Star Tracker Orientation Optimization Using Non-Dominated Sorting Genetic Algorithm (NSGA)

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Abstract— **One of the devices used to determine the attitude of a satellite is the star tracker, whose principle of operation is based on star position measurements on a specific inertial frame, allowing precise attitude determination and control of the satellite. Due to the high sensitivity of star camera, bright objects like Sun, Earth or Moon must be avoided in the sensor's field of view. This characteristic imposes a design constraint that shall be satisfied simultaneously by all the star trackers used in the specific satellite. Considering only the Sun exclusion case, the goal of this work is to find the star tracker orientation that maximizes simultaneously the Sun exclusion angle for both sensors in a way to ensure the proper equipment operation during a typical Earth pointing satellite mission. For this optimization problem, the search space is defined by the azimuth and elevation of each star tracker in the body centered coordinate frame system. Since the engineering goal implies a vectorial objective function optimization, the method used in this work was the Non-dominated Sorting Genetic Algorithm (NSGA), which allows a multi-optimization problem solution without the scalarization approach, in order to get a few optimal solutions along the non-dominated region. In order to get diversity in the optimal solutions, simulations used six different dummy fitness functions and compared the final results.**

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1. INTRODUCTION

The knowledge of the satellite orientation in space is extremely important and is typically associated with a very high pointing accuracy and stability requirements. This high precision of control in orbit is carried out by the satellite's Attitude and Orbit Control Subsystem (AOCS), which can use, for the case of low earth orbits (LEO) [1], coarse sun sensors, magnetometers and star trackers to obtain the platform attitude, such that the onboard computer can calculate the appropriate torques to be applied by the actuators (e.g., reaction wheels or thrusters) in a way to change the satellite attitude according to the specific mission goals.

In contrast to coarse attitude determination that is calculated based on coarse sun sensors and magnetometers measurements (e.g., using geometric methods such as TRIAD [2]), star trackers are fine attitude determination devices that take pictures of the sky, recognize them according to specific star patterns and use them to determine the current satellite attitude. In general, star trackers are built with high sensitivity cameras and, for this reason, bright objects like Sun, Moon or Earth must be avoided in their field of view. For the case of the Sun, Moon and Earth, constraints that indicate the minimum angle allowed between the attitude of the object and the star tracker's orientation are the solar, lunar and Earth exclusion angles, respectively. Considering a satellite architecture based upon the use of two star trackers for fine attitude determination and taking into account the solar exclusion angle only, it is easy to remark that there exists a multi-objective optimization problem related to star trackers orientations on the satellite structure.

A typical way of solving the star tracker orientation problem is a trial and error approach, where the system designer chooses a candidate solution and performs some analyses to see if the constraints are not violated. If the constraints are not violated the process ends, if not, the process is repeated with another candidate till the constraints are satisfied by them. The problems with this approach are the time required by the designer to perform the analyses and the potential lack of optimality related to maximum exclusion angles. If the satellite structure becomes more complex or if it changes during the design life cycle, the complexity of such analyses and the time required to perform them can be even greater. Therefore, an automatic approach, such as genetic algorithms (GAs), that deals with both optimality and mutability requirements related to this design problem is highly desirable [3].

Genetic algorithms have been successfully applied to many aerospace problems that range from feasibility study to mission operations phases. In [4], satellite conceptual design and launch vehicle selection where performed using GAs. The work [5] used GAs to determine sun sensors position in a spherical satellite while [6] used genetic algorithms to solve a multi-objective optimization concerning both the number and orientation of the sun sensors in a simplified planar scenario. In mission operations, GAs were also used for constellation maneuver planning [7] and for attitude path planing when the spacecraft has to deal with geometric, timed and dynamic constraints [8]. Good review papers of the applicability of GAs in the field of aeronautical and aerospace engineering are [9, 10]. For other feasible optimization techniques that can be applied to this problem class the reader is refereed to [11].

