Reliability of watershed area estimation using Digital Elevation Models

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Abstract

Digital elevation models (DEM) have become very useful for delineating catchment basins to obtain more accurate area estimations in the last decades. As a discretization of a continuous surface, DEM's ground sample data (GSD) delimit the elementary watershed boundary segments. The finer the GSD resolution, the closer tends to be the true drainage basin edge and its area to those estimated by DEM processing. Our goal is to establish the minimum area estimation that fits a desired degree of reliability for any catchment shape, considering the GSD of the DEM, the admitted tolerance and the shape of the basin. This approach starts with the area assessment of perfect circles which are then converted to raster format using a regular matrix of pixels. The squared Gravelius Index of a watershed is taken as a multiplicative factor that will allow the comparison between different shapes of basins, aiming to establish the minimum circular area compliant with the desired tolerance, in this case acknowledged as 0.5 percent and an equivalent GSD of 3-second-arc (90m) from SRTM. The estimation of mean response for the minimum area according to a least squares regression model is 0.641 Km².

Keywords: Watershed area, Gravelius Index, digital elevation models, SRTM.

1. Introduction

The determination of contribution areas of hydrography branches is crucial for many activities related to hydrology and water resources management. Since digital elevation models (DEM) have improved in vertical reliability and data availability in the last decades, they play an important role in watershed modelling.

One of the most used DEM in hydrological studies is the Shuttle Radar Topography Mission (SRTM) (Tulu, 2005; Rossetti and Valeriano, 2006; Pryde *et al.*, 2007; Mantelli *et al.*, 2009; Alcântara *et al.*, 2010.). World public free distribution of its finished dataset is available for a regular grid with 3-second-arc (approximately 90 m in Equator Line) resolution (USGS, 2011). Due the continental dimensions of Brazil, SRTM is widely adopted by Agência Nacional de Águas (National Water Agency) for delineating catchment basins.

Nevertheless, a DEM is a discretization of a continuous space and its ground sample data (GSD) delimits the elementary surface unit and watershed boundary segments. The finer the GSD resolution, the closer tends to be the true drainage basin edge and its area to those estimated by digital elevation model processing.

Considering the extremely irregular shapes of many watersheds with their recesses, it is needful to check if the estimated areas established by basin delineations from DEMs achieve a desired tolerance.

The Gravelius Index (K_G) represents the ratio of the perimeter of a shape to the perimeter of a circle having the same area and it is one of the several geomorphologic indexes (Barberis *et al.*, 2003).

Based on K_G, our working hypothesis is that the squared Gravelius Index of any shape can be taken as a multiplicative factor of a minimum area of a circle that achieve a desired tolerance in order to establish the smaller catchment basin area that complies with the same tolerance.

After testing the concept for general shapes, this work aims to estimate the minimum area of an equivalent DEM digitized circle that fits a desired degree of reliability. This value of area should be multiplied by for any watershed squared Gravelius Index, considering the same GSD and determined tolerance. We have chosen SRTM and 0.5% tolerance for practical reasons in most hydrological applications.

2. Methodology

The methodology includes two steps: the proof of concept and the estimation of the minimum area A^{*}_{min} for an equivalent ground sample data of 3-second-arc (90m) from Shuttle Radar Topography Mission.

2.1. Conceptual proof

Assuming the perimeter P_S and area A_S of a shape, the conceptual proof is based on the development of the general Gravelius Index formula $K_G = P_S/[2(\pi A_S)^{1/2}]$ to demonstrate the relationship between the minimum area A_{min} of a circle and the squared Gravelius Index of any other shape ($K_G{}^2$) to determinate its minimum area that can be estimated within a certain degree of reliability.

Squaring both terms of the general formula, we get:

$$K_G{}^2 . A_S = \frac{P_S{}^2}{4\pi} \tag{1}$$

The right term of Equation (1) is the area of a circle in function of its perimeter and, since the unity is the lowest value for K_G, we conclude that $A_S = A_{min}$ if it is a circular shape.

