OBJECTIVE DETECTION OF KINEMATIC AND MAGNETIC VORTICES IN ASTROPHYSICAL PLASMAS

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NO DOUBLE-GYRE HERE

LCS IN ASTROPHYSICAL PLASMAS

The Astrophysical Journal Letters, 735:L9 (7pp), 2011 July 1 doi:10.1088/2041-8205/735/1/L9

LAGRANGIAN COHERENT STRUCTURES IN NONLINEAR DYNAMOS

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Phys. Scr. **86** (2012) 018405 (9pp) doi:10.1088/0031-8949/86/01/

Lagrangian chaos in an ABC-forced nonlinear dynamo

Erico L Rempel[1], Abraham C-L Chian[2,3] and Axel Brandenburg[4,5]

J. Fluid Mech. (2013), *vol.* 729, *pp.* 309–329. © Cambridge University Press 2013
doi:10.1017/jfm.2013.290

Coherent structures and the saturation of a nonlinear dynamo

Erico L. Rempel[1,†], Abraham C.-L. Chian[1,2,3], Axel Brandenburg[4,5], Pablo R. Muñoz[1] and Shawn C. Shadden[6]

The Astrophysical Journal, 786:51 (13pp), 2014 May 1 doi:10.1088/0004-637X/786/1/51

DETECTION OF COHERENT STRUCTURES IN PHOTOSPHERIC TURBULENT FLOWS

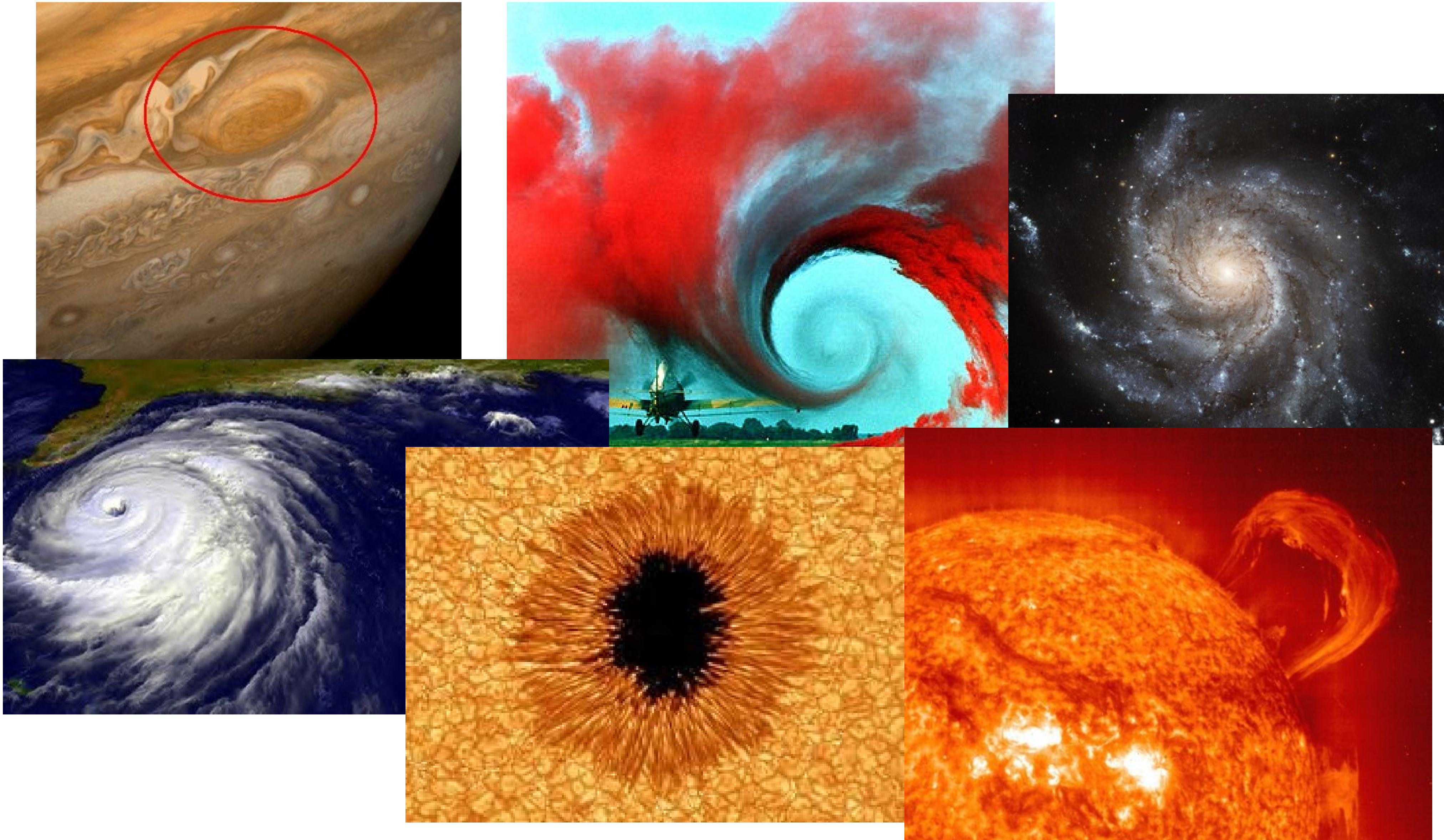
Abraham C.-L. Chian[1,2,3], Erico L. Rempel[1,4], Guillaume Aulanier[2], Brigitte Schmieder[2], Shawn C. Shadden[5], Brian T. Welsch[6], and Anthony R. Yeates[7]

Monthly Notices of the Royal Astronomical Society
MNRAS **466,** L108–L112 (2017) doi:10.1093/mnrasl/slw248
Advance Access publication 2016 December 10

Objective vortex detection in an astrophysical dynamo

E. L. Rempel,[1,2★] A. C.-L. Chian,[1,2,3] F. J. Beron-Vera,[4] S. Szanyi[5] and G. Haller[5]

WHAT ARE COHERENT STRUCTURES? A WAY OF REDUCTION



WHAT ARE VORTICES?

- Regions of high vorticity?
- Circular motion?
- Convex shape?
- Persistent in time?

LAGRANGIAN AVERAGED VORTICITY DEVIATION

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v} \qquad\qquad \dot{\boldsymbol{x}} = \boldsymbol{v}(\boldsymbol{x}, t)$$

$\bar{\omega}$ Instantaneous spatial mean vorticity

$$\mathrm{IVD}(\boldsymbol{x}, t) := |\boldsymbol{\omega}(\boldsymbol{x}, t) - \bar{\boldsymbol{\omega}}(t)|.$$

Instantaneous Vorticity Deviation (IVD)

$$\mathrm{LAVD}_{t_0}^{t_0+\tau}(\boldsymbol{x}_0) := \int_{t_0}^{t_0+\tau} |\boldsymbol{\omega}(\boldsymbol{x}(s), s) - \bar{\boldsymbol{\omega}}(s)| \mathrm{d}s.$$

Source: Haller et al., JFM 795 (2016) 136

LAGRANGIAN AVERAGED VORTICITY DEVIATION

LAVD is invariant under Euclidean frame transformations of the form

$$\boldsymbol{x} = Q(t)\boldsymbol{y} + \boldsymbol{b}(t),$$

where Q and $\boldsymbol{b}$ are arbitrary time-dependent rotation matrix and translation vector, respectively. The transformed vorticity $\tilde{\boldsymbol{\omega}}$ satisties

$$|\tilde{\boldsymbol{\omega}}(\boldsymbol{y}(s), s) - \bar{\tilde{\boldsymbol{\omega}}}(s)| = |\boldsymbol{\omega}(\boldsymbol{x}(s), s) - \bar{\boldsymbol{\omega}}(s)|,$$

Source: Haller et al., JFM 795 (2016) 136

LAGRANGIAN AVERAGED VORTICITY DEVIATION

Why LAVD?

1) It is simple

LAGRANGIAN AVERAGED VORTICITY DEVIATION

Why LAVD?

1) It is simple

2) It was not listed as one of the worst tools in George's talk

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Why LAVD?

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3) It is new

LAGRANGIAN AVERAGED VORTICITY DEVIATION

Why LAVD?

1) It is simple

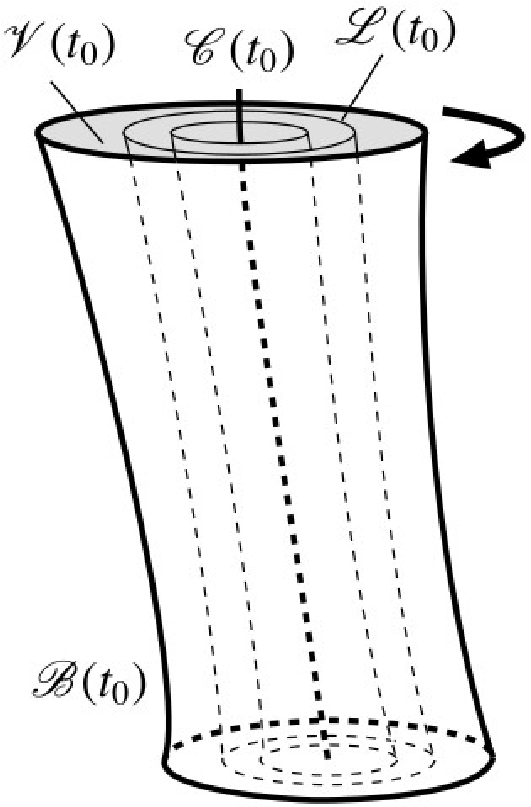
2) It was not listed as one of the worst tools in George's talk

3) It is new

4) It is easily adaptable to use in magnetic fields

LAGRANGIAN AVERAGED VORTICITY DEVIATION

A Lagrangian vortex is an evolving flow domain that is filled with a nested family of convex tubular level surfaces of LAVD with outward decreasing LAVD values. For Eulerian vortices, use IVD instead.



Source: Haller et al., JFM 795 (2016) 136

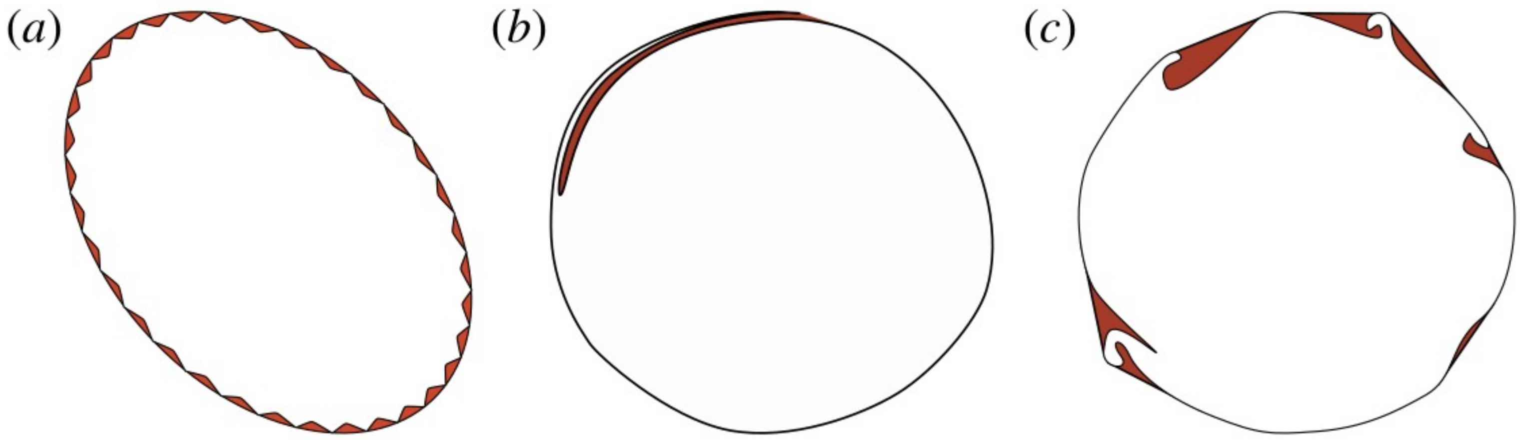
OBJECTIVE VORTEX DETECTION

- Compute the LAVD (IVD) field from a grid of initial particles on a plane in the domain,
- Detect the initial positions of vortex centers as local maxima of the LAVD (IVD) field,
- Seek vortex boundaries as outermost convex closed contours of LAVD (IVD) that encircle vortex centers.