Regarding star trackers orientation on the satellite structure, one way to solve this multi-objective optimization problem is to transform the vector of objectives, which is defined by the solar exclusion angles of each sensor, into a scalar function, by averaging the objectives with a weight vector. This process

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permits applying AGs as tools for mono-objective optimization to find the non-dominated solutions. However, although a mono-objective optimization problem is simpler than a multi-objective problem, the solutions found by scalarizing the vector of objectives depend on the weight vector used. In addition, a single objective optimization attempts to get a global maximum or global minimum (the best solution) relying on the fact that the optimization problem is a maximization or a minimization one. For this case, the designer is not supposed to get many alternative solutions.

In this paper, the aim is to capture a number of non-dominated solutions (i.e., set of solutions which are superior to the rest of solutions in the search space when all objectives are considered, but are inferior to other solutions with respect to one or more objectives isolated - *Pareto-optimal* [12–14]) to the star tracker orientation problem using Non-Dominated Sorting Genetic Algorithm (NSGA) for multi-objective optimization [15, 16]. The idea is basically to work with an initial population of solutions, determine the set of Pareto-optimal solutions in the initial population, reproduce them by crossover, taking into account that the non-dominated solutions have more chances to survive, and repeat the same process until a fixed number of generations is reached. It will be observed that the number of points that belong to the set of Pareto-optimal solutions converges to a certain value along the generations.

In the following sections details concerning the nature of the problem that was solved and the mechanics of the NSGA will be further explained. Some simulation results will also be presented to illustrate how the technique worked in this specific problem scenario when different dummy fitness function values were used.

2. Multi-objective Optimization Problem

Suppose that the satellite is based upon the Brazilian Multi-mission Platform (PMM) that is intended to operate in a sun-synchronous circular orbit at an altitude of 750 km (LEO) and inclination equal to 98.39^o [17]. In order to determine the star tracker orientation and sun attitude vectors, the body centered (satellite) coordinate frame system $\boldsymbol{oxyz}$ is considered: with origin $\boldsymbol{o}$ in the satellite's center of mass, and $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$ being the satellite vectors along the directions of measurements for the angles of roll, pitch and yaw, respectively. Spacecraft angular rate is assumed to be the same as the orbital angular rate in a way that the spacecraft-fixed frame remains aligned with orbital coordinate frame along the orbit as shown in Figure 1.

It is also necessary three coordinates (x, y, z) to define the orientation vector for each star tracker in the body centered coordinate frame. Therefore, if Cartesian coordinates are used, it can be concluded that the dimension of the search space would be equal to three for each star tracker. However, it is possible to reduce the dimension of the search space using the azimuth α and elevation β angles [18]. Figure 2 shows schematically two star trackers mounted on the satellite structure ($-z$ plane) and the corresponding orientation vectors.

Denoting by $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ the orientation vector of the first and second star tracker, respectively, the equipment's coordinates with respect to the $\boldsymbol{oxyz}$ frame are

$$\boldsymbol{T_i}(\alpha_i, \beta_i) = (\cos\alpha_i \cos\beta_i, \sin\alpha_i \cos\beta_i, \sin\beta_i), \quad (1)$$

where $-180° \leq \alpha_i \leq 180°$, $-90° \leq \beta_i \leq 90°$ and $i = 1, 2$.

Remark: There are no restrictions in α_i nor in β_i because it is supposed that star trackers can be placed anywhere on the satellite external surface. Moreover, solar panels and other appendage obstructions were not considered in this problem.

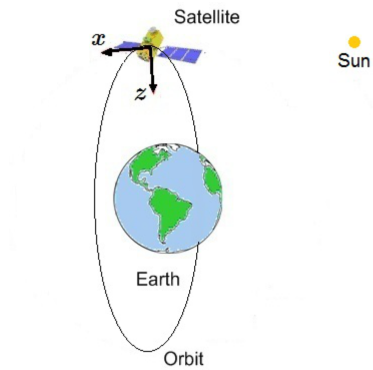

Figure 1. Body centered (satellite) and orbital coordinate frames remain aligned along the orbit (not to scale)

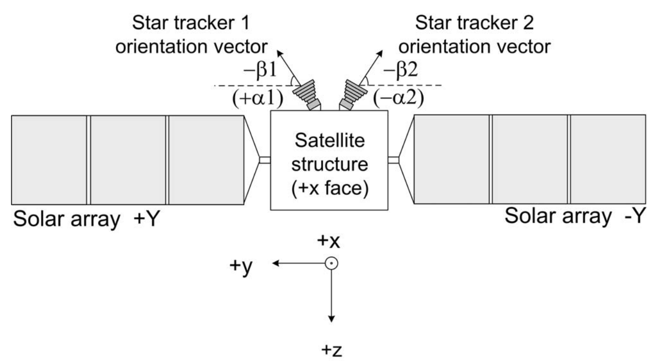

Figure 2. Star trackers 1 and 2 mounted on -z satellite structural plane(not to scale).

