LANGMUIR TURBULENCE AND SOLAR RADIO BURSTS

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Abstract. Langmuir waves and turbulence resulting from an electron beam-plasma instability play a fundamental role in the generation of solar radio bursts. We report recent theoretical advances in nonlinear dynamics of Langmuir waves. First, starting from the generalized Zakharov equations, we study the parametric excitation of solar radio bursts at the fundamental plasma frequency driven by a pair of oppositely propagating Langmuir waves with different wave amplitudes. Next, we briefly discuss the emergence of chaos in the Zakharov equations. We point out that chaos can lead to turbulence in the source regions of solar radio emissions.

Key words: chaos, instabilities, radio radiation, Sun, turbulence

1. Introduction

Solar radio bursts provide a powerful tool for predicting the space weather and monitoring the space environment. For example, type-III solar radio bursts are produced by energetic electron beams accelerated in the solar active regions. As the electron beams propagate out of the solar corona and across the solar wind, they interact with the background plasma, resulting in a beam-plasma instability. The growth of this instability leads to large-amplitude Langmuir waves and turbulence, which can generate electromagnetic radiation via nonlinear wave-wave coupling. Since Langmuir waves oscillate at frequencies close to the local plasma frequencies, type-III solar radio bursts may serve as a tracer of the space environment and as a remote sensing technique of Langmuir turbulence in the solar active regions and the interplanetary medium.

A sound interpretation of the observational data of solar radio bursts, from ground radio telescopes and interplanetary spacecrafts, requires a thorough theoretical study of Langmuir waves, instabilities and turbulence. We review in this paper a novel theory of the fundamental plasma emission of type-III solar radio bursts, and Langmuir turbulence driven by chaos in the source regions of solar radio emissions.

A theory of type-III solar radio bursts was formulated by Chian and Alves (1988) for the case of two counterpropagating Langmuir pump with the same wave amplitudes. Rizzato and Chian (1992) improved their model by self-consistently including a second grating that assures the symmetry of the wave kinematics and investigated the simultaneous generation of electromagnetic and Langmuir daughter waves. We generalize the model of Rizzato and Chian (1992) to the case of two pump waves with distinct wave amplitudes (Alves *et al.*, 2002). This theory will provide a better framework for understanding the observed features of type-III solar radio bursts which often indicate the signature of two populations of Lagmuir waves traveling in opposite directions with different amplitudes.

2. A Theory of the Fundamental Plasma Emission of Type-III Solar Radio Bursts

The nonlinear coupling of Langmuir waves (L), electromagnetic waves (T) and ion-acoustic waves (S) is governed by the generalized Zakharov equations (Rizzato and Chian, 1992; Alves *et al.*, 2002)

$$\left(\partial_t^2 - \nu_e \partial_t + c^2 \nabla \times (\nabla \times) - \gamma_e v_{th}^2 \nabla (\nabla \cdot) \ + \omega_p^2\right) \overrightarrow{E} = -\frac{\omega_p^2}{n_0} n \overrightarrow{E} \ , \tag{1}$$

$$\left(\partial_t^2 - \nu_i \partial_t - v_s^2 \nabla^2\right) n = \frac{\varepsilon_0}{2m_i} \nabla^2 < \overrightarrow{E}^2 > \ , \tag{2}$$

where $\overrightarrow{E}$ is the high-frequency electric field, n is the ion density fluctuation, $\omega_p^2 = n_0 e^2 / (m_e \varepsilon_0)$ is the electron plasma frequency, c is the velocity of light, $v_{th} = (k_B T_e / m_e)^{1/2}$ is the electron thermal velocity, $v_s = (k_B (\gamma_e T_e + \gamma_i T_i) / m_i)^{1/2}$ is the ion-acoustic velocity, $\nu_{e(i)}$ is the damping frequency for electrons (ions), $\gamma_{e(i)}$ is the ratio of the specific heats for electrons (ions), and the angle brackets denote the fast time average.

