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Associative memories based on fuzzy mathematical morphology and an application in prediction

Marcos Eduardo Valle and Peter Sussner

State University of Campinas (UNICAMP), Brazil
{mevalle,sussner}@ime.unicamp.br

1. Introduction

Fuzzy associative memories belong to the class of fuzzy neural networks that employ fuzzy operators such as fuzzy conjunctions, disjunctions, and implications in order to store associations of fuzzy patterns. Fuzzy associative memories are generally used to implement fuzzy rule-based systems. Applications of FAMs include backing up a truck and trailer, target tracking, human-machine interfaces, robot control, and voice cell control in ATM networks [3].

Recently, we observed that many well-known FAM models perform elementary operations of fuzzy mathematical morphology at every node [5, 8]. Therefore, many FAM models can be viewed as fuzzy morphological neural networks or - more precisely - as fuzzy morphological associative memories (FMAMs).

Fuzzy morphological neural networks and FMAMs, in particular, involve concepts from the areas of mathematical morphology, fuzzy set theory, and artificial neural networks. We intend to provide a detailed analysis of FMAMs in the near future. In particular, we plan to explore the mathematical morphology aspects of FMAMs by developping a more general recording strategy for FMAMs that is based on the notion of adjunction.

2. Basic concepts of fuzzy morphological associative memories

Morphological neural networks are equipped with *morphological neurons*, i.e. neurons that perform either a dilation, an erosion, a anti-dilation, or an anti-erosion [2, 5, 6]. In the fuzzy case, we simply speak of *fuzzy morphological neurons*.

Fuzzy morphological neurons can be defined in terms of fuzzy conjunctions, fuzzy disjunctions, or fuzzy implications [6]. Due to page constrainst, we will only introduce max-C morphological neurons since they represent the most important types of fuzzy neurons that occur in FMAM models and since other types of fuzzy morphological neurons can be obtained by means of a duality relationship such as adjunction or negation [8].

Recall that a *fuzzy conjunction* is an increasing mapping $C : [0,1] \times [0,1] \rightarrow [0,1]$ that satisfies $C(0,0) = C(0,1) = C(1,0) = 0$ and $C(1,1) = 1$. Examples of fuzzy conjunctions include the following operators: $C_M(x,y) = x \wedge y$ (minimum), $C_P(x,y) = xy$ (product), and $C_L(x,y) = 0 \vee (x + y - 1)$ (Lukasiewicz fuzzy conjunction). Note that C_M, C_P, and C_L also represent continuous t-norms.

If $x_1, \ldots, x_n$ are the fuzzy inputs, $w_1, \ldots, w_n$ are the fuzzy synaptic weights, and $\theta \in [0,1]$ is the bias of a *max-C neuron* then we compute the output $y \in [0,1]$ as follows:

$$y = \left[\bigvee_{j=1}^{n} C(w_j, x_j) \right] \vee \theta \,. \tag{1}$$

We may speak of a *max-C morphological neuron* if and only if $C(x, \cdot)$ is a dilation for every $x \in [0,1]$ [2]. In this case, Equation 1 corresponds to a fuzzy dilation [6].

A FMAM that consists of max-C_L neurons is called a Lukasiewicz FMAM. This FMAM model can be trained using *Lukasiewicz implicative learning* and in this case the Lukasiewicz FMAM coincides with the Lukasiewicz IFAM [7]. In general, the implicative learning scheme can be described as follows.

Suppose that we want to record the fundamental memory set $\{(\mathbf{x}^\xi, \mathbf{y}^\xi) : \xi = 1, \ldots, k\}$, where $\mathbf{x}^\xi \in [0,1]^n$ and $\mathbf{y}^\xi \in [0,1]^m$ by means of a synaptic weight matrix $W = (w_{ij}) \in [0,1]^{m \times n}$. The implicative fuzzy learning scheme consists in synthesizing the weight matrix W as follows.

$$w_{ij} = \bigwedge_{\xi=1}^{p} I_R(x_j^\xi, y_i^\xi) \,. \tag{2}$$

Here, I_R denotes the R-implication that corresponds to a certain continuous t-norm.

3. An application in prediction

In this section, we applied the Lukasiewicz FMAM to the problem of forecasting the average monthly streamflow of a large hydroelectric plant [4].

The time series prediction problem considered can be stated as follows: Given samples of the time series, s_ξ for $\xi = 1, \ldots, q-1$, we would like to obtain an

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Table 1. Mean Square, Mean Absolute, and Mean Relative Percentage Errors.

Methods	MSE ($\times 10^5$)	MAE (m^3/s)	MPE (%)
FMAM	1.38	221	21
PARMA	1.85	280	28
MLP	1.82	271	30
NFN	1.73	234	20
FPM-PRP	1.20	200	18

estimate $\hat{s}_q$ for the correct streamflow s_q based on a subset of the past values $s_1, \ldots, s_{q-1}$. The seasonality of the monthly streamflow suggests the use of 12 different models, one for each month of the year [4]. Furthermore, our FMAM based model only uses a fixed number of three antecedents. For example, the values of January, February, and March were taken into account to predict the streamflow of April.

The predictor based on the Lukasiewicz FMAM stores associations $(\mathbf{x}^\xi, \mathbf{y}^\xi)$, for $\xi = 1, \ldots, k$, where $\mathbf{x}^\xi$ and $\mathbf{y}^\xi$ are fuzzy sets that comprise some relevant information concerning the past values of the time series. Given an input $\mathbf{x}^q$ that takes into account the last three samples of the time series, the FMAM produces an output pattern $\mathbf{y}^q$. A defuzzification of $\mathbf{y}^q$ yields $\hat{s}_q \approx s_q$.

In this experiment, we employed the *subtractive clustering method* [1] to determine fuzzy sets $\mathbf{x}^\xi$ and $\mathbf{y}^\xi$ with Gaussian-type membership functions from streamflow data from 1931 to 1990 [4]. For computational reasons, $\mathbf{x}^q$ was modeled as a discrete Dirac-δ function. A defuzzification of $\mathbf{y}^q$ using the center of mass yielded $\hat{s}_q$.

Figure 1 shows the forecasted streamflows estimated by the prediction model based on the FMAM for the Furnas reservoir from 1991 to 1998. The continuous line corresponds to the actual values and the dashed line corresponds to the predicted values. Table 1 compares the errors that were generated by the FMAM model and several other models [4]. In contrast to the FMAM-based model, the MLP, NFN, and FPM-PRP models were initialized by optimizing the number of the parameters for each monthly prediction. For example, the MLP considers 4 antecedents to predict the streamflow of January and 3 antecedents to predict the streamflow for February. Moreover, the FPM-PRP model also takes into account slope information which requires some additional "fine tuning". We experimentally determined a variable number of parameters (including slopes) for the FMAM model such that MSE $= 0.88 \times 10^5$, MAE $= 157$, and MPE $= 15$.

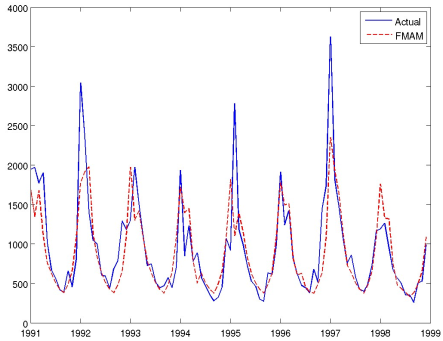

Figure 1. The streamflow prediction for the Furnas reservoir from 1991 to 1998.

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