Source: Haller et al., JFM 795 (2016) 136

CONVEXITY DEFICIENCY

One may wish the following non-convex curves to be classified as convex:



The outer line defines the convex-hull, the smallest convex set that contains the inner set.

Source: Haller et al., JFM 795 (2016) 136

CONVEXITY DEFICIENCY

Convexity deficiency, ε: the ratio of the area difference between the curve and its convex hull to the area enclosed by the curve.

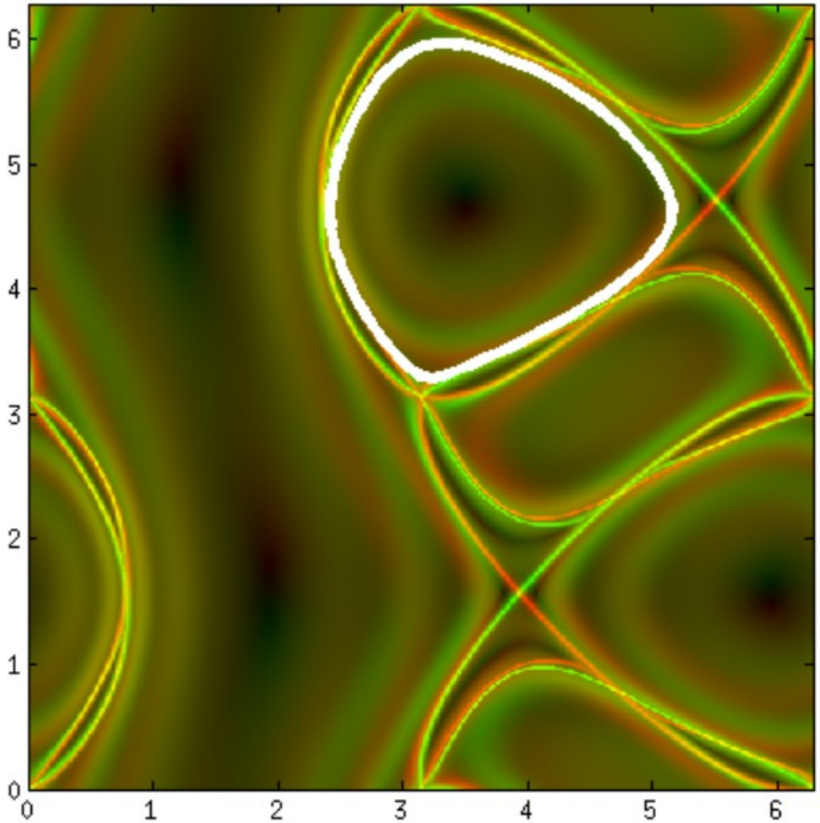
$\varepsilon = (Acc - Ach) / Acc$

Acc – Area of the closed contour
Ach – Area of its convex-hull

LAGRANGIAN AVERAGED VORTICITY DEVIATION

$$\boldsymbol{u}_{ABC} = \frac{A_f}{\sqrt{3}} \begin{pmatrix} \sin k_f z + \cos k_f y \\ \sin k_f x + \cos k_f z \\ \sin k_f y + \cos k_f x \end{pmatrix} \qquad \begin{aligned} k_f &= 1 \\ A_f &= \sqrt{3} \end{aligned}$$

Z=0



LAGRANGIAN AVERAGED VORTICITY DEVIATION

$$\boldsymbol{u}_{ABC} = \frac{A_f}{\sqrt{3}} \begin{pmatrix} \sin k_f z + \cos k_f y \\ \sin k_f x + \cos k_f z \\ \sin k_f y + \cos k_f x \end{pmatrix} \qquad \begin{aligned} k_f &= 1 \\ A_f &= \sqrt{3} \end{aligned}$$

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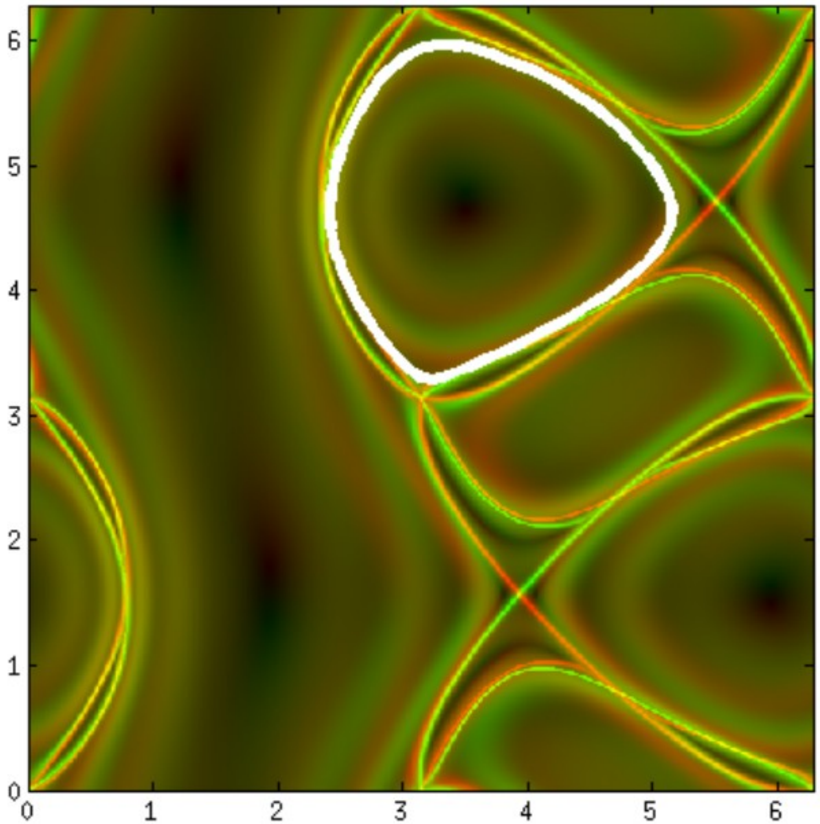
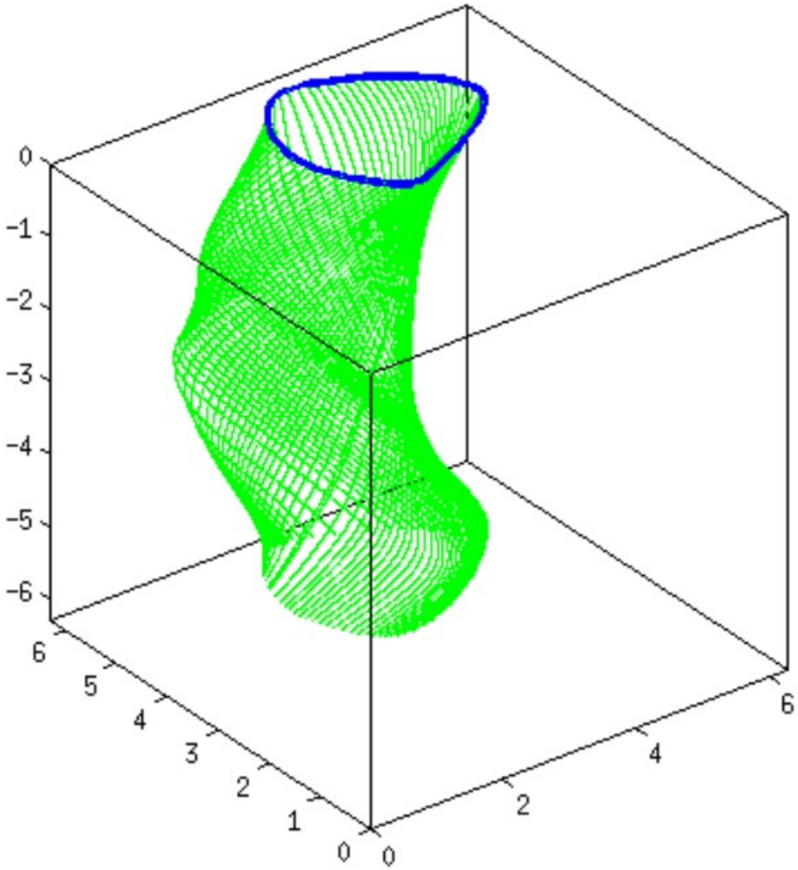
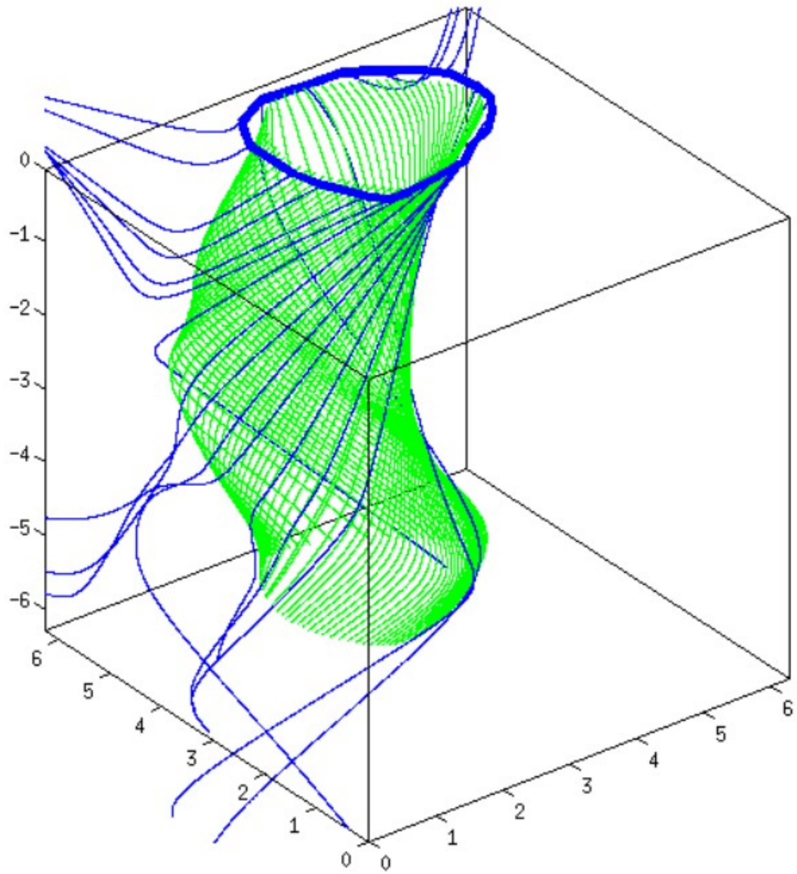
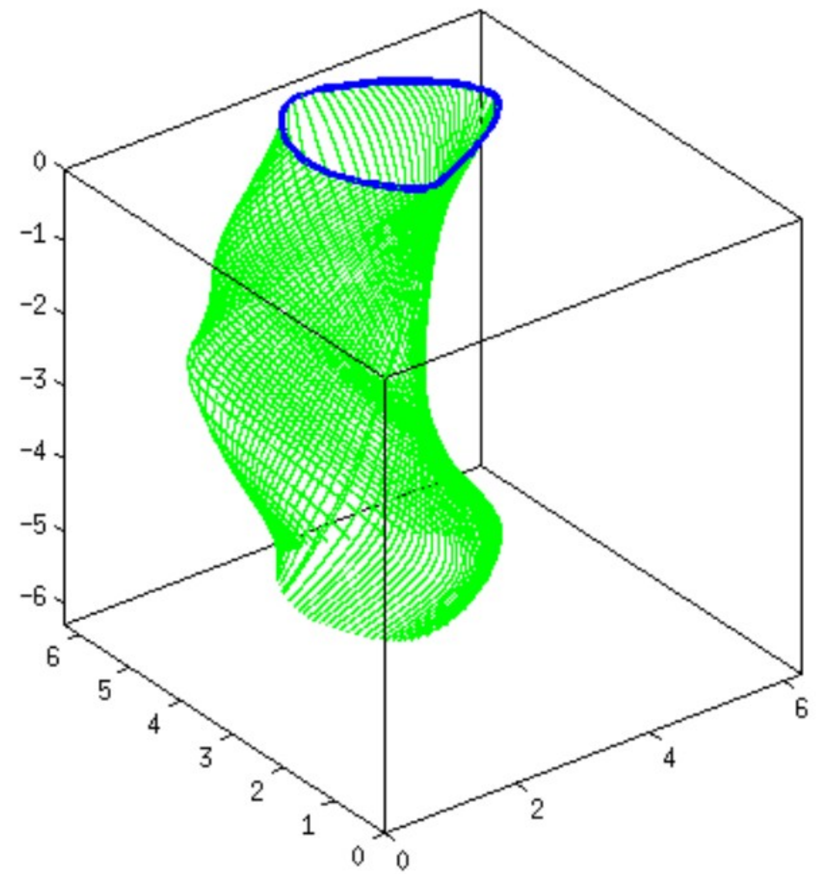




LAGRANGIAN AVERAGED VORTICITY DEVIATION

$$\boldsymbol{u}_{ABC} = \frac{A_f}{\sqrt{3}} \begin{pmatrix} \sin k_f z + \cos k_f y \\ \sin k_f x + \cos k_f z \\ \sin k_f y + \cos k_f x \end{pmatrix} \qquad \begin{aligned} k_f &= 1 \\ A_f &= \sqrt{3} \end{aligned}$$





WHAT IS A MAGNETIC FLUX ROPE?