Now denote by $\boldsymbol{S}(t)$ the sun unit vector at instant t. By definition of scalar product, the algebraic operation $\boldsymbol{T_i} \cdot \boldsymbol{S}(t)$ permits to obtain the cosine of the angle between the vectors $\boldsymbol{T_i}$ and $\boldsymbol{S}(t)$ for every $t \geq 0$. Similarly, denote by θ^* the solar exclusion angle. Therefore, the restriction that prevents sun in the field of view of the star tracker camera implies

$$\boldsymbol{T_i} \cdot \boldsymbol{S}(t) \leq \cos\theta^*, \quad (2)$$

for every $t \geq 0$ and $i = 1, 2$. Therefore, the goal for the space mission designer is to determine between all possible orientation vectors the coordinates that minimize the maximum dot product $\boldsymbol{T_i} \cdot \boldsymbol{S}(t)$ reached during a certain time of operation denoted by τ such that Eq. (2) is always satisfied. Note that to minimize the maximum dot product between two vectors is equivalent to maximize the minimum angle between them. This minimum angle will be denoted by ϕ_i.

In this manner, let us define the function $f_i(\alpha_i, \beta_i)$ as

$$f_i(\alpha_i, \beta_i) = \max_{t \leq \tau} \{\boldsymbol{T_i}(\alpha_i, \beta_i) \cdot \boldsymbol{S}(t)\} \quad (3)$$

where $i = 1, 2$. Mathematically, the corresponding minimization multi-objective problem is formulated as follows:

$$\min \boldsymbol{F}(\alpha_1, \beta_1, \alpha_2, \beta_2) = (f_1(\alpha_1, \beta_1), f_2(\alpha_2, \beta_2)) \quad (4)$$

subject to

$$f_1 \leq \cos\theta^*, \tag{5}$$
$$f_2 \leq \cos\theta^*, \tag{6}$$
$$|\boldsymbol{T_1} \cdot \boldsymbol{T_2}| < \varepsilon. \tag{7}$$

where $0 << \varepsilon < 1$. Constrain (7) represents the condition that two star trackers shall not be aligned, which comes from the fact that accuracy of attitude determination is improved this way. Mathematically, two vectors are aligned when the absolute value of scalar product between them is equal to 1. Nevertheless, in engineering, it is important to define a tolerance that indicates if two vectors can be considered aligned. This tolerance is given by the term ε and shall be defined by the AOCS and mission engineers. The angle between star trackers will be denoted by φ.

In a minimization multi-objective problem, a vector $\boldsymbol{F^{(1)}}$ is partially less than another vector $\boldsymbol{F^{(2)}}$ $\left(\boldsymbol{F^{(1)}} \prec \boldsymbol{F^{(2)}}\right)$ in the objective space when no value of $\boldsymbol{F^{(2)}}$ is less than $\boldsymbol{F^{(1)}}$ and at least one value of $\boldsymbol{F^{(2)}}$ is strictly greater than $\boldsymbol{F^{(1)}}$. It is said that the solution the solution $\boldsymbol{F^{(1)}}$ *dominates* $\boldsymbol{F^{(2)}}$ or the solution $\boldsymbol{F^{(2)}}$ is *inferior* to $\boldsymbol{F^{(1)}}$ [19]. The optimal solutions for this multi-objective optimization problem are called non-dominated solutions, also known as *Pareto-optimal* solutions. For the problem discussed in this paper, the non-dominated solutions are those that correspond to star trackers with orientation vectors that satisfy the sun exclusion angle constraint and that are superior than many other alternatives that also satisfy this constraint.