In order to derive a dispersion relation from Equations (1)–(2) we assume that the electric field of the Langmuir pump wave is given in the form

$$\overrightarrow{E}_0 = \frac{1}{2} \Big(\overrightarrow{E}_0^+ exp(i(\overrightarrow{k}_0 \cdot \overrightarrow{r} - \omega_0 t)) + \overrightarrow{E}_0^- exp(i(-\overrightarrow{k}_0 \cdot \overrightarrow{r} - \omega_0 t)) \Big) + c.c.,$$

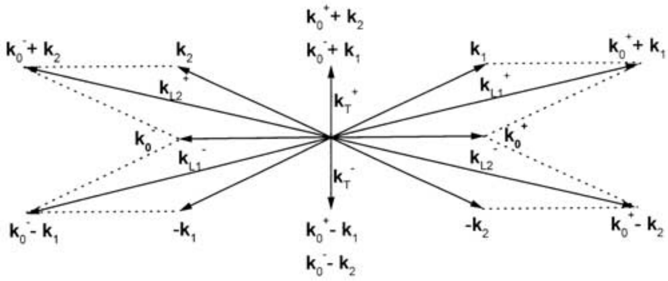

Figure 1. Wave-vector kinematics for our model: $k_0^{+(-)}$ are related to the pump Langmuir waves, $k_{1(2)}$to ion acoustic waves, $k_0^{+(-)} - k_{1(2)}$ are the electrostatic (oblique to $k_0^{+(-)}$) or electromagnetic ($\perp$ to $k_0^{+(-)}$) Stokes modes and $k_0^{+(-)} + k_{1(2)}$ are the electrostatic (oblique to $k_0^{+(-)}$) or electromagnetic ($\perp$ to $k_0^{+(-)}$) anti-Stokes modes.

which represents two oppositely propagating waves of equal frequencies and opposite wave vectors, the amplitude of two pump waves can be different, $\left|\overrightarrow{E}_0^-\right|^2 = r\left|\overrightarrow{E}_0^+\right|^2$, with $0 \le r \le 1$. Each of the two pump waves can generate the Stokes $(\omega_0 - \omega^*)$ and the anti-Stokes $(\omega_0 + \omega)$ modes, either electrostatic or electromagnetic, as known in three-wave processes. We consider the coupling of four triplets. Each two triplets have in common the Langmuir pump wave (forward or backward wave) and a pair of independent density gratings, $n_{1(2)}$, given by

$$n = \frac{1}{2} n_{1(2)} exp\left(i(\overrightarrow{k}_{1(2)} \cdot \overrightarrow{r} - \omega t)\right) + c.c.,$$

with $\left|\overrightarrow{k}_1\right| = \left|\overrightarrow{k}_2\right|$. The electric fields of the daughter waves obey the relation

$$\overrightarrow{E} = \frac{1}{2}\left(\overrightarrow{E}_w^{+(-)} exp(i(\overrightarrow{k}_w^{+(-)} \cdot \overrightarrow{r} - \omega_w^{+(-)} t))\right) + c.c.,$$

where the subscript w represents either electromagnetic (T) or Langmuir (L) daughter wave, respectively; the superscript $+(-)$ represents the anti-Stokes (Stokes) mode, respectively. Figure 1 illustrates this coupling and also shows the wave-vector matching conditions we have assumed.

The assumption of two oppositely propagating pumps implies the need of some process to cause the E_0^+-waves to be scattered into the backward direction. High time resolution electric field waveform measurements from the Galileo spacecraft have shown that Langmuir waves associated with type-III solar radio bursts have a characteristic beat pattern (Gurnett *et al.*, 1993). This beat pattern is produced by two closely spaced narrowband components. One of these components is believed to be the beam-driven Langmuir wave, E_0^+, and the other is believed to be the oppositely propagating Langmuir wave generated by the parametric decay of E_0^+. For the sake of simplicity, we assume in this paper the two pump waves are anti-parallel. It is worth pointing out that the Langmuir decay process can generate a spectrum of $\vec{k}$ not collinear with $\vec{k}_0$. This difference can influence the results and should be studied in the future.

The dispersion relation, considering the wave number and frequency matching conditions, can be obtained using either the propagator technique or the Fourier transform technique of the assumed electric fields. It has been shown that these two techniques lead to the same results (Rizzato and Chian, 1992).