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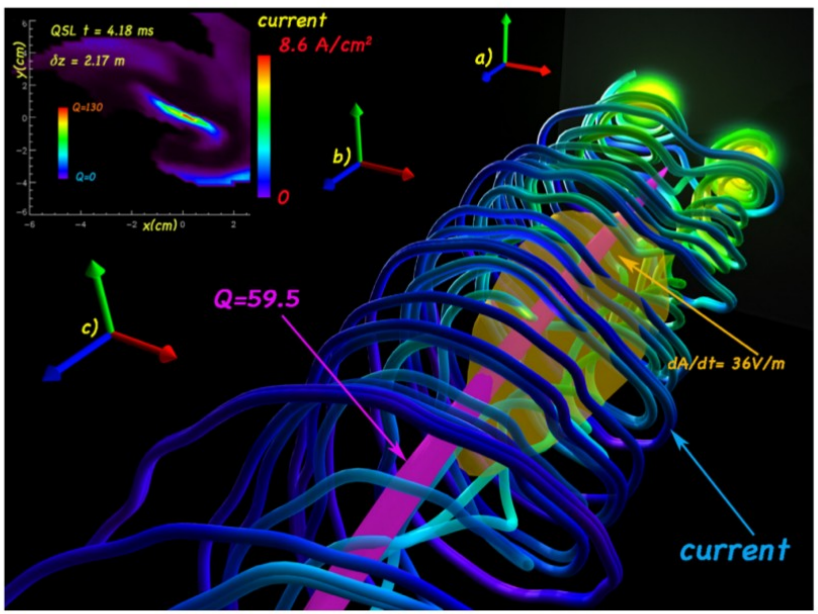
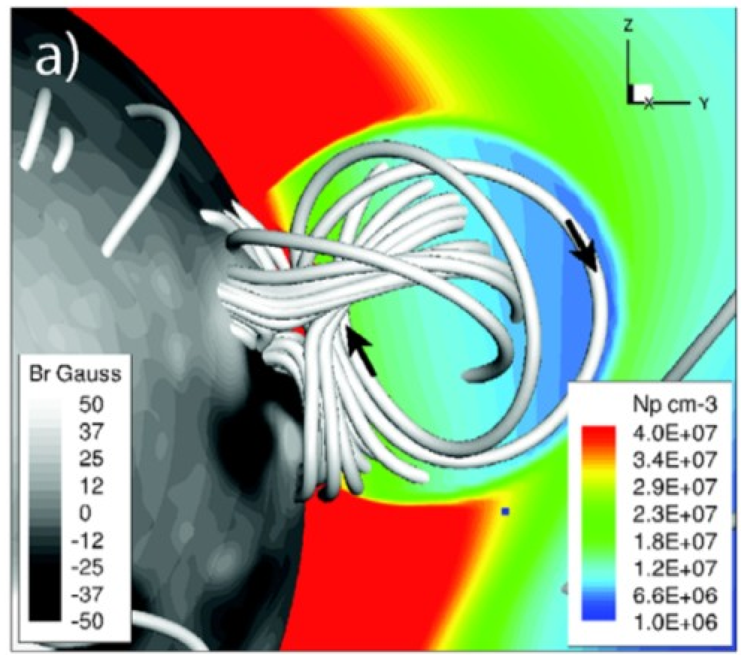
Plasma Physics and Controlled Fusion
Plasma Phys. Control. Fusion **56** (2014) 060301 (2pp)
doi:10.1088/0741-3335/56/6/060301

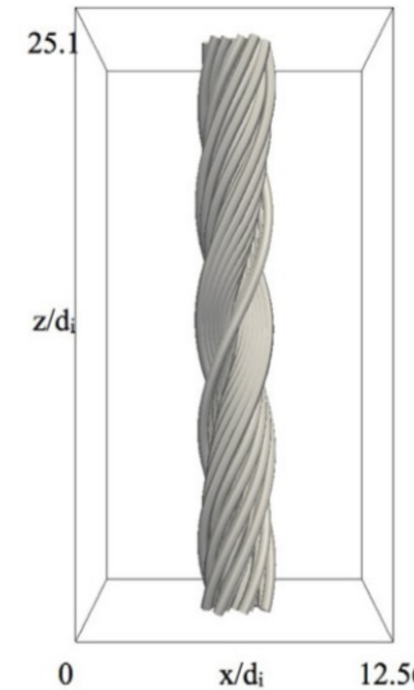
Preface

Guest Editor
Vyacheslav S Lukin

Self-organization in magnetic flux ropes







They can be as narrow as a few larmor radii or as wide as the Sun!

WHAT IS A MAGNETIC FLUX ROPE?

• A magnetic flux tube is a bundle of magnetic field lines; it is a cylindrical region inside which the axial magnetic field is much larger than the magnetic field outside **(but what is the threshold?)**.

• A magnetic flux rope is a twisted flux tube, with helical field lines

• Flux tubes and ropes need not be straight, and their cross-sections can be neither circular, nor uniform along their lengths.

Source: Paul Lorrain et al., Magneto-Fluid Dynamics, Astronomy and Astrophysics Library, 2006

WHAT IS A MAGNETIC FLUX ROPE?

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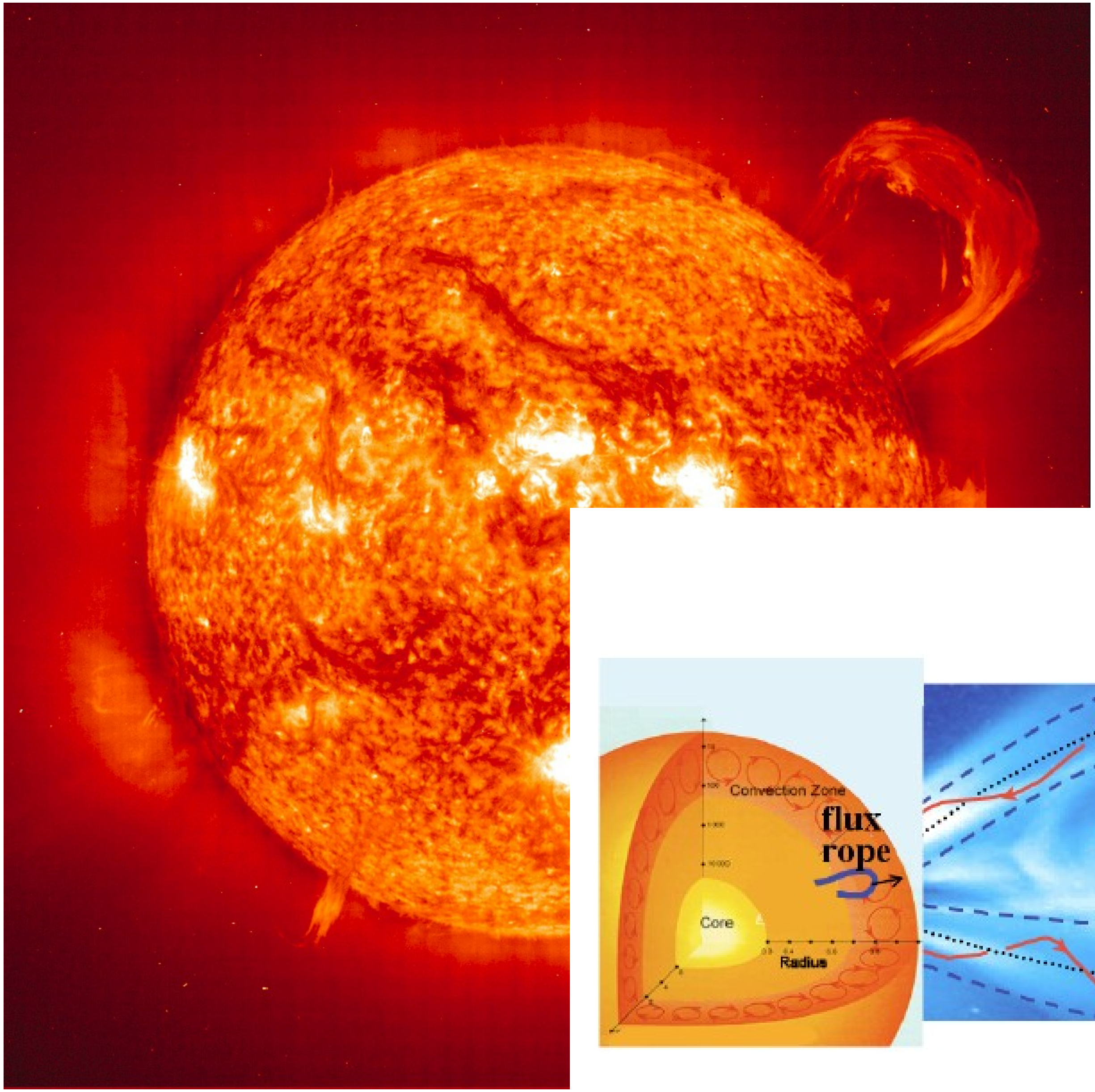
Preface

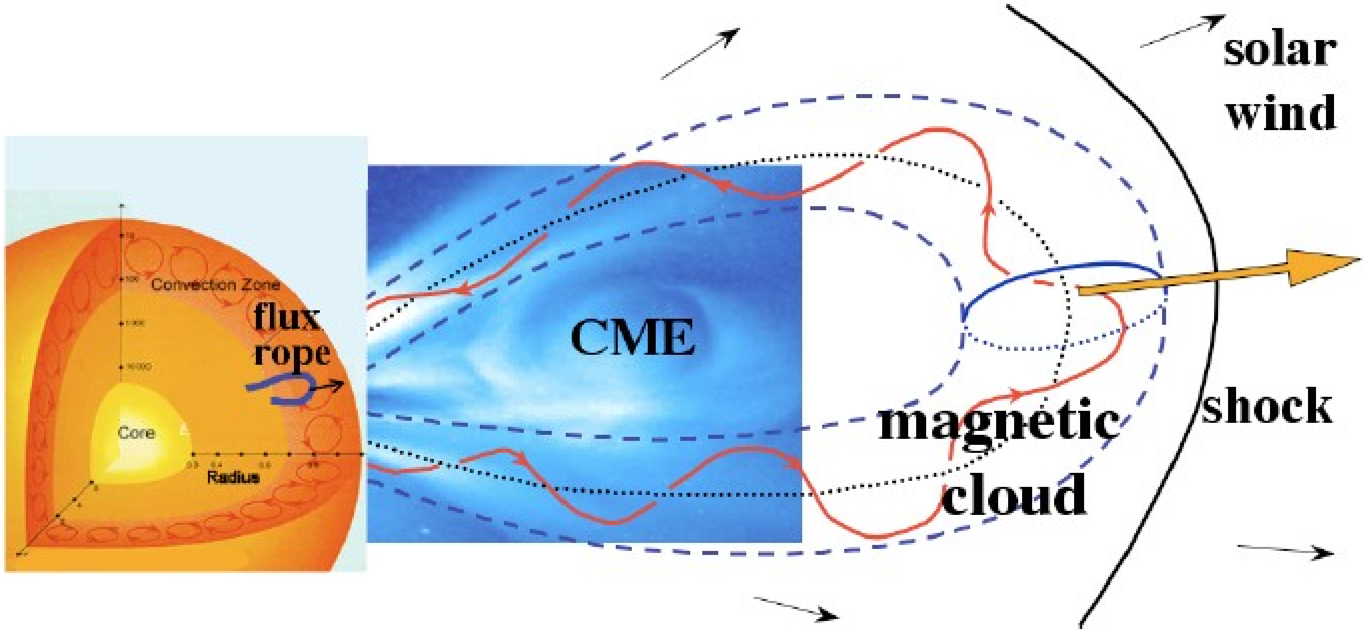
Guest Editor
Vyacheslav S Lukin

Self-organization in magnetic flux ropes

There may not be a strict definition of a magnetic flux rope that everyone can agree on. Nonetheless, the ingredient common to all magnetic flux ropes is that the magnetic field lines that thread nearby plasma elements at one location along the flux rope must wind around and not diverge away from each other over a sufficiently long distance to look like a piece of an ordinary rope. In a way, it is similar to turbulence—you know it when you see it.