3. NSGA Algorithm Details

The method used in this work minimizes the vector function (4) through a non-dominated sorting procedure in conjunction with a sharing technique [20]. In fact, the idea behind the algorithm implemented in [21] differs from the simple genetic algorithm only in the way the selection operator works. In fact, before the selection is performed, the non-dominated individuals present in the population are first identified from the current population based on the non-dominant criterion described previously. It is assumed that all these individuals constitute the first non-dominated front in the population and the same dummy fitness value (large enough) is assigned. This process guarantees that the non-dominated individuals have the same reproductive potential.

In order to avoid bias towards some Pareto-optimal solutions and maintain the diversity in the population, these classified individuals are then *shared* with their dummy fitness value, i.e., dividing the original fitness value of an individual by a quantity proportional to the number of individuals around it. Expression (8) describes the sharing function value between two individuals i and j in the same front:

$$Sh\left(d_{ij}\right) = \begin{cases} 1 - \left(\frac{d_{ij}}{\sigma_{\text{share}}}\right)^2, & \text{if } d_{ij} < \sigma_{\text{share}}; \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

where: d_{ij} corresponds to the euclidean distance in the objective space between individuals i and j in the current front, in other words it measures the difference of sun exclusions that exists between two candidate solutions; σ_{share} is the maximum euclidean distance allowed between any two individuals to become members of a niche and, for this search procedure, is related to the user definition of solutions that are considered as having similar sun exclusion properties taking into account both sensors.

A parameter niche count denoted by m_i is calculated as follows:

$$m_i = \sum_{j=1}^{K} Sh\left(d\left(i,j\right)\right). \tag{9}$$

where K is the number of individuals in the current front (i.e., that have similar sun exclusion angles). Thus, the shared fitness value of each individual is calculated dividing its dummy fitness value (9) by its niche count. After sharing, these first ranked individuals are ignored temporarily such that in the remaining individuals of the population non-dominated individuals are identified for the second non-dominated front. Similarly, it is assigned a new dummy fitness value which is smaller than the minimum shared dummy fitness of the previous front. The process continues until the entire population is ranked in several fronts. It is important to point out that the sharing procedure permits a good exploration of the search space, allowing the user the access to many star tracker orientation solutions with desirable sun exclusion properties.

The population is then reproduced according to the dummy fitness values and using the roulette wheel selection [22]. The crossover and mutation operators remain as usual.

The next section shows simulation results that were obtained using different values for σ_{share}.

4. Simulation Results

In this section, NSGA is applied on the minimization problem (4). In all simulations, the Genetic Algorithm (GA) parameters that were used are:

Maximum generation:	100
Population size:	500
String length (binary code):	52
Probability of crossover:	0.9
Probability of mutation:	$0.01 * \exp\left(-G\right)$

where G represents the number of current generation. In this way, the mutation probability converges rapidly to zero along the generations in order to observe the effectiveness of NSGA. On the other hand, the orientation of two star trackers are defined by a four dimension vector $(\alpha_1, \beta_1, \alpha_2, \beta_2)$. Since an individual in the population is represented by a binary vector of length 53, then each angle consists of a binary vector of length 13 which permits a angular resolution of 0.01^o. The tolerance used is $\varepsilon = \cos 10^o$, i.e., two star trackers are considered aligned if the angle between them is less than 10° or greater than 170°.

Software was used to simulate a sun-synchronous circular orbit of altitude and inclination equal to 750 km and 98.39^o, respectively, with $J2$ perturbation, and obtain the coordinates of sun attitude vector with respect to the body centered (satellite) frame during a time $\tau = 1$ year with a step size of 10 minutes. Finally, the star tracker ALTAIR HB+ made by Surrey Satellite Technology Ltd [23] was the model chosen across all runs which possess a solar exclusion angle (θ^*) equal to 60^o.

