FLYBY FOR OPTIMAL SOLUTIONS FOR NEAR-EARTH ASTEROID

Denilson Paulo Souza dos Santos, Antônio Fernando Bertachini de Almeida Prado

[1,2] *Division of Space Mechanics and Control – INPE*
C.P. 515, 12227-010 São José dos Campos - SP, Brasil
denílson.paulo@gmail.com; denílson@dem.inpe.br; prado@dem.inpe.br

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ABSTRACT

The spacecraft propulsion system has passed for diverse evolutions, leaving combustion engines and arriving at ion propulsion. The necessity of more efficient rockets stimulated the research in this scope. In this work ΔV will be analyzed proceeding from an electric propellant acting in set with gravitational maneuvers. The optimization of maneuvers will be approached in interplanetary missions using solar electric propulsion and Gravity Assisted Maneuver attended to reduce the costs of the mission. The high specific impulse of electric propulsion makes a Gravity Assisted Maneuver 1 year after departure convenient. Missions for several Near Earth Asteroids will be considered. The analysis suggests criteria for the definition of initial solutions demanded for the process of optimization of trajectories.

1. Introduction

Indirect optimization methods are suitable for the low thrust trajectories that are used in simulations. A finite force is applied during a finite interval of time and it is necessary to integrate the state equation along the time to know its effect. Several results exist in literature, starting with the works of Tsien (1953) and Lawden (1955). Other results and references can be found in Prado (1989), Prado and Rios-Neto (1993), Casalino and Colasurdo [1], Santos [2]. The most used method in this model is the so called "primer-vector theory", developed by Lawden (1953 and 1954) [4], [5], according to Prado [6], [7], Santos [8], [9]. In this paper, theory of optimal control is applied and a procedure based on the Newton Method to decide the boundary problems is developed. The Pontryagin's Maximum Principle (PMP) is used to maximize the Hamiltonian associated to the problem and evaluates the optimal structure of the "switching function".
The spacecraft leaves the Earth's sphere of influence with a hyperbolic velocity whose optimal magnitude and the direction will be supplied by the optimization procedure. The initial mass is directly related to the magnitude of the hyperbolic velocity, assuming that a chemical thruster is used to leave a low Earth orbit (LEO). Out of the Earth's sphere of influence, the electric propellants is activate and the available power is proportional to the square of the distance from the sun; the propulsion is provided by one or two "PPS 1350 ion thrusters and Phall1 thrusters (UNB)".

2. Description of the Problem

The spacecraft will be considered a point with variable mass m and trajectory will be analyzed using the patched-conics approach. The time required by the spacecraft to leave the Earth's sphere of influence is neglected and, in this formulation, only equations of motion in the heliocentric reference system will be considered. The spacecraft is influenced by the Sun gravitational acceleration $\vec{g}(r)$ and the propulsion system of the vehicle implements a thrust T. With this

formulation, a maneuver of Earth flyby can be used to gain energy and velocity, which provokes a discontinuity in the relative state variables in the velocity.
The variables are normalized using the radius of the Earth's orbit, the corresponding circular velocity, and the mass of the spacecraft in stationary orbit as values of reference.
The solar electric Propulsion will be considered, therefore, the available power and thrust varies with the square of the distance from the sun.
In the problem, the thrust is the only control during the heliocentric arcs, and it will be optimized to get the minimum consumption, that is measured by the final mass of the spacecraft. Since the thrust appears linearly in the equation of motion, a bang-bang control, which consists of alternating ballistic arcs with arcs of maximum thrust, will be required. The trajectory is composed by a succession of ballistic arcs (zero-thrust) and arcs of maximum thrust, where the optimal direction will be supplied by the optimization procedure.
The boundary conditions are imposed in satisfactory way at the junctions between trajectory arcs.
The integration initiates when the spacecraft leaves the Earth's sphere of influence, at the position $\vec{r}_i = \vec{r}_\oplus(t_i)$ that coincides with the Earth is position, considering the velocity $\vec{v}_i$ free. The hyperbolic velocity is given by $\vec{v}_{\infty i} = \vec{v}_i - \vec{v}_\oplus(t_i)$, assuming that a rocket thruster is used to leave the Low Earth Orbit (LEO) with an impulsive maneuver; the vehicle mass on LEO is specified. The increment of velocity (ΔV) demanded to provide the hyperbolic velocity is $\Delta V = \sqrt{v_{\infty i}^2 + v_e^2} - v_c$, where v_e and v_c are the escape and circular velocity at the LEO radius (Santos et. all., 2009).
The initial mass at the exit from the Earth's sphere of influence is,