ICMEs AS FLUX ROPES





Sources: SOHO ultraviolet image, https://apod.nasa.gov, Demoulin, SW 12 (2010)

MAGNETIC FIELD LINES

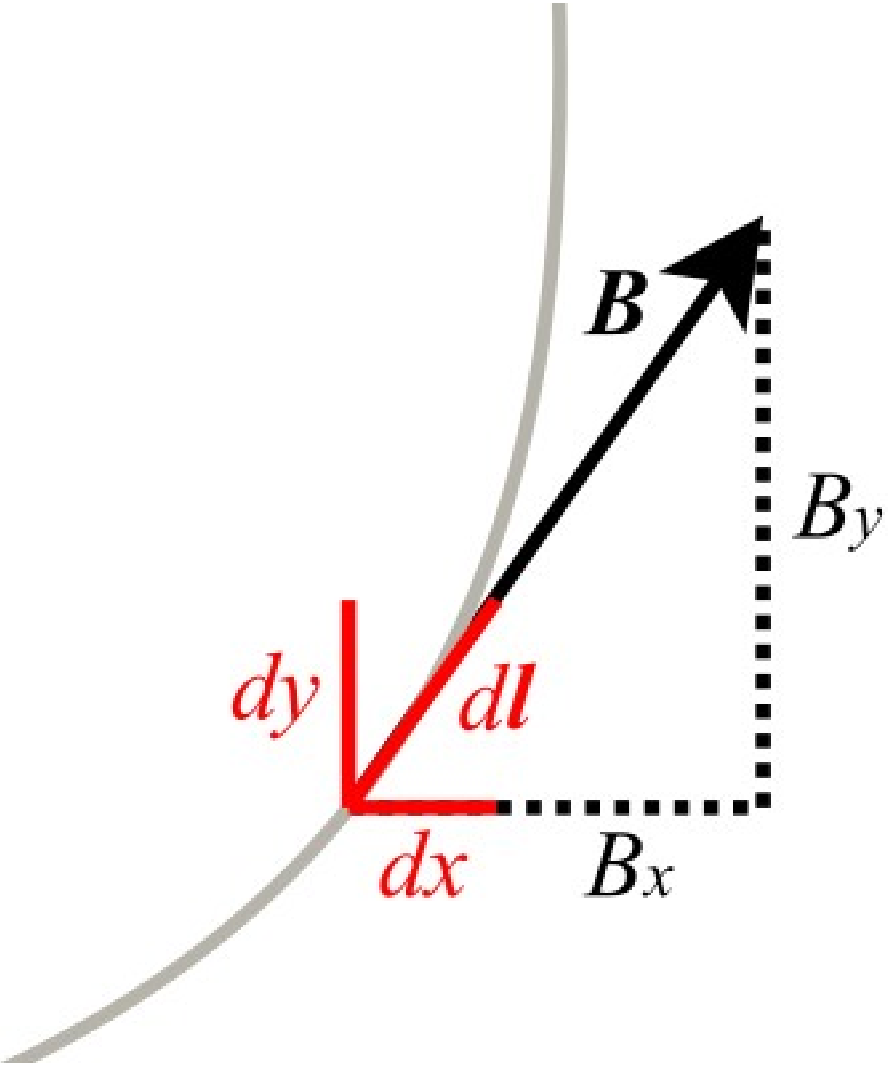
$$\boldsymbol{B} \parallel d\boldsymbol{l}$$

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} = \frac{dl}{B}$$

$$\frac{dx}{dl} = \frac{B_x}{B}$$

$$\frac{dy}{dl} = \frac{B_y}{B}$$

$$\frac{dz}{dl} = \frac{B_z}{B}$$



MAGNETIC FIELD LINES

Adopting $dl = B ds$

$$\frac{dx}{ds} = B_x$$

$$\frac{dy}{ds} = B_y \quad \Rightarrow \quad \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \boldsymbol{B}(\boldsymbol{x}(s), t_0), \qquad \boldsymbol{x}(s_0) = \boldsymbol{x}_0.$$

$$\frac{dz}{ds} = B_z$$

INTEGRATED AVERAGED CURRENT DEVIATION (IACD)

We compute the IACD for ***B*** in essentially the same way as LAVD, but fixing the time and using the current density in place of the vorticity:

$$\boldsymbol{J} = \nabla \times \boldsymbol{B}/\mu_0$$

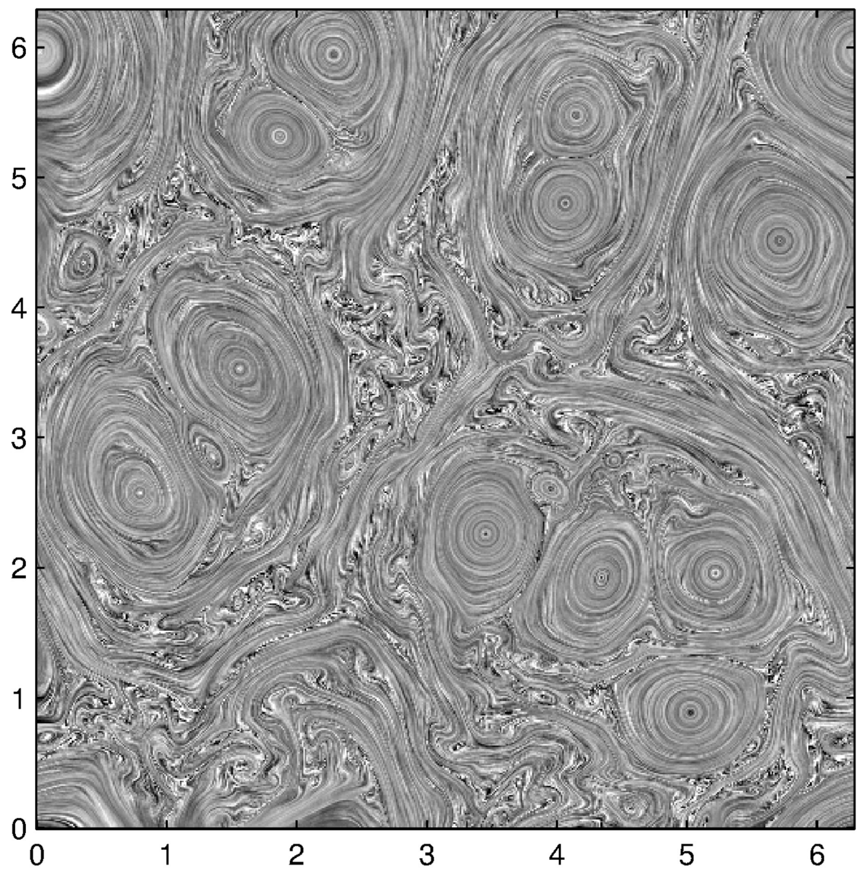
$$\mathrm{IACD}_{s_0}^{s_0+\xi}(\boldsymbol{x}_0) := \int_{s_0}^{s_0+\xi} |\boldsymbol{J}(\boldsymbol{x}(s), t_0) - \bar{\boldsymbol{J}}(t_0)| \mathrm{d}s$$

$\bar{\boldsymbol{J}}(t_0)$ is the mean current density of the box and

$\boldsymbol{x}(s)$ is a solution of $\dfrac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}s} = \boldsymbol{B}(\boldsymbol{x}(s), t_0), \quad \boldsymbol{x}(s_0) = \boldsymbol{x}_0.$

IACD is invariant under changes of the form: $\boldsymbol{x} = Q(s)\boldsymbol{y} + \boldsymbol{b}(s)$

MAGNETIC VORTICES IN 2D MHD TURBULENCE



$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla p - \nabla \cdot (\mathbf{vv} - \mathbf{BB})$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \cdot (\mathbf{vB} - \mathbf{Bv})$$

$$\nabla \cdot \mathbf{v} = 0,$$

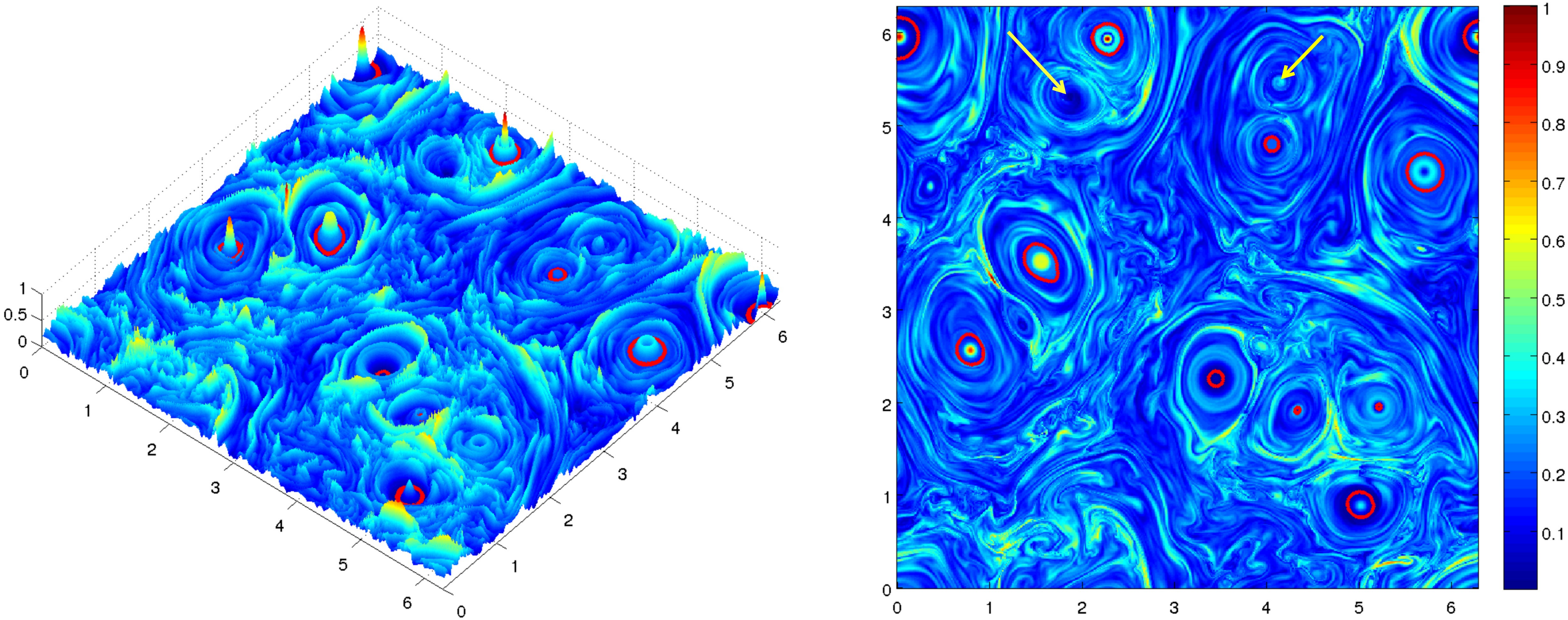
$$\nabla \cdot \mathbf{B} = 0,$$

Periodic boundary conditions, 512 x 512, Finite-differences, 4^{th} order in time, 5^{th} order in space.