Six unbiased initial populations spread over entire variable space in consideration are generated randomly. Figure 3 shows the evolution of each population in the objective space where the first, second and third column show the population distribution at 1-st, 60-th and 100-th generations, respectively. Additionally, NSGA is applied with σ_{share} equal to 0.01, 0.02, 0.03, 0.04, 0.05 and 0.06 in the black, purple, green, gray, red and blue populations respectively. It is interesting to note that at generation 60, the six populations practically converged towards the non-dominated region. There is no much difference performance of NSGA with the different σ_{share} values.

Figure 4 shows the non-dominated individuals at generation 100. It can be noted that although NSGA's performance was the same in the six cases, the distribution of population over the non-dominated region was better when $\sigma_{\text{share}} = 0.06$ and $\sigma_{\text{share}} = 0.05$; with low sharing parameter, $\sigma_{\text{share}} = 0.01$ and $\sigma_{\text{share}} = 0.02$, NSGA obtained two optimal solutions very close to each other. This is due to the fact that, with low sharing parameter, the number of individuals inside of a niche decreases such that the niche count (9) converges to 1. This results optimal points that converges to a certain solution have equal reproductive potential than optimal points that spread across the non-dominated region. This is a characteristic of NSGA that does not depend on the initial population [21]. In the other intermediate cases, $\sigma_{\text{share}} = 0.03$ and $\sigma_{\text{share}} = 0.04$, the optimal solutions are not close to each other but they do not spread across the non-dominated region. Therefore, this analysis shows the large dependence in the NSGA regarding the σ_{share} parameter to maintain diversity in the population. Six different σ_{share} values had to be tested to obtain a set of optimal solutions well spread in the non-dominated region. Although there exists some guidelines [24], the need for specifying the σ_{share} value is always a difficulty [25]. This situation can be noted not only in the wide variety of non-dominated solutions but also in the quantity of them as shown in Fig. 4. The optimal solutions converge to a few number of points in Pareto frontier.

In order to investigate how NSGA behaves with different σ_{share} values, Figure 5 shows the performance of function $\min(f_1 + f_2)$ for NSGA with the six initial populations shown in Fig. 4 and their respective σ_{share} values. Note that vectorial function (4) was scalarized setting the same weights to each function f_i. Similar observations can be made in this case, although in the six cases the function converges rapidly from generation 10, NSGA with $\sigma_{\text{share}} = 0.02$ converges better and to lower value than the other two cases. It is important to note that with $\sigma_{\text{share}} = 0.04$, NSGA's performance is poorer than the other sharing parameters. Additionally, NSGA with $\sigma_{\text{share}} = 0.05$ converges more slowly to the same value with $\sigma_{\text{share}} = 0.02$. However, the solutions found with sharing parameter equal to 0.05 spread much better than the solutions found with 0.02.

In this manner, Tables 1-6 show the azimuth (α_i) and elevation (β_i) for the non-dominant solutions shown in Fig. 3 and the minimum angle (ϕ_i) between each solution and sun attitude during one year period with the three different sharing parameters tested in this study. Similarly, Tables 7-12 show the corresponding coordinates (T_i) of star trackers orientation vectors with respect to the body centered (satellite) frame system as well as the angle (φ) between them. As it can be seen in Tables 1 - 12, all non-dominated solutions satisfy constrains (5)-(7). However, the choice of a specific orientation for each star tracker between the set of optimal solutions found in this study will depend on the project designer. For example, once the optimal solutions were known, if the goal of the project was the orientation that maximize the minimum angle between the first device and sun attitude vector, the best choice would be the 3-rd solution in Table 1. On the other hand, if the same purpose was applied but in the second device, the best choice would be the 2-nd solution in the previous Table. Finally, if the goal was reducing the alignment between the star tracker cameras, the best solution would be 3-th solution in Table 12.

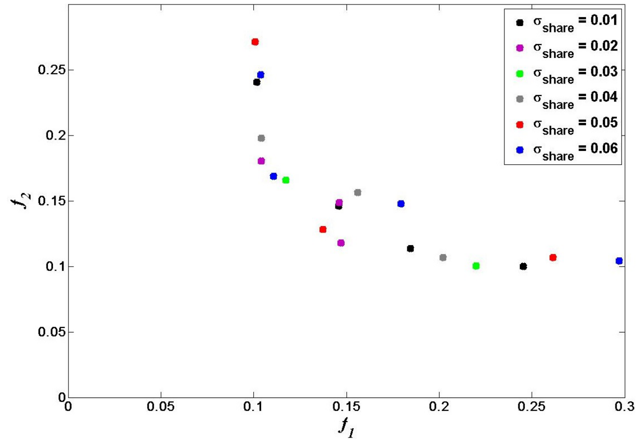

Figure 4. Nondominan individuals at generation 100.

