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COSMIC RAY ACCELERATION BY STRONG SPACE-CHARGE WAVES

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Abstract

Strong space-charge waves may accelerate cosmic ray in pulsar magnetospheres. The nonlinear dispersion relation of strong space-charge waves in a relativistic electron (positron) beam is derived. Wave-breaking conditions of strong space-charge waves are obtained. The discussion includes both negative-energy and positive-energy modes as well as wave propagation parallel and anti-parallel to the particle beam.

Introduction Space-charge waves resulting from the longitudinal bunching of space-charges in a relativistic electron or positron beam may be of interest for cosmic ray acceleration in astrophysical plasmas. According to the current polar-cap pulsar models, the pulsar magnetosphere is composed of secondary electrons and positrons produced by pair production induced by high-energy curvature radiation photons which are emitted by primary electron or positron beam coming from the pulsar surface (Chian and Kennel, 1983). The interaction of the primary electron and positron beams with the pulsar magnetosphere may give rise to unstable space-charge waves. Moreover, strong waves with the wave intensity so large that $eE/m\omega c \gtrsim 1$ may propagate in the pulsar magnetosphere (Chian, 1981; Chian, 1982; Chian and Kennel, 1983). In the presence of strong space-charge waves, the charged particles may be accelerated to relativistic energies (Chian, 1989).

As the space-charge wave reaches large amplitudes, higher harmonics of the fundamental wave oscillation are generated which leads to nonlinear steepening of the wave. The steepening of the space-charge wave proceeds until the wave attains a critical amplitude beyond which wave breaking occurs. The aim of this paper is to determine the wave-breaking condition of space-charge waves in a relativistic electron (positron) beam (Chian, 1989). The discussion is generalized to include both negative-energy and positive-energy modes. In particular, both cases of wave propagation parallel and anti-parallel to the electron (positron) beam are treated.

Nonlinear Dispersion Relation Before carrying out a rigorous derivation of the nonlinear dispersion relation for space-charge waves, we shall first give a heuristic derivation that indicates the relation between a stationary plasma and a relativistic electron (positron) beam. In the beam (primed) frame in which $v_b'=0$, the amplitude-dependent frequency of relativistic nonlinear plasma waves in a cold plasma is independent of k' and is approximately given by (Chian, 1979)

$$\omega' \doteq \pm\omega_p'/\gamma_0'^{1/2} \equiv \pm\Omega_p' , \qquad (1)$$

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where $\omega_p' = (n_0' e^2/m\varepsilon_0)^{1/2}$, $m_e = m_p \equiv m$, $\gamma_0' = (1 - v_0'^2/c^2)^{-1/2}$, and v_0' is the peak particle quiver velocity. Thus the wave oscillates at the proper plasma frequency with the rest mass m replaced by the relativistically corrected mass $\gamma_0 m$. A Lorentz transformation of (1) to the laboratory frame (where the beam has a velocity v_b), using the relations $\omega' = \gamma_b(\omega - v_b k)$ and $\Omega_p' = \Omega_p/\gamma_b^{1/2}$, where $\gamma_b = (1 - v_b^2/c^2)^{-1/2}$, yields the following nonlinear dispersion relation for space-charge waves in a relativistic electron (positron) beam:

$$\omega - v_b k = \pm\Omega_p/\gamma_b^{3/2}, \tag{2}$$

where the amplitude-dependent plasma frequency $\Omega_p = (n_0 e^2/\gamma_0' m\varepsilon_0)^{1/2}$ and $\gamma_0' = \gamma_b(\gamma_0 - v_b\gamma_0 v_0/c^2)$. In the limit $\gamma_0' = 1$, (2) reduces to the linear dispersion relation.

Now, we derive the exact nonlinear dispersion relation of space-charge waves in a relativistic electron (positron) beam. The relativistic momentum and continuity equations, together with Poisson's equation, describe the cold electron (positron) beam

$$\frac{\partial p}{\partial t} + v\frac{\partial}{\partial x}p = qE \ , \tag{3}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \tag{4}$$

$$\varepsilon_0\frac{\partial E}{\partial x} = q(n_0 - n), \tag{5}$$

where $p = m\gamma v$, $\gamma = (1 - v^2/c^2)^{-1/2}$, and one-dimensional fluid is considered since the wave motion of space-charge waves is longitudinal. Seeking periodic traveling wave solutions which depend only on the variable $\tau = \omega_p(t - x/v_{ph})$, we obtain from (4) the following equation for the electron-beam density:

$$\frac{n}{n_0} = \frac{v_{ph} - v_b}{v_{ph} - v} \ , \tag{6}$$

where the constant of integration is chosen so that, $v_b \equiv \langle nv\rangle/\langle n\rangle$ and $n_0 \equiv \langle n\rangle$, where $\langle\ \rangle$ denotes averaging over a period in τ. Taking the derivative of the momentum equation and substituting (5) and (6) we arrive at the following nonlinear wave equation:

$$\frac{d^2}{d\tau^2}(\gamma - \beta_{ph}u) = \beta_{ph}^2\left(\frac{\beta - \beta_b}{\beta_{ph} - \beta}\right), \tag{7}$$

where $u = \gamma\beta$, $\beta = v/c$, $\beta_{ph} = v_{ph}/c$, and $\beta_b = v_b/c$. Integration of (7) gives the following exact nonlinear dispersion relation of space-charge waves in a relativistic electron (positron) beam:

$$\omega - v_b k = \pm\frac{\sqrt{2}\pi\omega_p}{\displaystyle\int_{u_1}^{u_2}\frac{\beta_{ph} - \beta}{(\beta_{ph} - \beta_b)(W - \gamma + \beta_b u)^{1/2}}\,du}, \tag{8}$$

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where

$$u_{1,2} = \gamma_b^2[\beta_b W \pm (W^2-1/\gamma_b^2)^{1/2}] ,$$

and the plus (minus) signs refer, respectively, to the positive- (negative-) energy mode if $\omega>0$ and the reverse if $\omega<0$.

