable in the BN model (*Temperature* and *Precipitation* variables source: <http://www.worldclim.org/>).

Sugarcane expansion in the South-central region of Brazil has occurred mainly on pasturelands and eventually on annual crops [Adami et al. 2012; Castro et al. 2010]. Taking this account, the land use maps produced by IBGE (12 classes) were rasterized to represent the *Land Use* variable. Land use classes were grouped in five classes: agriculture; planted pasture; forest; grassland; and natural pasture. Agriculture, planted and natural pasture represent 11%, 45% and 40% of the land use, respectively. Considering that each land use class sets limits to the sugarcane expansion, *Land Use* variable is considered as *Suitable Area* variable's parent. We define *Land Use* variable as descendent of *Terrain Slope* and *Soil Aptitude* variables (*Land Use* variable source: <http://portaldemapas.ibge.gov.br/>).

Another context variable is the *Distance to the nearest Water Body*, which was computed using the drainage network. This variable was selected to take into account conservation of natural vegetation around water bodies/streams. This suggests that sugarcane cultivation does not occur in areas close to water bodies. Therefore, *Distance to the nearest Water Body* variable is set as parent of *Suitable Area* variable. It is noteworthy that small distances from water bodies are generally associated with higher slope. Therefore, *Distance to the nearest Water Body* variable is also descendent of *Terrain Slope* variable (Drainage network source: <http://portaldemapas.ibge.gov.br/>).

Installation of new mills was also a driver for sugarcane expansion in Goiás and Mato Grosso do Sul states, usually located close to the roads to facilitate the production transport [Granço et al. 2015]. Hence, the *Distance to the nearest Mill* and *Distance to the nearest Road* were selected as context variables, and they were built from highways network and points of mills location, respectively. For logistical and economic reasons, it is expected that the sugarcane expansion occurs near mills and roads. Thus, the *Distance to the nearest Mil* and *Distance to the nearest Road* variables can be defined as parents of the *Suitable Area* variable in BN model (Highways network source: <http://portaldemapas.ibge.gov.br/> and points of mills location source: <http://ctbe.cnpem.br/pesquisa/producao-biomassa/cana-info/>).

4.3. BN graphical models and discretization phase

Two BN models were created. The first one identifies sustainable areas for sugarcane expansion taking into account the variables *Terrain Slope*, *Soil Aptitude*, *Precipitation*, *Temperature*, *Land Use* and *Distance to the nearest Water Body*. The second one specifies logistically appropriate areas as well, considering the variables *Distance to the nearest Mill* and *Distance to the nearest Road*. Additionally, the *Probability Image* produced by the first BN model was incorporated into the second BN model as a context variable. Figure 6 shows the first and second BN models.

The limits of the intervals should be appropriately chosen to describe as best as possible the context variable according to the target variable *Suitable Area*. Table 1 presents the interval limits and the categories defined for context variables, which were chosen based on our knowledge and experience.

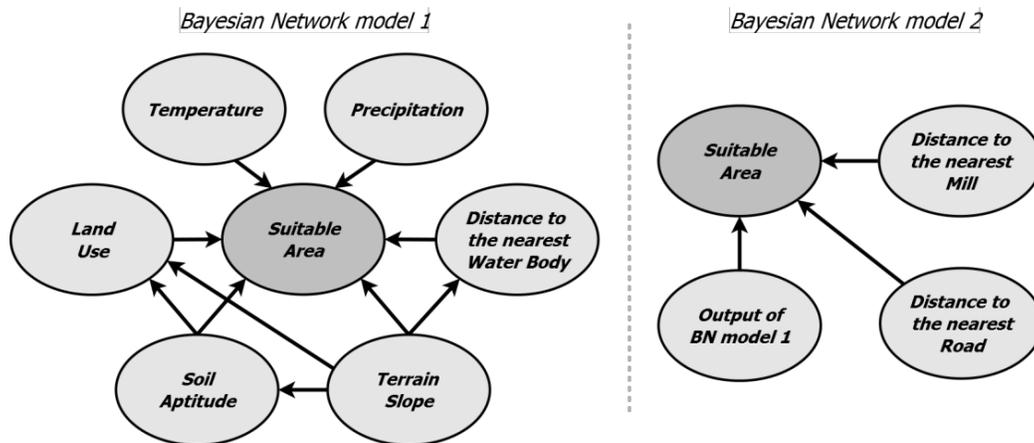


Figure 6. BN model 1 (left): identify sustainable areas; BN model 2 (right): identify sustainable and logistically appropriate areas.