Thanks: C.C. Wu, UCLA

KINEMATIC VORTICES IN 2D MHD TURBULENCE

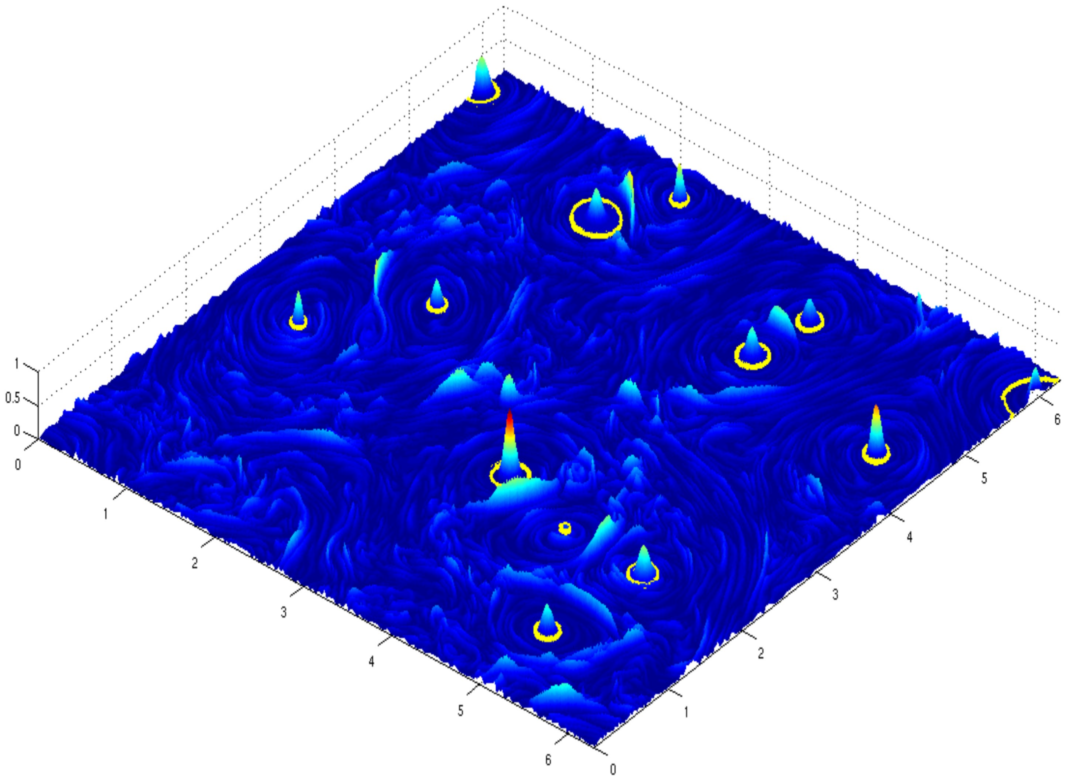
Velocity field vortices, from t=300 to t=386

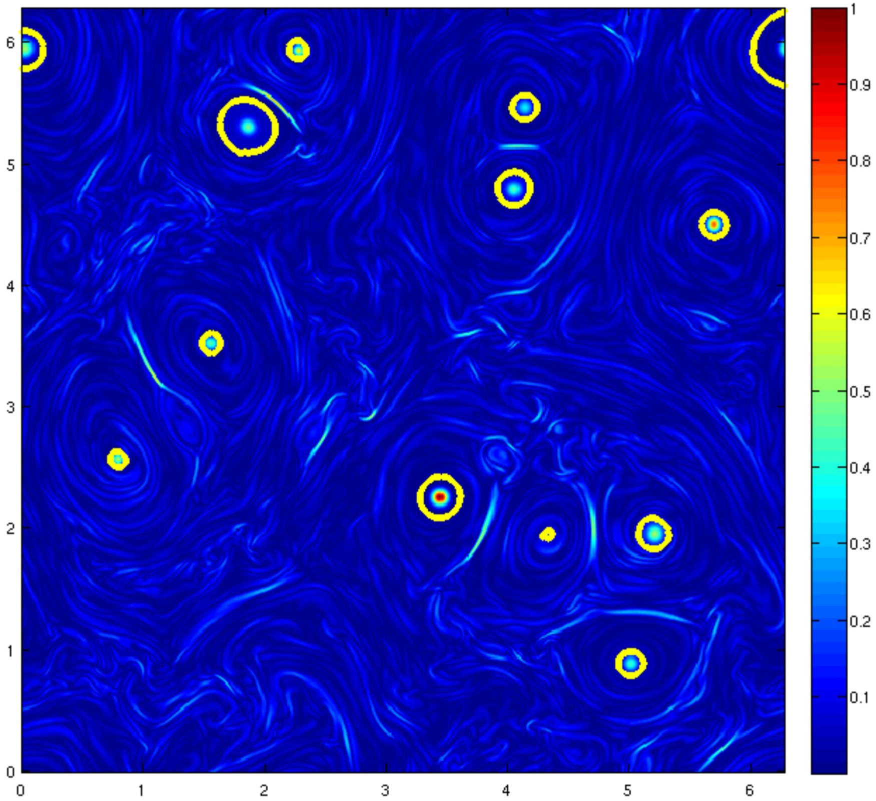


Third order interpolation in space and time. Fourth order Runge-Kutta for particles Integration.