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In order to calculate A_{min}, we have to solve $[\pi(r + \Delta r)^2/\pi r^2] \leq 1 + Tol$, assuming Δr as a little gap for the radius r of a circle in the continuous surface that should include or exclude a boundary pixel in the equivalent DEM. So, we can establish the minimum radius r_{min} for which a new gap causes no relative variation on its previous area greater then a tolerance Tol and calculate A_{min}.

Thus, the minimum area A_{min} is:

$$A_{min} = \pi \left(\frac{\Delta r}{\sqrt{1 + Tol} - 1} \right)^2 \tag{2}$$

According to K_G formula, for an equal sides l quadrilateral and an equilateral triangle, we have squared Gravelius Indexes QK_G^2 and TK_G^2, respectively. Following the same reasoning presented for the circle, in order to calculate the minimum quadratic and triangular shape areas QA_{min} and TA_{min} for which a new gap Δr causes no relative variation on their previous area greater then a tolerance Tol, we get:

$$\frac{(l + 2\Delta r)^2}{l^2} \leq 1 + Tol \tag{3}$$

$$QA_{min} = \left(\frac{2\Delta r}{\sqrt{1 + Tol} - 1} \right)^2 \tag{4}$$

$$\frac{\frac{1}{2} \left[(l + 2\sqrt{3}\Delta r)(\frac{\sqrt{3}}{2} l + 3\Delta r) \right]^2}{\frac{\sqrt{3}}{4} l^2} \leq 1 + Tol \tag{5}$$

$$TA_{min} = 3\sqrt{3} \left(\frac{\Delta r}{\sqrt{1 + Tol} - 1} \right)^2 \tag{6}$$

From the regular quadratic shape and equilateral triangle Gravelius Indexes QK_G^2 and TK_G^2; the Equations (2), (4) and (6), we conclude that $QA_{min} = QK_G^2.A_{min}$, and $TA_{min} = TK_G^2.A_{min}$, the conceptual proof.

2.2. Estimation of the minimum raster area A^*_{min} in the discrete space

Since the discretization of circles causes boundary pixels which compensate elementary areas themselves and must not be completely overestimated, we need an appropriate approach to evaluate the minimum circular area in the discrete (raster) space A^*_{min} through least squares regression model technique.

Due the positional variability of central point of circles inside a pixel at the grid of digital elevation model (Figure 1), Monte Carlo experiments will be conducted in order to generate 100 randomly positions for each radius origin in a group of 300 circles ($n = 300$). The initial radius r_l is set to be 30m and the gap Δr, also 30m. These are the inputs to determinate the best fit regression function in the next processes.
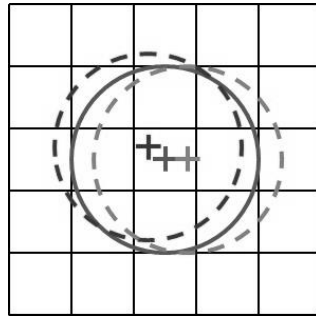
Figure 1: Positional variability of circles considered in Monte Carlo experiments.

The area, the perimeter and the Gravelius Index for the n circles (A_n, P_n, K_{Gn}) and their respective discretized polygons mean parameters (A_n*, P_n*, $K_{Gn}*$) according to SRTM ground sample data are then calculated.

Following the procedures, all the absolute relative difference areas ($AbsRelDifArea_n = |(A_n/A_n*) - 1|$) are computed to construct the regression model with A_n* as response variable.