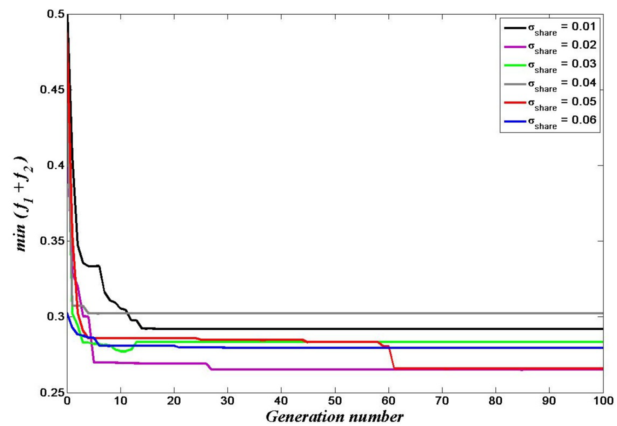

Figure 5. Performance of function $\min(f_1 + f_2)$ for NSGA with σ_{share} equal to 0.01, 0.02, 0.03, 0.04, 0.05and 0.1.

Table 1. Azimuth (α_i) and elevation (β_i) for the non-dominant individuals and minimum angle (ϕ_i) between them and the sun attitude during one year with $\sigma_{\text{share}} = 0.01$

α_1	β_1	α_2	β_2	ϕ_1	ϕ_2
88.47°	4.76°	93.90°	−3.87°	81.61°	81.60°
93.28°	−11.10°	90.17°	−0.23°	75.79°	84.26°
90.47°	−0.77°	80.56°	3.63°	84.16°	76.07°
93.28°	−7.53°	88.06°	2.69°	79.36°	83.47°