The chosen kinematics implies that the two Langmuir pumps propagate oppositely along the longitudinal x-axis, generating two opposite induced electromagnetic modes that primarily propagate along transverse y-axis, plus induced Langmuir modes that mainly propagate along the x-axis. Writing the total high-frequency fluctuating field in terms of its transverse and longitudinal component, $\vec{E} = \vec{E}_L + \vec{E}_T$, imposing perfect $\vec{k}$-matching but allowing for frequency mismatches between the interacting waves and using the kinematic conditions presented in Figure 1 in Equation (1), we can write the variations of the induced waves as follows

$$\mathcal{D}_{L1}^{-}E_{L1}^{-} = \frac{\omega_p^2}{n_0}n_1^* E_0^- \,,\ \mathcal{D}_{L1}^{+}E_{L1}^{+} = \frac{\omega_p^2}{n_0}n_1 E_0^+ , \tag{3}$$

$$\mathcal{D}_{L2}^{-}E_{L2}^{-} = \frac{\omega_p^2}{n_0}n_2^* E_0^+ , \mathcal{D}_{L2}^{+}E_{L2}^{+} = \frac{\omega_p^2}{n_0}n_2 E_0^- , \tag{4}$$

$$\begin{aligned} \mathcal{D}_T^+ E_T^+ &= \frac{\omega_p^2}{n_0}\left(n_1 E_0^- + n_2 E_0^+\right) , \\ \mathcal{D}_T^- E_T^- &= \frac{\omega_p^2}{n_0}\left(n_1^* E_0^+ + n_2^* E_0^-\right) . \end{aligned} \tag{5}$$

In Equations (3–5) sub-indexes $L1(2)$ refer to the Langmuir daughter wave due do the first (second) grating.

Introducing Equations (3–5) in Equation (2), we obtain the following equations for the density fluctuations

$$\begin{aligned} \mathcal{D}_{s1}n_1 \;=& \frac{\varepsilon_0 k_1^2\omega_p^2}{2m_i n_0}\left(\frac{n_1\left|E_0^+\right|^2}{\mathcal{D}_{L1}^+} + \frac{n_1\left|E_0^-\right|^2}{\mathcal{D}_{L1}^{-*}} + \frac{n_1\left|E_0^-\right|^2}{\mathcal{D}_T^+}\right. \\ &\left. + \frac{n_2E_0^+E_0^{-*}}{\mathcal{D}_T^+} + \frac{n_1\left|E_0^+\right|^2}{\mathcal{D}_T^{-*}} + \frac{n_2E_0^+E_0^{-*}}{\mathcal{D}_T^{-*}}\right) , \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{D}_{s2}n_2 \;=& \frac{\varepsilon_0 k_2^2\omega_p^2}{2m_i n_0}\left(\frac{n_2\left|E_0^+\right|^2}{\mathcal{D}_{L2}^{-*}} + \frac{n_2\left|E_0^-\right|^2}{\mathcal{D}_{L2}^{+}} + \frac{n_2\left|E_0^+\right|^2}{\mathcal{D}_T^+}\right. \\ &\left. + \frac{n_1E_0^{+*}E_0^{-}}{\mathcal{D}_T^+} + \frac{n_2\left|E_0^-\right|^2}{\mathcal{D}_T^{-*}} + \frac{n_1E_0^{+*}E_0^{-}}{\mathcal{D}_T^{-*}}\right) . \end{aligned} \tag{7}$$

In Equations (6) and (7) we have:

$$\begin{aligned}
\mathcal{D}_{s1(2)} &= \omega^2 + i\nu_i\omega - v_s^2 k_{1(2)}^2, \\
\mathcal{D}_T^{\pm} &= \left((\omega_o \pm \omega)^2 + i\nu_T(\omega_o \pm \omega) - c^2 k_T^{\pm 2} - \omega_p^2\right), \\
\mathcal{D}_{L1(2)}^{\pm} &= \left((\omega_o \pm \omega)^2 + i\nu_L(\omega_o \pm \omega) - v_{th}^2 k_{L1(2)}^{\pm 2} - \omega_p^2\right).
\end{aligned}$$

Observe that $\left|\overrightarrow{k}_{L1}^{\pm}\right| = \left|\overrightarrow{k}_{L2}^{\pm}\right|$, and $\left|\overrightarrow{k}_T^{+}\right| = \left|\overrightarrow{k}_T^{-}\right|$, as shown in Figure 1.