Finally, the best fit function can be established and the point of mean response for A_n^* when the absolute relative difference area (required tolerance) equals to 0.5% ($A_n^* = A_{min}^*$), the goal of this work, is estimated.
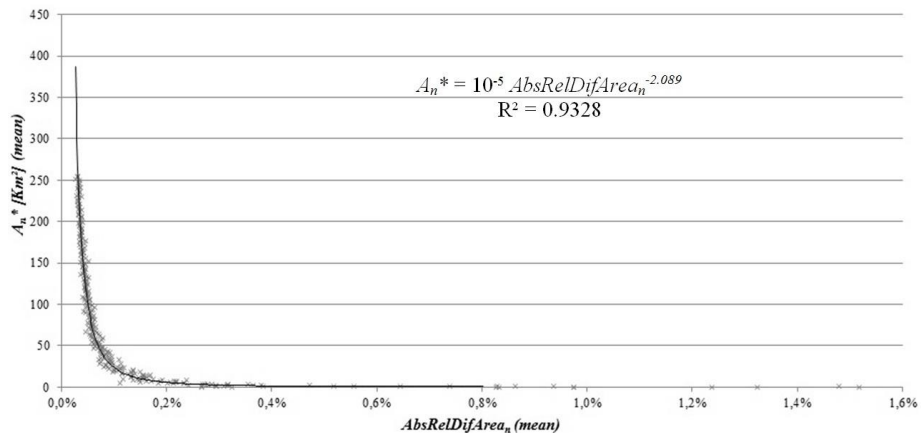
3. Results

An example of vector circle – dashed line – and its equivalent SRTM raster polygon is illustrated in the (Figure 2).
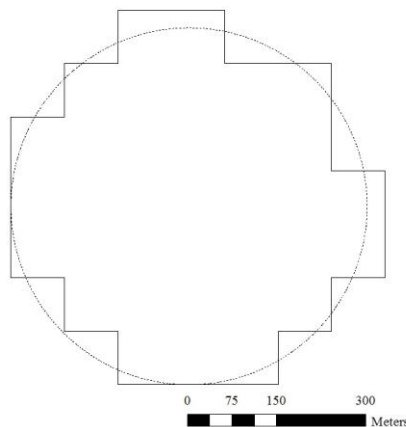
**Figure 2:** Vector circle ($r = 300$m) and its equivalent SRTM raster polygon.

(Figure 3) illustrates the best fit function for all processed data – fulfilled line, its regression equation and the coefficient of determination R².

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$$A_n^* = 10^{-5} \, AbsRelDifArea_n^{-2.089}$$
$$R^2 = 0.9328$$

Figure 3: Best fit function (fulfilled line), regression equation and R².

Considering the tolerance of 0.5% ($AbsRelDifArea_n = 0.005$), the estimation of the point of mean response for A^*_{min} according to the regression equation is 0.641 Km².

It was also noted that the greater the radius of Monte Carlo simulated circles, the closer is the Gravelius Index K_{Gn}^* to 1.27 for their minimum, mean and maximum equivalent SRTM raster polygons (Figure 4). It indicates a correction for Gravelius Index of watershed shapes obtained from SRTM before applying this squared index as a multiplicative factor of the minimum area A^*_{min}.
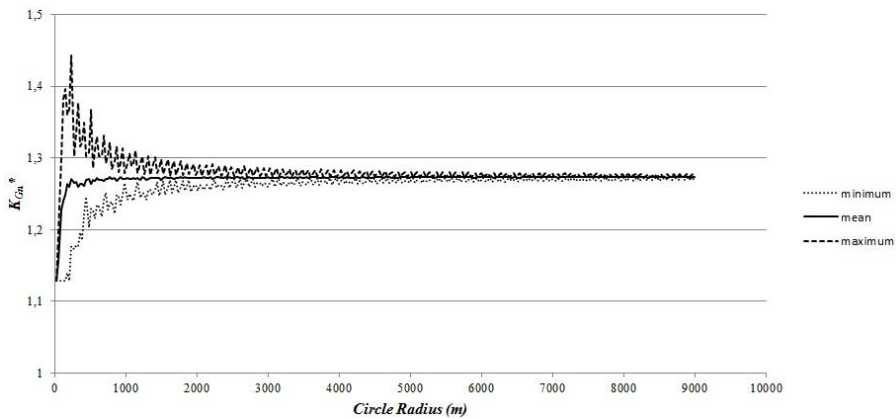
Figure 4: Convergence of Gravelius Indexes.