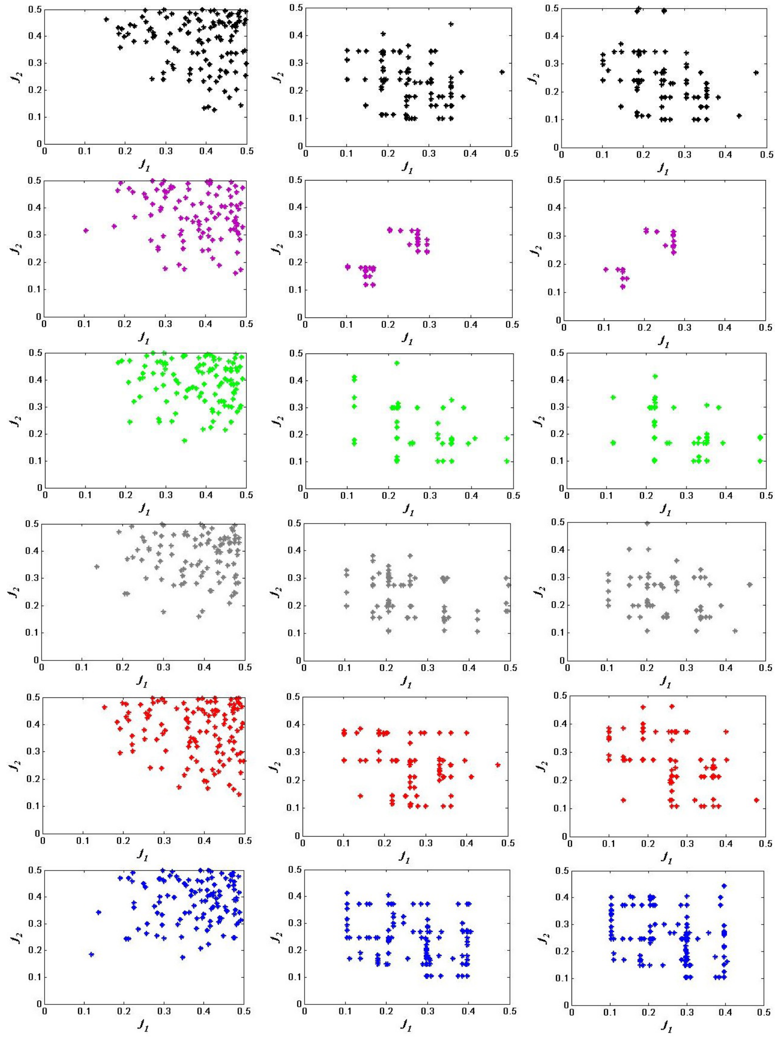

Figure 3. Population at 1-st (first column), 60-th (second column) and 100-th (third column) generations. NSGA is applied with σ_{share} equal to 0.01 (black population), 0.02 (purple population), 0.03 (green population), 0.04 (gray population), 0.05 (red population) and 0.06 (blue population).

Table 2. **Azimuth (α_i) and elevation (β_i) for the non-dominant individuals and minimum angle (ϕ_i) between them and the sun attitude during one year with $\sigma_{\text{share}} = 0.02$**

α_1	β_1	α_2	β_2	ϕ_1	ϕ_2
$92.93°$	$-5.30°$	$87.79°$	$3.29°$	$81.53°$	$83.23°$
$91.06°$	$-1.49°$	$84.74°$	$6.57°$	$84.02°$	$79.60°$
$92.93°$	$-5.24°$	$85.95°$	$3.48°$	$81.60°$	$81.44°$

Table 3. **Azimuth (α_i) and elevation (β_i) for the non-dominant individuals and minimum angle (ϕ_i) between them and the sun attitude during one year with $\sigma_{\text{share}} = 0.03$**

α_1	β_1	α_2	β_2	ϕ_1	ϕ_2
$88.67°$	$2.73°$	$94.75°$	$-5.66°$	$83.26°$	$80.44°$
$95.80°$	$-9.16°$	$89.64°$	$0.49°$	$77.30°$	$84.24°$

5. CONCLUSIONS

In this paper we have applied a genetic algorithm known as NSGA to maximize simultaneously the angle between the sun attitude and star tracker orientation. The multi-objective function was not scalarized such that instead of to find a unique global optimal solution, NSGA permitted to determine from the vectorial function few optimal solutions using the criterion of non-dominance. This approach permitted the automation of this design problem in a way to furnish many feasible solutions to the mission and AOCS engineers.

The NSGA's performance depends on the sharing parameter which denotes the largest value of the distance metric within which any two solutions share each others fitness. This parameter is usually set by the user producing a large dependency of the spread of the optimal solutions across the non-dominated region on the σ_{share} value. Six different values between 0.01 and 0.06 were tested to obtain a sufficient number of well-diversified optimal solutions. The performance of NSGA depended on the sharing parameter used in each run. The results showed that for this particular case NSGA with sharing parameter much greater than 0.04 obtains solutions

Table 4. Azimuth (α_i) and elevation (β_i) for the non-dominant individuals and minimum angle (ϕ_i) between them and the sun attitude during one year with $\sigma_{\text{share}} = 0.04$

α_1	β_1	α_2	β_2	ϕ_1	ϕ_2
91.06°	−1.49°	88.11°	8.11°	84.02°	78.58°
87.31°	5.78°	94.51°	−3.78°	81.02°	81.00°
95.59°	−8.07°	89.54°	−0.03°	78.34°	83.87°

Table 5. Azimuth (α_i) and elevation (β_i) for the non-dominant individuals and minimum angle (ϕ_i) between them and the sun attitude during one year with $\sigma_{\text{share}} = 0.05$

α_1	β_1	α_2	β_2	ϕ_1	ϕ_2
86.69°	12.05°	88.69°	1.81°	74.85°	83.88°
86.71°	4.78°	91.92°	−3.81°	82.10°	82.63°
90.45°	−0.58°	101.26°	−3.81°	84.21°	74.25°