MAGNETIC VORTICES IN 2D MHD TURBULENCE

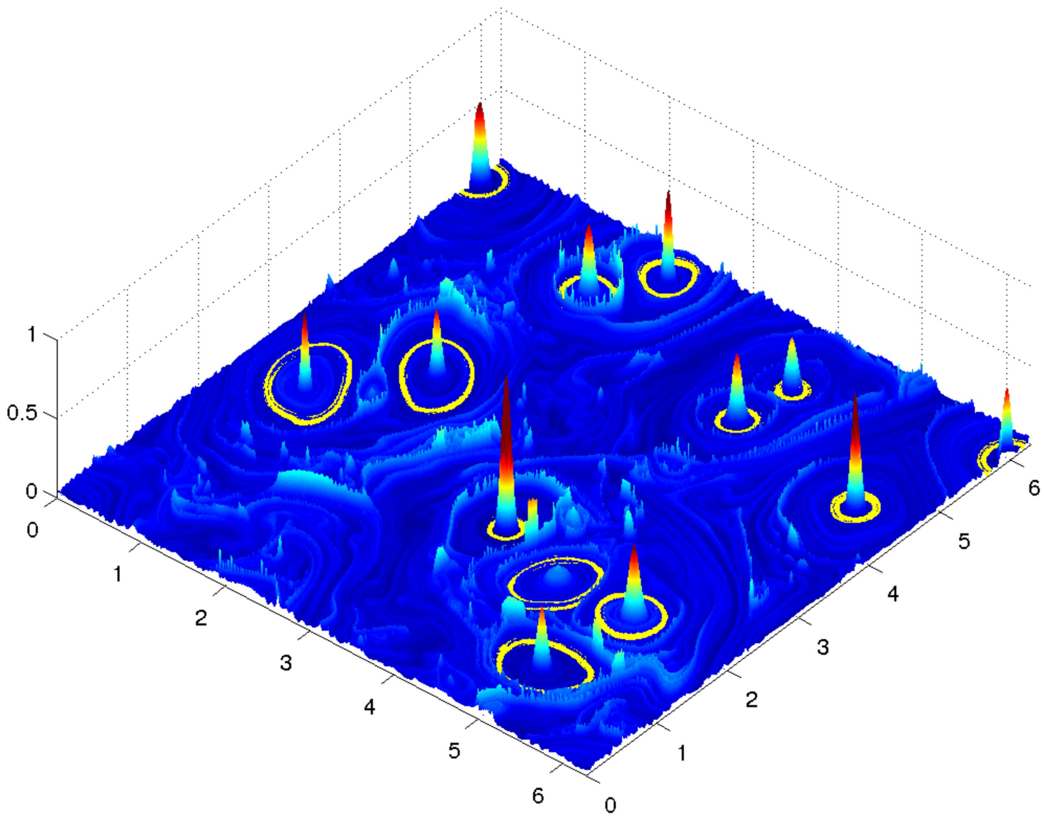
Current density vortices at t=300

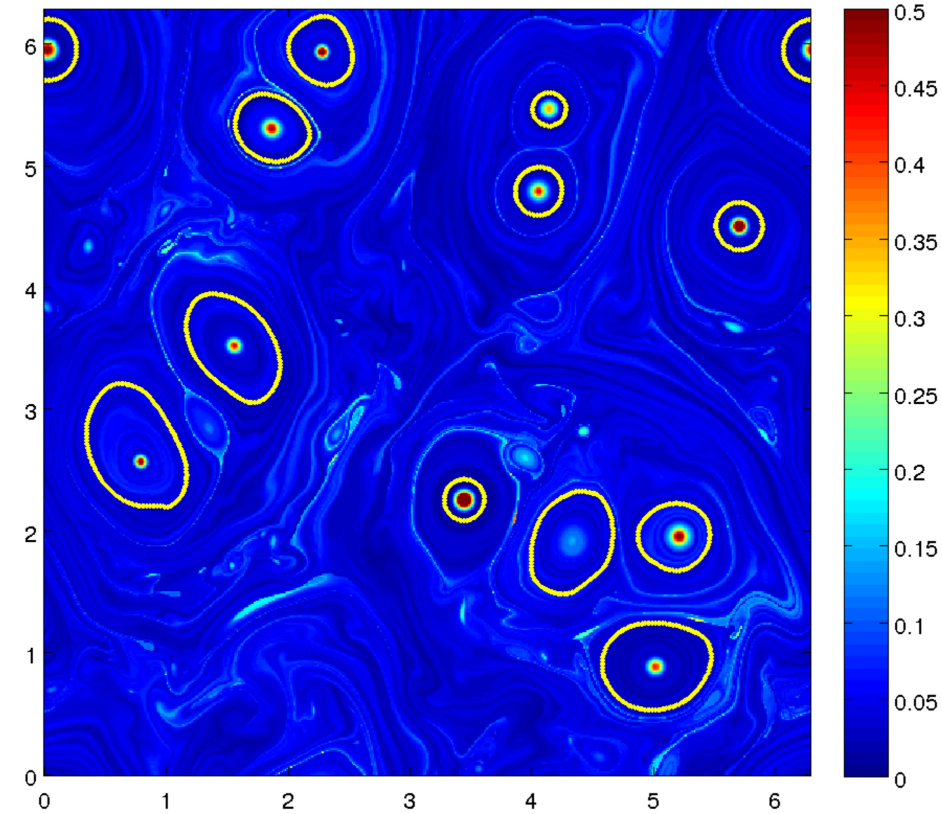




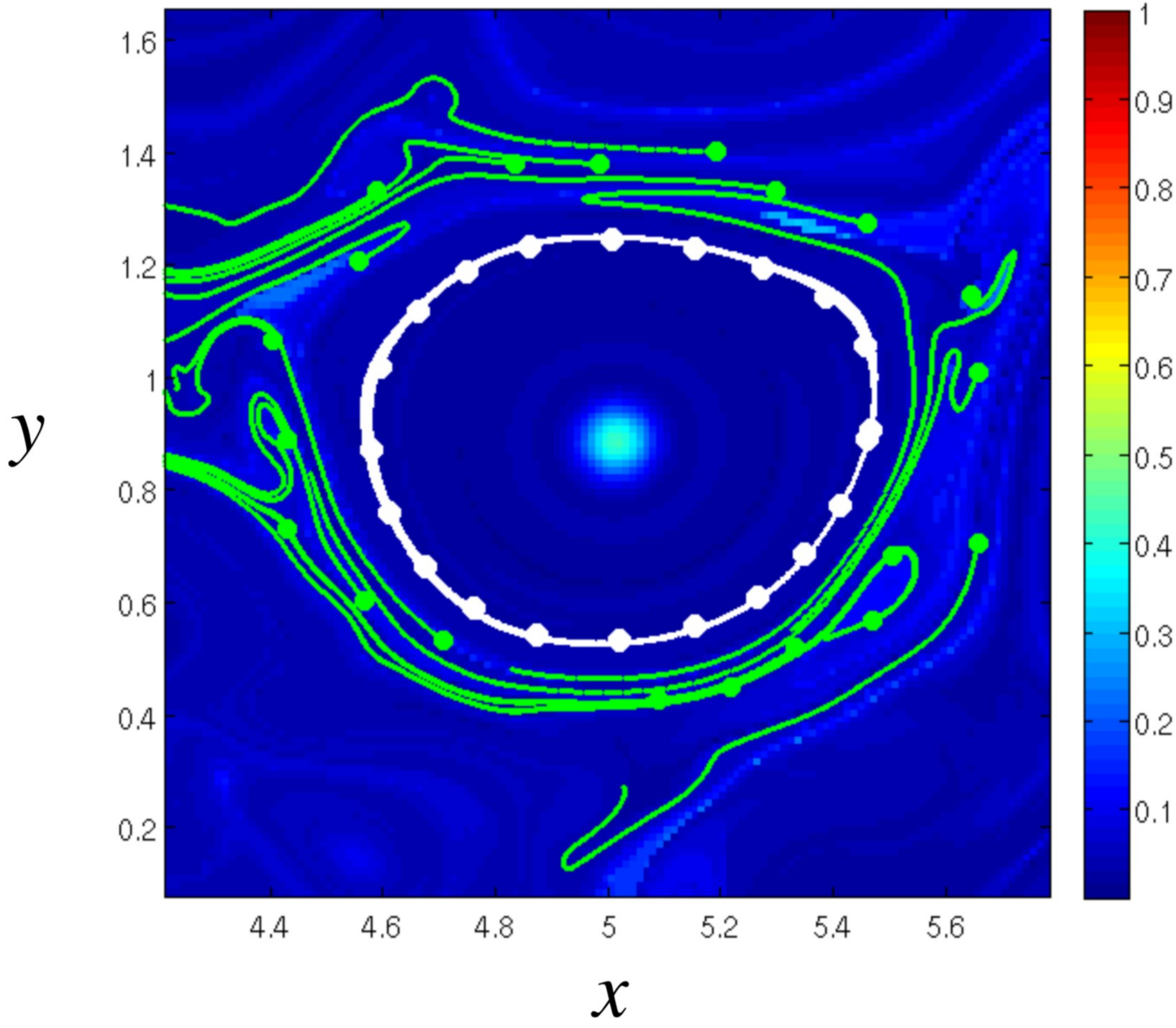
MAGNETIC VORTICES IN 2D MHD TURBULENCE

IACD vortices at t=300, ξ = 100, $\varepsilon \sim 10^{-3}$





MAGNETIC VORTICES IN 2D MHD TURBULENCE



NUMERICAL SIMULATION OF A NONLINEAR DYNAMO

We consider a compressible gas ($\nabla\cdot\mathbf{u} \neq 0$) with constant sound speed c_s, constant dynamical viscosity μ, constant magnetic diffusivity η, and constant magnetic permeability μ_0

Compressible, resistive MHD equations:

$$\partial_t \ln\rho + \mathbf{u}\cdot\nabla\ln\rho + \nabla\cdot\mathbf{u} = 0 \qquad \text{(Continuity eq.)}$$

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -c_s^2\nabla\ln\rho + (\mathbf{J}\times\mathbf{B})/\rho + \mu/\rho(\nabla^2\mathbf{u} + \nabla\nabla\cdot\mathbf{u}/3) + \mathbf{f} \qquad \text{(Momentum eq.)}$$

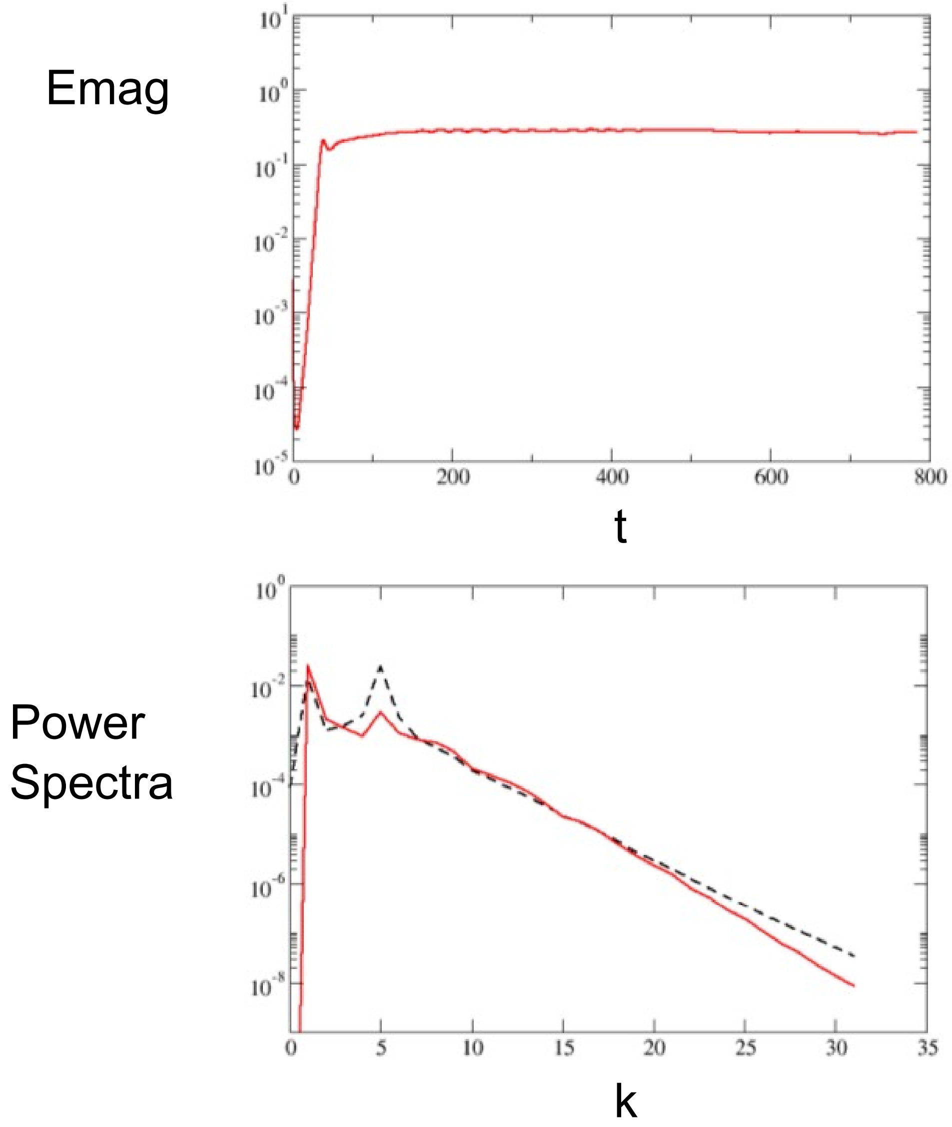
$$\partial_t \mathbf{A} = \mathbf{u}\times\mathbf{B} - \eta\mu_0\mathbf{J} \qquad \text{(Induction eq.)}$$

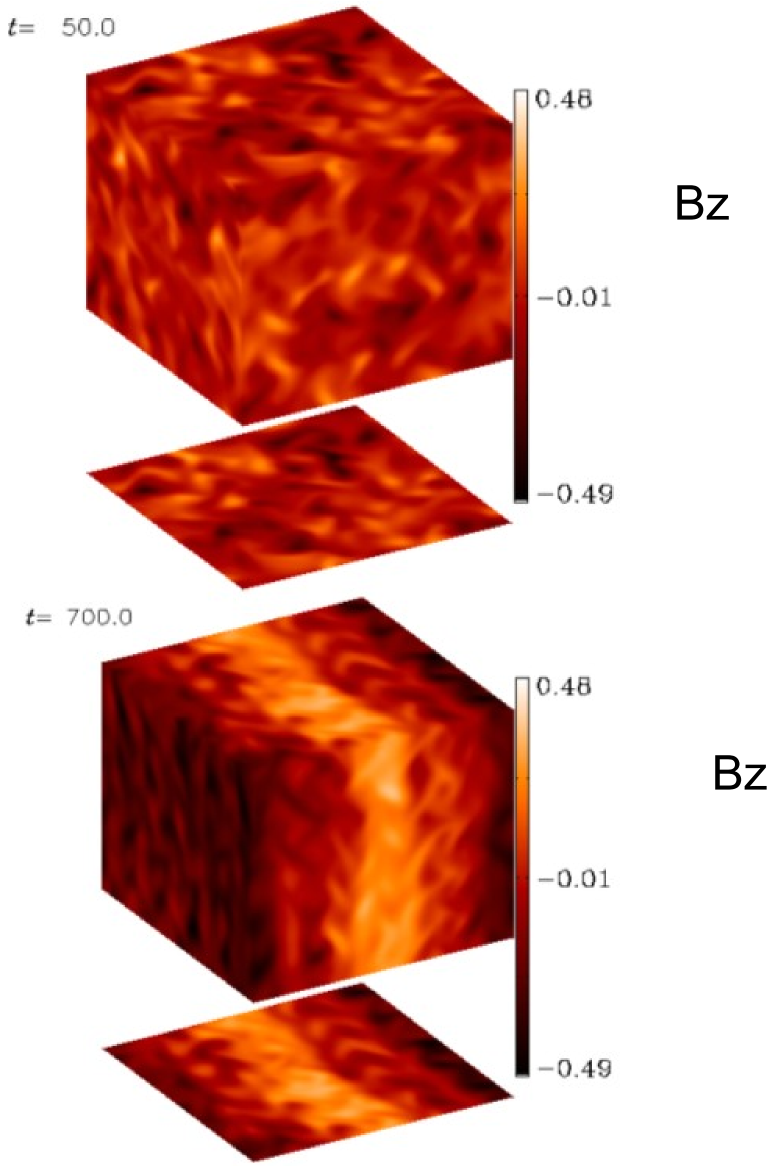
where $\mathbf{J} = \nabla\times\mathbf{B}/\mu_0$ is the current density, $\mathbf{B} = \nabla\times\mathbf{A}$, and the gas is isothermal.

$f = u_{ABC}$

Pencil Code: third order in time, sixth order in space.