4. Conclusion

This work is aimed at estimating the minimum area A^*_{min} of raster SRTM polygons equivalent to digitized perfect circles that achieve a 0.5% relative difference area tolerance and its relationship with any other watershed shape.

It has been proved that the squared Gravelius Index of a shape is a multiplicative factor for a minimum area of a circle which satisfies the same relative difference area tolerance.

Data from Monte Carlo experiments vector circles and their equivalent SRTM digitized raster polygons have established a least squares regression equation which contemplates highly correlated response (A_n*) and explanatory ($AbsRelDifArea_n$) variables – $R^2 = 0.9328$.

Considering the tolerance of 0.5%, the estimation of the point of mean response for A^*_{min} is 0.641 Km².

The Gravelius Indexes tend to 1.27 as the areas of raster circles increase. This constant should be a correction for Gravelius Index of a watershed shape before applying this index's squared value as a multiplicative factor in order to obtain the minimum area that fits the desired tolerance for that shape.

The focus of this paper is the reliability of watershed area estimation using SRTM, but the same reasoning can be followed for another digital elevation model, like ASTER/GDEM (Advanced Spaceborne Thermal Emission and Reflection Radiometer/Global Digital Elevation Model), and for others natural resources necessities, like analysis of deforestation area reliability using digital images from satellites.

References

Alcântara, E.H., Novo, E.M.L.M., Stech, J.L., Assireu, A., Nascimento, R., Lorenzzetti, J.A., Souza, A. (2010), "Integrating historical topographic maps and SRTM data to derive the bathymetry of a tropical reservoir". *Journal of Hydrology*, Vol. 389, n. 3-4, pp. 311-316.

Barberis, C., Molnar, P., Claps, P., Burlando, P. (2003), *Hydrologic Similarity of River Basins through Regime Stability*. Working Paper 2003-03, Politecnico di Torino, Italy. Available at: <http://www.idrologia.polito.it/~barberis/PDF/WorkingPaperChiara.pdf>. Access in: 15 March 2012.

Mantelli, L.R., Rossetti, D.F., Albuquerque, P.G., Valeriano, M.M. (2009), "Applying SRTM digital elevation model to unravel Quaternary drainage in forested areas of Northeastern Amazonia". *Computers and Geosciences*, Vol. 35, pp. 2331-2337.

Pryde, J.K., Osorio, J., Wolfe, M.L., Heatwole, C., Benham, B., Cardenas, A. (2007), "Comparison of watershed boundaries derived from SRTM and ASTER digital elevation datasets and from a digitized topographic map". *ASABE Annual International Meeting*, Minnesota, United States of America, ASABE Paper Number 072093.

Rossetti, D.F., Valeriano, M.M. (2006), *Evolution of the lowest amazon basin modeled from the integration of geological and srtm topographic data*. São José dos Campos, Brazil. Available at: <http://urlib.net/sid.inpe.br/ePrint@80/2006/07.31.20.37>. Access in: 13 March 2012.

Tulu, M.D. (2005), *SRTM Suitability in Runoff Studies*, Master thesis, International Institute for Geoinformation Science and Earth Observation, The Netherlands. Available at: <http://www.itc.nl/library/papers_2005/msc/wrem/mesay.pdf>. Access in: 15 March 2012.

USGS (2011), *Shuttle Radar Topography Mission (SRTM) – Finished*, Earth Resources Observation and Science Center, United States of America. Available at: http://eros.usgs.gov/#/Find_Data/Products_and_Data_Available/SRTM. Access in: 13 March 2012.