Wave-breaking Conditions Eq. (8) admits both superluminous ($v_{ph}>0$) and subluminous ($v_{ph}<0$) space-charge waves. Wave breaking does not occur for superluminous waves since in that case the particle velocity may never reach the wave phase velocity. However for subluminous waves there is the possibility of wave breaking, as shown below. In the beam frame, the particle density is

$$n' = \frac{v'_{ph} n'_0}{v'_{ph}-v'} , \tag{9}$$

which combined with the fact that v' oscillates between the turning points $\pm v'_0$ shows that wave solutions exist provided

$$-v'_{ph}<v'<v'_{ph} . \tag{10}$$

In the laboratory frame, it is evident from (6) that wave solutions for space-charge waves are bounded by

$$\begin{aligned} v<v_{ph} \quad &\text{if} \quad v_{ph}>v_b , \\ v>v_{ph} \quad &\text{if} \quad v_{ph}<v_b . \end{aligned} \tag{11}$$

Eq. (11) corresponds to the upper bound of (10) in the beam frame, namely, $v'=v'_{ph}$. Another limitation on the particle velocity can be obtained by Lorentz transforming the lower bound of (10), $v'=-v'_{ph}$, to the laboratory frame using the relation $v' = (v-v_b)/(1-v_b v/c^2)$, which gives

$$v = \frac{2v_b-v_{ph}-\beta_b^2 v_{ph}}{1+\beta_b^2-2\beta_b\beta_{ph}} \equiv v_{cr}. \tag{12}$$

Hence the existence condition for space-charge waves in a relativistic electron (positron) beam, valid for both negative-energy and positive-energy modes, is

$$\begin{aligned} v_{cr}<v<v_{ph} \quad &\text{if} \quad v_{ph}>v_b , \\ v_{ph}<v<v_{cr} \quad &\text{if} \quad v_{ph}<v_b . \end{aligned} \tag{13}$$

Violation of (13) implies wave breaking.

Next, we derive an expression for the wave-breaking amplitude of space-charge waves. It follows, from (3) and (7), that the electric field inside the beam is

$$E=\pm\sqrt{2}(m\omega_p c/e)(W-\gamma+\beta_b u)^{1/2}. \tag{14}$$

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Eq. (14) shows that the maximum of E (i.e., dE/du=0) is given by $v=v_b$, thus

$$E_{max} = \sqrt{2}(m\omega_p c/e)(W-1/\gamma_b)^{1/2}. \quad (15)$$

Now, W is related to the turning points of u through the equation $W-\gamma+\beta_b u=0$, and according to (13) the critical turning points for which wave breaking occurs are given by $v=v_{cr}$ or $v=v_{ph}$. Hence, at the onset of wave breaking, the value W is

$$W = \gamma_{cr} - \beta_b u_{cr} = \gamma_{ph} - \beta_b u_{ph}. \quad (16)$$

The final expression for the maximum electric field is obtained by substituting (16) in (15), yielding

$$E_{max} = \sqrt{2}(m\omega_p c/e)[\gamma_{ph}(1-\beta_{ph}\beta_b)-1/\gamma_b]^{1/2}. \quad (17)$$

Eq. (17), valid for both negative-energy and positive-energy space-charge waves, determines the critical wave amplitude above which wave breaking takes place.

Eqs. (13) and (17) provide alternative means for analyzing the wave-breaking condition of space-charge waves in a relativistic electron (positron) beam. It can be concluded that for a given v_b: (i) for either the negative-energy or positive-energy modes, $E_{max}\to 0$ as $v_{ph}\to v_b$; (ii) for the negative-energy mode E_{max} decreases as v_{ph} increases, and (iii) for the positive-energy mode, E_{max} increases as v_{ph} departs from v_b and $E_{max}\to\infty$ as $|v_{ph}|\to c$.

Conclusions The breaking of space-charge waves studied in this paper is of fundamental relevance for cosmic ray acceleration by strong space-charge waves since it determines the largest accelerating field physically permissible. Our results indicate that wave breaking imposes severe limitation on space-charge waves, either negative-energy or positive-energy modes, with phase velocity close to the particle-beam velocity since in that case the normalized maximum wave amplitude $eE_{max}/m\omega_p c \ll 1$. On the other hand, wave breaking presents little (or no) restriction for positive-energy space-charge waves, either parallel or anti-parallel propagating, with phase speed near (or greater than) the speed of light, since in that case extremely large wave amplitudes, such that $eE_{max}/m\omega_p c \gg 1$, are attainable.

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