Table 1. Interval limits and categories defined for each context variable.

Context variable	Terrain Slope (%)	Soil Aptitude	Land Use	Temperature (°C)
	[0, 8)	High	Agriculture	[20, 23.6)
	[8, 13)	Moderate	Planted pasture	[23.6, 24.9)
	[13, max]	Low	Forest	[24.9, max]
	-	-	Grassland	-
	-	-	Natural pasture	-
Context variable	Precipitation (mm)	Distance to the nearest Water Body (km)	Distance to the nearest Mill (km)	Distance to the nearest Road (km)
	[920, 1600)	[0, 0.25)	[0, 25)	[0, 1)
	[1600, max]	[0.25, 0.5)	[25, 50)	[1, 2)
	-	[0.5, 1)	[50, max]	[2, 5)
	-	[1, max]	-	[5, max]

5. Results and Discussion

Probability Image is the main e-BayNeRD outcome, in which each pixel value represents the probability of such area be a sugarcane growing area given the observed context variables values. The Probability Images produced by both BN models are shown in Figures 7.

Green colored pixels in Figure 7 represent areas with high probability for sugarcane expansion. High probability values were achieved when context variables exhibited good conditions for sugarcane plantation: *Terrain Slope* < 8%; high *Soil Aptitude*; *Temperature* < 24.9 °C; *Distance to the nearest Water Body* > 1km; agriculture or grassland as *Land Use*; *Distance to the nearest Mill* < 50km; *Distance to the nearest Road* < 5km; and *Precipitation* had no much influence. This result is similar to those obtained by Ribeiro et al. (2015). This result can be explained by two factors: agricultural aptitude and logistical issues.

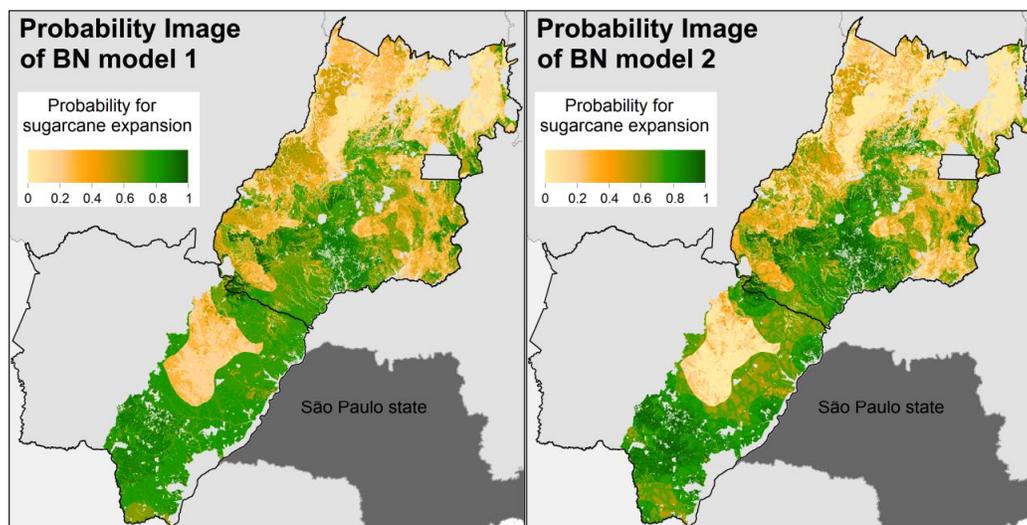


Figure 7. Probability Images of: BN model 1 (left) and BN model 2 (right). Green colored pixels represent areas with high probability for sugarcane expansion.