Using the high-frequency approximation

$$\begin{aligned}
\mathcal{D}_T^{\pm} &\cong \pm 2\omega_p\left(\omega \pm (\omega_0 - \omega_T) + i\nu_T/2\right), \\
\mathcal{D}_{L1(2)}^{\pm} &\cong \pm 2\omega_p\left(\omega \pm (\omega_0 - \omega_L) + i\nu_L/2\right),
\end{aligned}$$

with $\omega_{L(T)}$ representing the linear relation for the Langmuir (electromagnetic) wave, $\left|\overrightarrow{k}_1\right| = \left|\overrightarrow{k}_2\right|$, and normalizing ω by ω_s and k by $k\lambda_D$ we introduce

$$\begin{aligned}
D_s &= \left(\omega^2 - 1 + i\nu_i/2\right) \\
D_T^{\pm} &= \omega \pm \frac{3}{2}\frac{k_0}{(\mu\tau)^{1/2}} \mp \frac{1}{2}\frac{c^2}{v_{th}^2}\frac{k_T^2}{(\mu\tau)^{1/2}k_0} \\
D_L^{\pm} &= \omega \mp \frac{9}{2}\frac{k_0}{(\mu\tau)^{1/2}}
\end{aligned}$$

where $\mu = m_e/m_i$, and $\tau = (\gamma_e T_e + \gamma_i T_i)/T_e$. Finally, by writing $\left|\overrightarrow{E}_0^{-}\right|^2 = r\left|\overrightarrow{E}_0^{+}\right|^2$, $W_0 = \varepsilon_0\left|\overrightarrow{E}_0^{+}\right|^2/(2n_0k_BT_e)$, and $W_{T0} = (1+r)W_0$, we obtain the general dispersion relation

$$\begin{aligned}
D_s^2 &- \frac{W_{T0}}{4\tau(\mu\tau)^{1/2}k_0}D_s\left(\frac{1}{D_L^+} - \frac{1}{D_L^-} + \frac{1}{D_T^+} - \frac{1}{D_T^-}\right) \\
&+ \frac{W_{T0}^2}{16\mu\tau^3k_0^2(1+r)^2}\left[\frac{-(1-r)^2}{D_T^+D_T^-}\right. \\
&+ (1+r^2)\left(\frac{1}{D_T^+D_L^+} - \frac{1}{D_L^+D_L^-} + \frac{1}{D_T^-D_L^-}\right) \\
&\left. -2r\left(\frac{1}{D_L^+D_T^-} + \frac{1}{D_T^+D_L^-}\right) + r\left(\frac{1}{D_L^{+2}} + \frac{1}{D_L^{-2}}\right)\right] = 0\,.
\end{aligned} \tag{8}$$

In order to verify the relative importance of the second wave pump amplitude, we solve Equation (8) for different values of r and typical physical parameters measured in type-III events in the solar wind ($W_0 = 10^{-5}, k_0 = 0.0451, \mu = 1/1836, v_{th} = 2.2 \times 10^6 m/s, T_e = 1.6 \times 10^5 K, T_i = 5 \times 10^4 K$). Figure 2 shows