$$m_i = (1+\varepsilon)e^{-\frac{\Delta V}{C'}} - \varepsilon \quad (1)$$

where,
$\varepsilon(1-m_i)$ is the jettisoned mass of the exhausted motor, which is proportional to the propellant mass.
The spacecraft intercepts the Earth and accomplishes Gravity Assisted Maneuvers (Santos et al., 2005). The position of the vehicle $\vec{r}_\pm = \vec{r}_\oplus(t_\pm)$ is constrained and the magnitude of the hyperbolic excess velocity $\vec{v}_{\infty\pm} = \vec{v}_\pm - \vec{v}_\oplus(t_\pm)$ is continuous $v_{\infty+}^2 = v_{\infty-}^2$ [2].
If the minimum height constraint on the flyby is requested, a condition on the velocity turn angle is added:

$$\vec{v}_{\infty+}^T \vec{v}_{\infty-} = -\cos(2\phi) v_{\infty-}^2 \quad (2)$$

where,

$$\cos(\phi) = \frac{v_p^2}{\left(v_{\infty-}^2 + v_p^2\right)} \quad (3)$$

v_p is the circular velocity at the low distances allowed for a planet.

$$\vec{v}_{\infty\pm} = \vec{v}_{i\pm} - \vec{v}_4 \quad (4)$$

At the final point (subscript f), the position and velocity vectors of the spacecraft and the asteroid coincide,

$$r_f = r_A(t_f) \quad (5)$$
$$v_f = v_A(t_f) \quad (6)$$

The theory of optimal control provides the control law and necessary boundary conditions for optimality.

3. Optimization Procedures

The objective is to use the theory of optimal control to maximize the spacecraft final mass.
Dynamical equations are,

$$\dot{\vec{r}} = \vec{v}$$
$$\dot{\vec{v}} = \vec{g}(\vec{r}) + \frac{\vec{T}}{m} \qquad (7)$$
$$\dot{m} = -\frac{\vec{T}}{c}$$

Applying the theory of optimal control, the Hamiltonian function is defined as (Lawden, 1954) :

$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t (\vec{g} + \frac{\vec{T}}{m}) - \lambda_m \frac{\vec{T}}{c} \qquad (8)$$

An indirect optimization procedure is used to maximize the payload. According to Pontryagin's Maximum Principle the optimal controls maximize H.
The nominal thrust T_o at 1 AU, and the electrical power are (Casalino, L. and Colasurdo, G., 2002),

$$P_0 = \frac{T_0 c}{2\eta}$$
$$T_{Max} = \frac{T_0}{r^2} \qquad \mathbf{(9)}$$

Optimal control theory provides differential equation for the adjoint equations of the problem (Euler-Lagrange).
Adjoint equations are,

$$\dot{\vec{\lambda}}_r^t = \vec{\lambda}_v^t \frac{\partial \vec{g}}{\partial \vec{r}} - S_f \frac{\partial T}{\partial \vec{r}} \qquad (10)$$

$$\dot{\vec{\lambda}}_v^t = -\vec{\lambda}_r^t \qquad (11)$$

$$\dot{\vec{\lambda}}_r = \lambda_v^t \frac{\vec{T}}{m^2} \qquad (12)$$

where, $G = \frac{\partial \vec{g}}{\partial \vec{r}}$.
Optimal control: thrust direction an magnitude are,