NUMERICAL SIMULATION OF A NONLINEAR DYNAMO

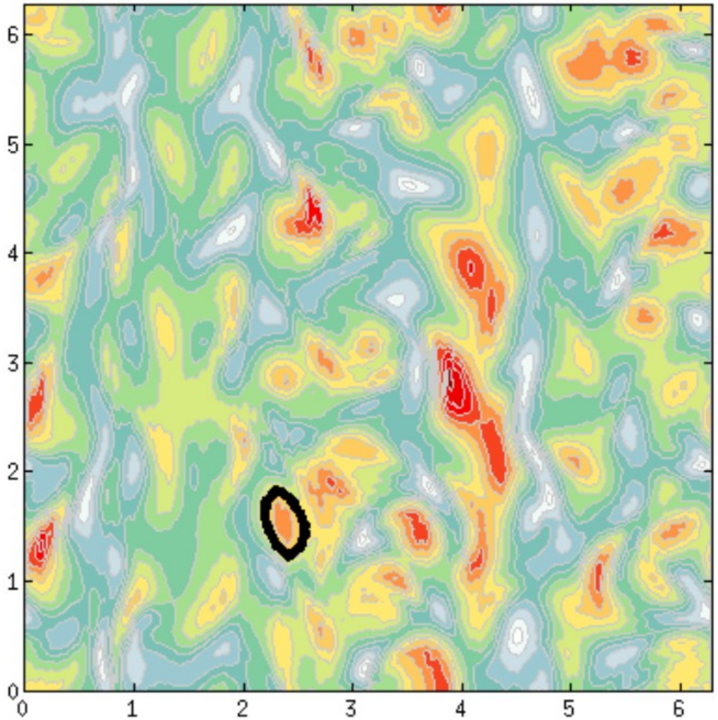


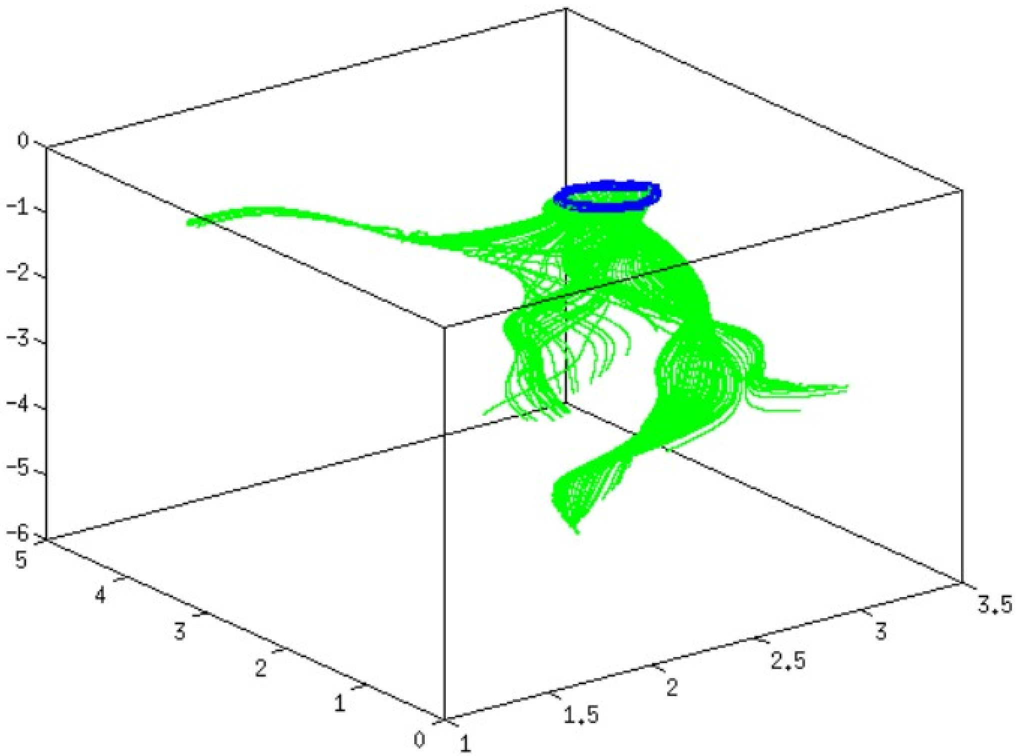


Source: Rempel et al., MNRAS 466, L108–L112 (2017)

INSTANTANEOUS VORTICITY DEVIATION

t = 700
$\varepsilon \sim 10^{-3}$





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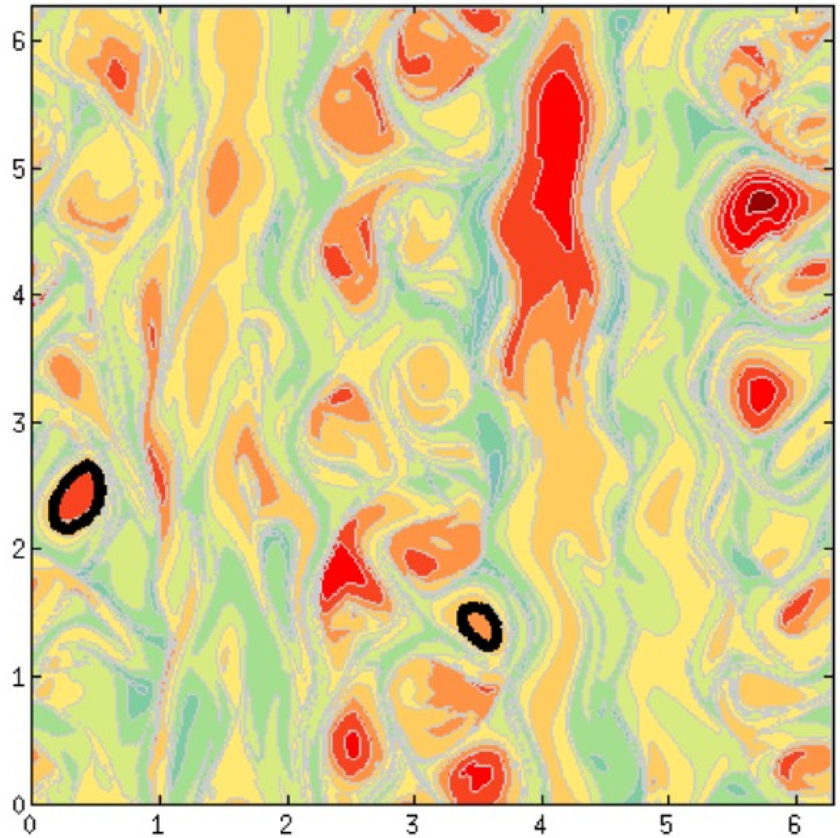
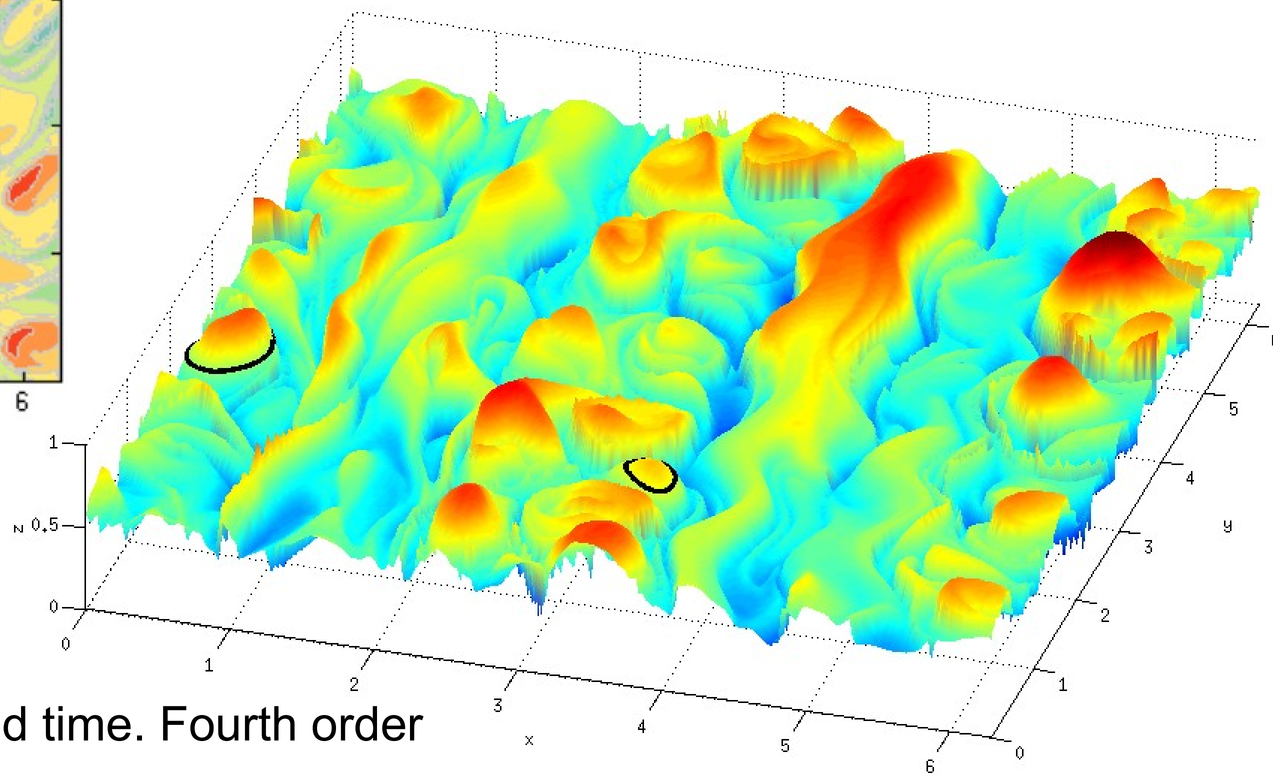
LAGRANGIAN AVERAGED VORTICITY DEVIATION



$t_0 = 700$

$\tau = 10$

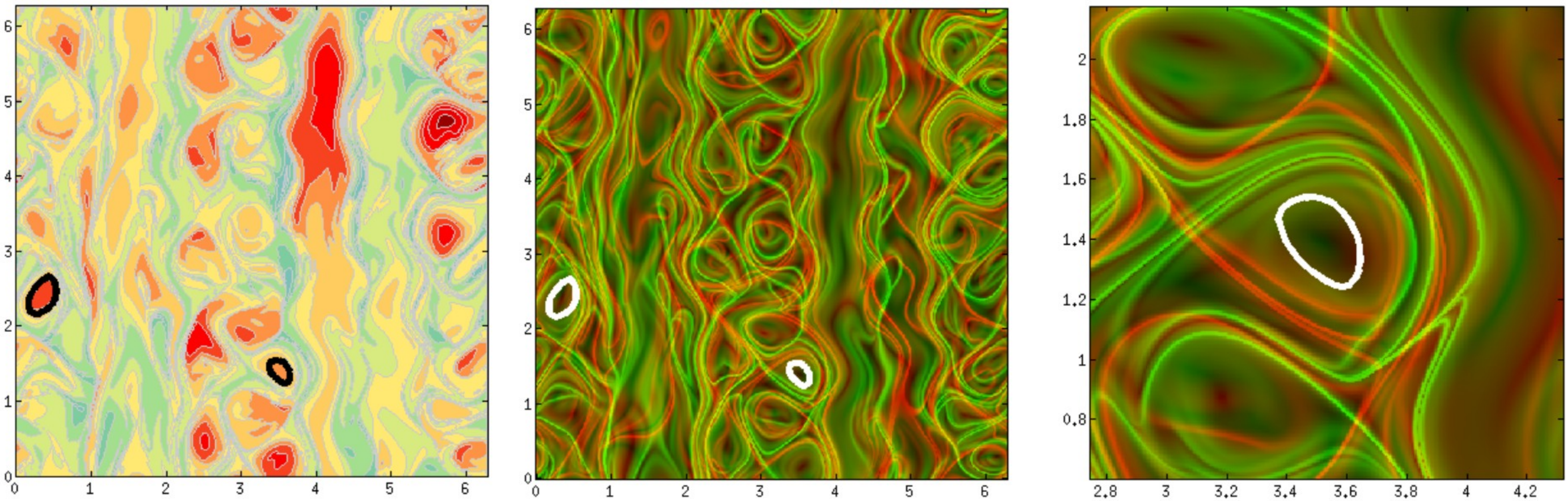
$\varepsilon \sim 10^{-3}$



Third order interpolation in space and time. Fourth order Runge-Kutta for particles Integration.

Source: Rempel et al., MNRAS 466, L108–L112 (2017)