Table 6. Azimuth (α_i) and elevation (β_i) for the non-dominant individuals and minimum angle (ϕ_i) between them and the sun attitude during one year with $\sigma_{\text{share}} = 0.06$

α_1	β_1	α_2	β_2	ϕ_1	ϕ_2
88.66°	2.26°	94.93°	−5.64°	83.64°	80.28°
86.79°	7.23°	94.00°	−3.77°	79.66°	81.05°
86.79°	14.17°	89.72°	−0.02°	72.72°	84.03°
91.04°	−1.49°	99.11°	−7.52°	84.04°	75.73°

Table 7. Optimal star trackers orientation vector coordinates (T_i) with respect to the body centered (satellite) frame system and the angle (φ) between them with $\sigma_{\text{share}} = 0.01$.

x	y	z	φ
0.0266	0.9962	0.0829	10.19°
−0.0679	0.9954	−0.0675	
−0.0561	0.9797	−0.1925	11.3°
−0.0030	1.0000	−0.0040	
−0.0081	0.9999	−0.0134	10.83°
0.1637	0.9845	0.0633	
−0.0567	0.9898	−0.1310	11.47°
0.0338	0.9983	0.0470	

more distributed over the non-dominated region.

Finally, it was shown that if the project designer desired to maximize the angle between the star tracker cameras, it would be possible to find a global optimal solution. However, it must be emphasized that despite the fact that all solutions satisfied the constrains, specially the solar exclusion angle, to be considered a complete design tool, this approach must also take into account Earth exclusion angle, Moon exclusion angle, the accuracy criterion related to star trackers alignment, satellite appendages and platform shape as constraints that must be satisfied simultaneously.

Table 8. Optimal star trackers orientation vector coordinates (T_i) with respect to the body centered (satellite) frame system and the angle (φ) between them with $\sigma_{\text{share}} = 0.02$.

x	y	z	φ
−0.0510	0.9944	−0.0924	10.00°
0.0385	0.9976	0.0573	
−0.0185	0.9995	−0.0260	10.23°
0.0910	0.9893	0.1144	
−0.0510	0.9945	−0.0913	11.17°
0.0706	0.9957	0.0608	

Table 9. Optimal star trackers orientation vector coordinates (T_i) with respect to the body centered (satellite) frame system and the angle (φ) between them with $\sigma_{\text{share}} = 0.03$.

x	y	z	φ
0.0232	0.9986	0.0476	10.35°
−0.0824	0.9917	−0.0986	
−0.0997	0.9822	−0.1592	11.44°
0.0062	0.9999	0.0086	

Table 10. Optimal star trackers orientation vector coordinates (T_i) with respect to the body centered (satellite) frame system and the angle (φ) between them with $\sigma_{\text{share}} = 0.04$.

x	y	z	φ
−0.0185	0.9995	−0.0260	10.04°
0.0326	0.9895	0.1410	
0.0467	0.9938	0.1008	11.97°
−0.0784	0.9947	−0.0660	
−0.0965	0.9854	−0.1404	10.05°
0.0080	1.0000	−0.0006	

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Table 11. **Optimal star trackers orientation vector coordinates (T_i) with respect to the body centered (satellite) frame system and the angle (φ) between them with $\sigma_{\text{share}} = 0.05$.**

x	y	z	φ
0.0564	0.9764	0.2087	$10.42°$
0.0228	0.9992	0.0316	
0.0572	0.9949	0.0833	$10.04°$
-0.0334	0.9972	-0.0665	
-0.0079	0.9999	-0.0102	$11.28°$
-0.1949	0.9786	-0.0665	

Table 12. **Optimal star trackers orientation vector coordinates (T_i) with respect to the body centered (satellite) frame system and the angle (φ) between them with $\sigma_{\text{share}} = 0.06$.**

x	y	z	φ
0.0233	0.9990	0.0394	$10.08°$
-0.0856	0.9915	-0.0983	
0.0556	0.9905	0.1258	$13.14°$
-0.0695	0.9954	-0.0657	
0.0543	0.9680	0.2448	$14.48°$
0.0049	1.0000	-0.0003	
-0.0181	0.9995	-0.0260	$10.05°$
-0.1569	0.9789	-0.1309	

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