In according to Castro et al. (2010) and Silva and Miziara (2011), there is sugarcane expansion tendency in the south-central of Goiás state. Areas in the south of Goiás present appropriate conditions for sugarcane cultivation. The Sugarcane Agroecological Zoning [Manzatto et al. 2009] pointed out that most areas in Goiás have conditions for sugarcane expansion. Our BN models showed high probabilities for sugarcane expansion in the south-central of Goiás. Silva and Miziara (2011) argue that sugarcane spatial dynamics is directly related to installation of new mills. Indeed, our results showed a similar spatial distribution between the mills locations (context variable *Distance to the nearest Mill* in Figure 5) and the potential areas for sugarcane expansion. This distribution is known as the south-central expansion axis [Castro et al. 2010], which can be observed in our results showed in Figure 7.

On the other hand, sugarcane expansion is more restricted in the north of Goiás, where *Terrain Slope* (above 13%) and *Soil Aptitude* (low) are not favorable. Results also presented low probability values in the west and east of Goiás. In this case, the limiting factor in the modeling was high *annual Temperature* in the west and high *Terrain Slope* and low *Soil Aptitude* in east.

Regarding the state of Mato Grosso do Sul, Probability Images in Figure 7 show that more suitable regions for sugarcane expansion are located in the eastern and southern regions, which are near to São Paulo state, the main domestic market for ethanol. This result is in according to those presented by Manzatto et al. (2009) and Ribeiro et al. (2015). Mato Grosso do Sul state presents favorable agricultural and environmental conditions for sugarcane cultivation. However, low probability values in the central region of the state occurred mainly due to low *Soil Aptitude* in this region. Pasture represents most land use in Mato Grosso do Sul, but the highest probability values in Mato Grosso do Sul were achieved in agriculture areas in the southern of the state.

In addition to the good agriculture and climate conditions in the Mato Grosso do Sul state, it is important to observe that most mills are concentrated in the southern state (context variable *Distance to the nearest Mill* in Figure 5). It is possible to note that potential areas for sugarcane expansion stated more specific in the result of second BN model, which considered the *Distance to the nearest Mill* and *Distance to the nearest Road* context variables. Mills distribution in the southern of Mato Grosso do Sul facilitates the logistics and transportation of ethanol, since this region has better road infrastructure connecting it with São Paulo state [Granco et al., 2015].

5.1. BN models assessment

We analyzed the Probability Images (Figure 7) for sugarcane areas only in 2012/2013 crop year, which correspond to 955.100 hectares approximately, according to Canasat project [Rudorff et al. 2010]. Sugarcane was monitored in the South-central region of Brazil by the Canasat Project from 2005 to 2013 [Rudorff et al. 2010].

Probability Images can be used to indicate the best regions for sugarcane expansion. Based on these probabilities, we sliced the range of probability values in aptitude categories for sugarcane expansion, as illustrated in Figure 8. We verified that the expansion has mostly occurred in areas in which BN models considered as high probability of sugarcane cultivation. For BN model 1, about 75% of sugarcane areas have probability ≥ 0.7 ; for BN model 2, about 85% of sugarcane areas have probability

≥ 0.7 . These results indicates that second BN model, which infer about sustainable and logistically appropriate areas for sugarcane expansion, it is better than the first BN model.

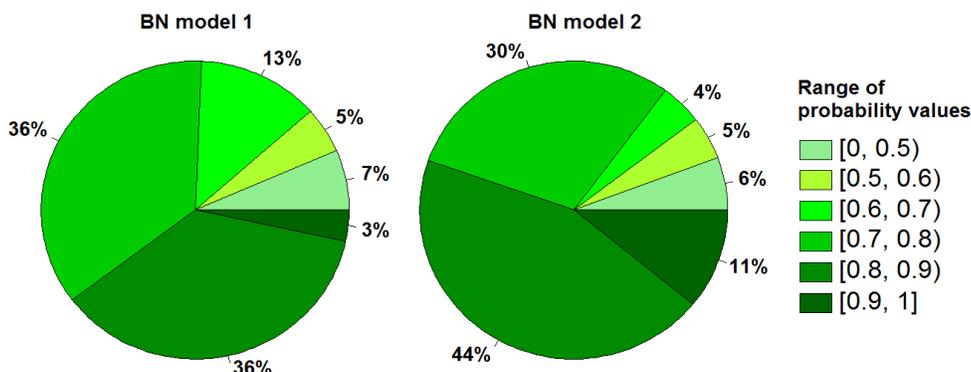


Figure 8: Percentage of sugarcane areas in the 2012/2013 crop year according to the range of probability values for each BN model.