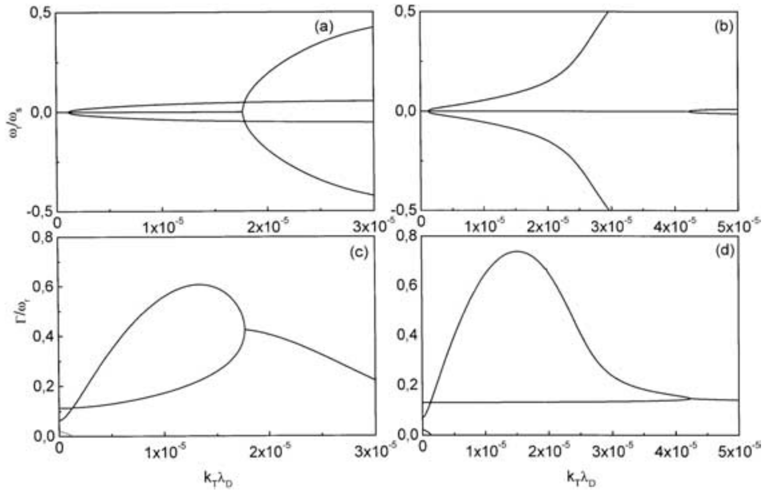

Figure 2. Numerical solutions for the present model with different values of r; (a) and (b) show the real part of the solution and (b) and (d) the growth rates, with $k_0 = 10^{-4}$, within the limit $k_0 < (1/3)W_0^{1/2}$; (a) and (c) refers to $r = 0.5$, and (b) and (d) to $r = 0.95$.

the results for $r = 0.5$ ((a) and (c)), and for $r = 0.95$ (b) and (d)). The presence of a second pump wave with different amplitude from the first one introduces a region of convective instability ($\omega_r \neq 0$), not present when $r = 1$. For small values of $k_T \lambda_D$, the instability is purely growing, $\omega_r = 0$. For a critical value of $k_T \lambda_D$ the real part of the frequency bifurcates off the line $\omega_r = 0$, while still presenting an imaginary part $\Gamma \neq 0$. The critical value of $k_T \lambda_D$ decreases as r decreases.

3. Langmuir Turbulence driven by Chaos in the Source Regions of Solar Radio Emissions

The Zakharov equations, Equations (1)–(2), can model the Langmuir turbulence driven by chaos in the emission regions of solar radio bursts.

De Oliveira, Rizzato and Chian (1995) analysed regular and chaotic dynamics of one-dimensional weakly relativistic Zakharov equations. They first locate a certain region of parameter space where the nonlinear dynamics of Langmuir waves is low-dimensional and regular. They then study the transition to chaos as a function of the system length scale, which follows initial inverse pitchfork bifurcations. The transition includes resonant and quasiperiodic features, as well as separatrix crossing phenomena. Rizzato, de Oliveira and Erichesen (1998) investigated the process of energy transfer in the Zakharov equations. Starting with a low-dimensional quasi-integrable regime where solitons are formed via a modulational instability, they showed that if the largest length scale of the linearly excited modes is much longer than the most unstable one, the interaction of these solitons with ion-acoustic waves leads to energy transfer to smaller length scales due to the stochastic dynamics.

The transition from order to chaos may occur in 3-wave interaction and triplet-triplet interaction in the Zakharov equations. De Oliveira, de Oliveira and Rizzato (1997) found that as the wave amplitude increases a triplet undergoes a transition from a quasi-integrable regular regime to a nonintegrable chaotic regime. Lopes and Rizzato (1999) showed that the coupling of two triplets destroys integrability, leading to spatiotemporal chaos.

The aforementioned works have demonstrated that as the beam-excited Langmuir waves evolve nonlinearly, a variety of dynamical processes may lead to Langmuir turbulence via chaos. Since the electrostatic Langmuir waves are in general coupled to electromagnetic waves, as seen in Equations (1)–(2), solar radio emissions provide an interesting means to probe the turbulent state of the plasma regions traversed by the energetic electron beams emanated from the solar active regions. Further details on this subject can be obtained in the reviews by Chian (1999) and Chian *et al.* (2000).

4. Conclusions

In conclusion, we derived a general dispersion relation for the parametric generation of fundamental plasma emissions due to two counterstreaming Langmuir waves with different wave amplitudes. The presence of a second pump wave with different amplitude from the first one, excites a region of convective instability, not present for equal wave amplitudes. In addition, we discussed the transition from order to chaos which can explain the onset of Langmuir turbulence. There are a number of observational reports of a close correlation of Langmuir and ion-acoustic waves in connection with type-III radio bursts (Lin *et al.*, 1986; Gurnett *et al.*, 1993). Recently, Thejappa and MacDowall (1998) presented experimental results for type-III radio bursts with clear evidence of occurrence of ion-acoustic waves in association with Langmuir waves. Hence, the theory presented in this paper provides a detailed picture of nonlinear wave-wave interaction processes which may be responsible for the excitation of type-III solar radio emissions.

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