LAGRANGIAN AVERAGED VORTICITY DEVIATION



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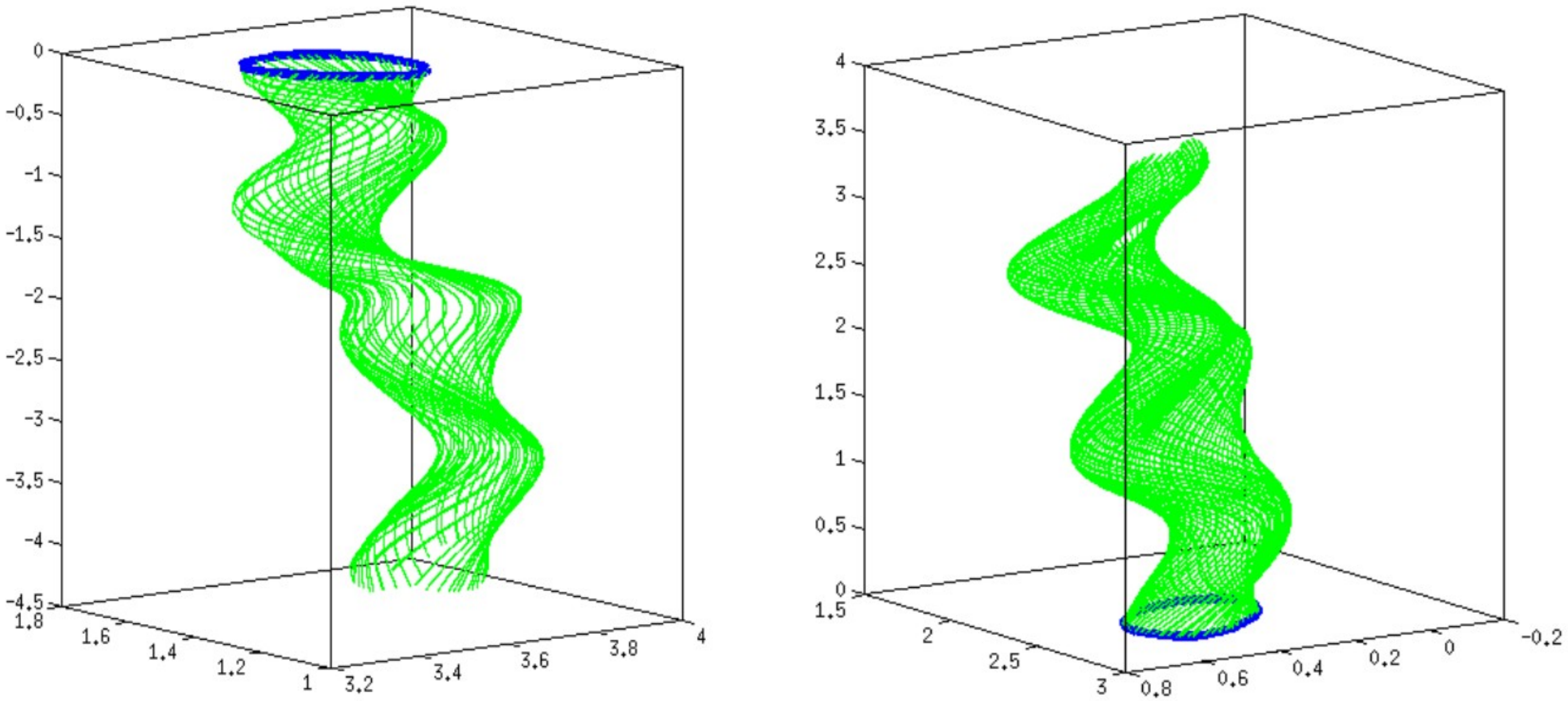
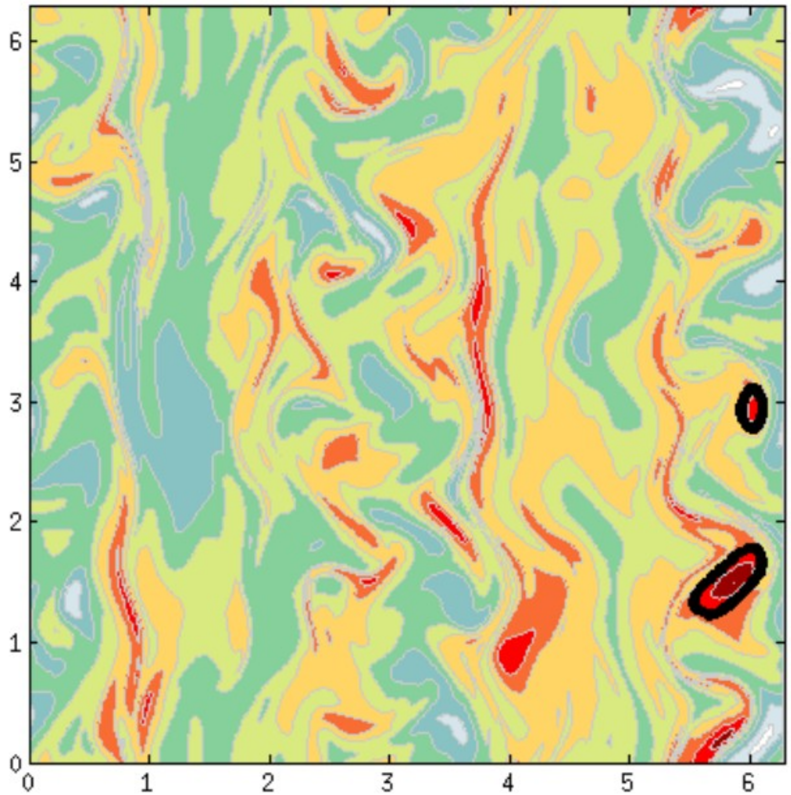
LAGRANGIAN VELOCITY VORTICES



Source: Rempel et al., MNRAS 466, L108–L112 (2017)

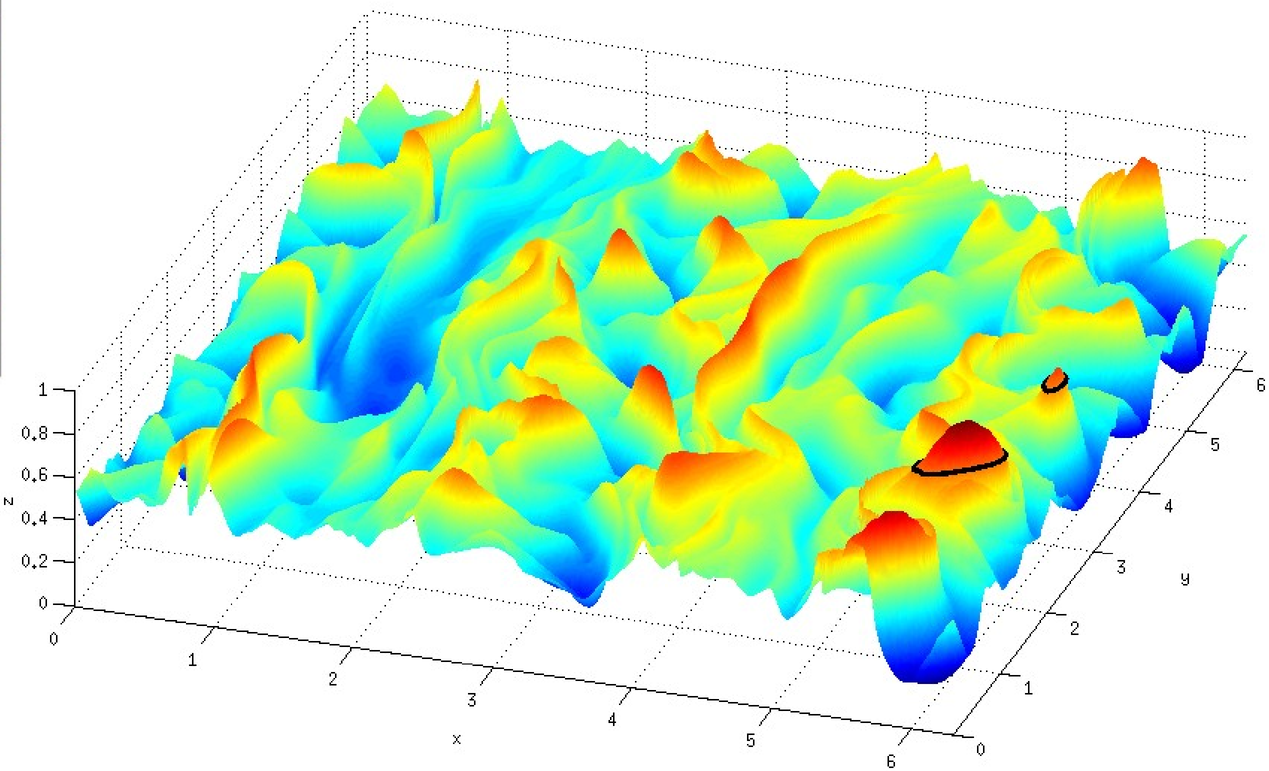
OBJECTIVE MAGNETIC VORTICES



$t_0 = 700$

$\xi = 10$

$\varepsilon \sim 10^{-3}$



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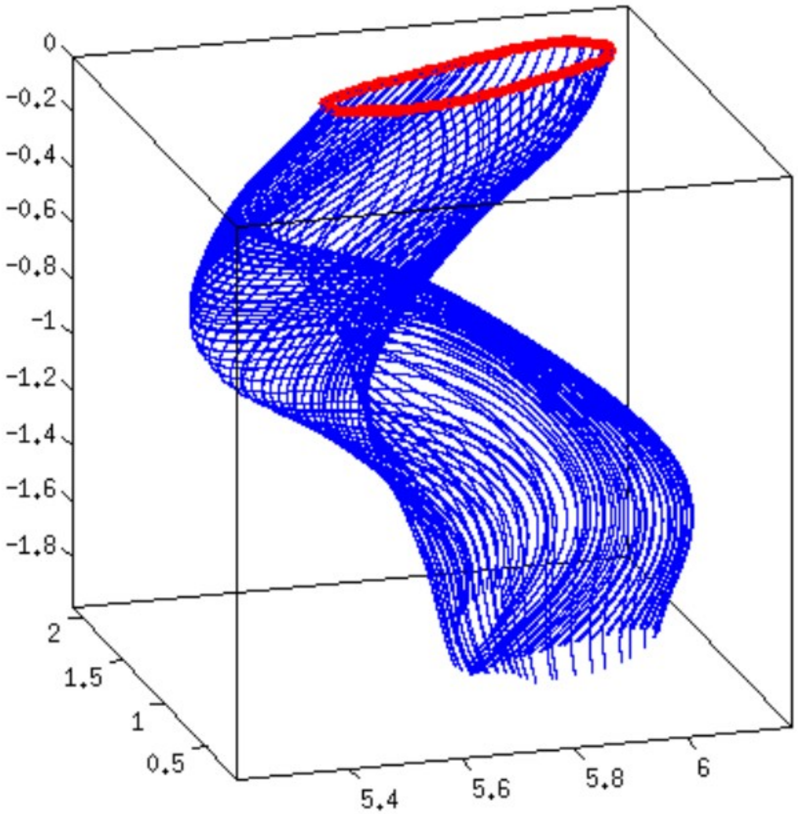
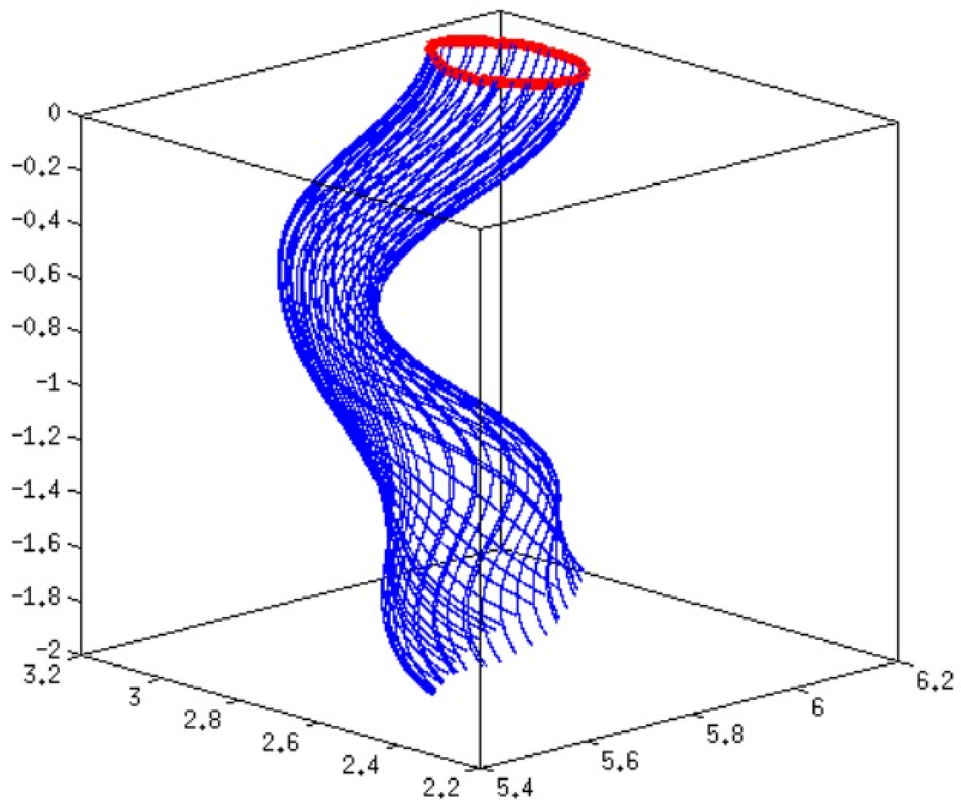
OBJECTIVE MAGNETIC VORTICES





Source: Rempel et al., MNRAS 466, L108–L112 (2017)

CONCLUSIONS

In magnetic fields, LCS identify regions of greater or smaller dispersion of field lines;

The Integrated Averaged Current Deviation is an objective way to define magnetic vortices.

Techniques for reconstruction of photospheric velocity and magnetic fields from satellite data render the method applicable to study magnetic reconnection in observable solar plasmas.

CONCLUSIONS

Thank you, very much

CURRENT VORTICES