Two complementary indices are commonly used in binary classifications: sensitivity, which indicates true positives rate (target areas labeled as target), and specificity, which indicates true negatives rate (non-target areas labeled as non-target). As BN models output a predicted probability, several thematic maps were produced by varying the probability threshold from 0 to 1. For example, if threshold = 0, all pixels are classified as suitable area. If all pixels that are suitable for sugarcane cultivation are correctly classified, then the sensitivity value is equal to 100%. On the other hand, pixels that are not suitable but are misclassified lead to specificity value equals to 0%.

The Receiver Operating Characteristic (ROC) curve [Hanley and McNeil 1982] is a graph that shows performance of a binary classifier in terms of two indices. ROC curve graph is plotted with sensitivity (Y-axis) versus 1-specificity (X-axis). To generate the ROC curve it is necessary to plot sensitivity versus 1-specificity for all possible classification thresholds. The upper left corner is the best point, where both indexes are equal to 100%.

Figure 9 shows ROC curve for both BN models. In ROC curve, points plotted above diagonal represent a classification better than random (random guess). This means that BN models did a good job separating suitable areas and non-suitable areas classes. In the Probability Image for BN model 1, threshold = 0.45 resulted in a point closest to the upper left corner with sensibility = 84% and specificity = 67%. In the BN model 2, threshold = 0.51 resulted in a point with sensibility = 85% and specificity = 72%.

Area under the curve (AUC) is used to quantify BN model performance. AUC is the percentage of area under the ROC curve [Fawcett 2006]. Therefore, points in the ROC curve closest as possible to the upper left corner represent the best accuracy. AUC for BN model 1 and BN model 2 was 82% and 84%, respectively. This indicates that the BN models are able to differentiate suitable area and non-suitable area classes.

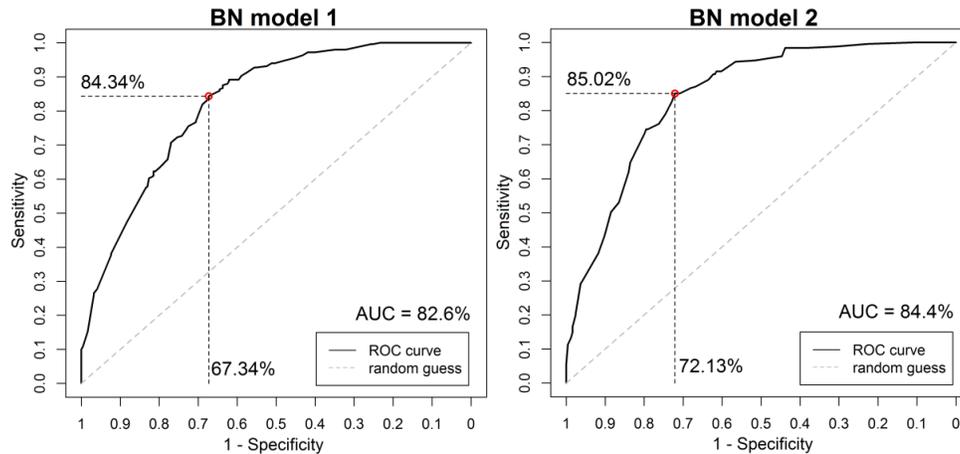


Figure 9. ROC curve of the first (left) and second (right) BN model.

6. Conclusion

This work used Bayesian Network method based on raster data observations to identify adequate areas for sugarcane expansion in the Brazilian Cerrado in the states of Goiás and Mato Grosso do Sul. Most of new sugarcane areas in recent crop year occurred in regions in which the model assigned to them high probability values ($> 70\%$). This indicates that the Probability Image output of BN models can be used to direct the sugarcane expansion to most suitable areas. Bayesian Network model was able to distinguish suitable and non-suitable areas for sugarcane expansion and it can be used as a planning decision tool.

In general, the study area presented favorable agro-environmental conditions for sugarcane plantation. Among all constraining factors, *Terrain Slope* and *Soil Aptitude* variables were the main drivers for sugarcane expansion. The highest probability values for expansion were achieved in agricultural areas and grassland. Future research should consider only pasture areas for sugarcane expansion, since there is a concern about competition with other agricultural areas; and also compare the e-BayNeRD method with others similar algorithms.

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