$$\vec{T} \,//\, \vec{\lambda}_v$$
$$H = \vec{\lambda}_r^t \vec{v} + \vec{\lambda}_v^t G + \vec{T}\left(\frac{\lambda_v}{m} - \frac{\lambda_m}{c'}\right) \qquad (13)$$
$$S_f = \frac{\lambda_v}{m} - \frac{\lambda_m}{c'}$$

where, c' - is the effective exhaust velocity of the rocket thruster;

$$T_{Max} = \begin{cases} \frac{T_0}{r^2} \rightarrow S_f > 0 \\ 0 \rightarrow S_f < 0 \end{cases} \quad (14)$$

The necessary optimal conditions.

$$\left(H_{j_-} + \frac{\partial \varphi}{\partial t_{j_-}} + \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_-}} \right) \delta t_{j_-} = 0 \quad (15)$$

$$\left(H_{j_+} - \frac{\partial \varphi}{\partial t_{j_+}} - \mu^t \frac{\partial \vec{\chi}}{\partial t_{j_+}} \right) \delta t_{j_+} = 0 \quad (16)$$

$$\left(\lambda^t_{j_-} - \frac{\partial \varphi}{\partial \vec{x}_{j_-}} - \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_-}} \right) \delta \vec{x}_{j_-} = 0 \quad (17)$$

$$\left(\lambda^t_{j_+} + \frac{\partial \varphi}{\partial \vec{x}_{j_+}} + \mu^t \frac{\partial \vec{\chi}}{\partial \vec{x}_{j_+}} \right) \delta \vec{x}_{j_+} = 0 \quad (18)$$

Where:

$\vec{X}$: the vector collecting the constraining boundary conditions (see eq. 15 - 18)

$\varphi = m_f$

At the initial point:

1. $\vec{r}_0 = \vec{r}_\oplus$;
2. $m_o = 1 - bV_\infty - cV_\infty^2$
3. $(\vec{v}_0 - \vec{v}_\oplus)^2 = \vec{v}_{\infty 0}^2$;
4. Equations 16 and 18 provide optimal control with λ_{ro} and T_{ro} free;
5. the necessary condition optimal of the state is $\vec{\lambda}_{v0}$ (primer vector) be parallel to the hyperbolic velocity;

At flyby []

1. the equations (15 and 16) are used to obtain the transversality conditions, that implicates in determining the arc time used;
2. at the equations (17 and 18) the $\vec{\lambda}_{vi}$ is parallel to the hyperbolic velocity, before and after of free flyby maneuver; the magnitude is continuous;
3. the states of Hamiltonian remain continuous through the *flyby* maneuvers;
4. when the minimum height constraint of the *flyby* is requested, a condition on the velocity turn angle is added (Eq. 2 and 3).

At the final point:

1. $\vec{\lambda}_{vf}$ is parallel to the hyperbolic velocity, $\vec{\lambda}_{rf}$ is parallel to the radius and $\vec{\lambda}^t_{rf} \vec{v}_f + \vec{\lambda}^t_{vf} \vec{g} = 0$;
2. the final values of $\vec{\lambda}_{mf}$ and H_f depends on the control model that was considered in the maneuver;
3. the adjoint variable $\vec{\lambda}_v$ is zero during the whole trajectory.

4. Mission Asteroid 2002TC70

4.1 Numerical Analysis With PPS1350 (ESA)

The characteristics of the spacecraft propulsion system that have been assumed are:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg;
2. specific impulse $I_s = 1550s$, specific energy $\varepsilon = 0.06$;
3. $T = 2 \cdot 70mN$ (thruster PPS 1350 used for the SMART-1 mission to the moon);
4. The time: *time* = 0 corresponds to the date 01/01/2000.

The necessary optimal condition were formulated in agreement with the problem; the bang-bang control was used in the formularization with limited power and constraint in the time of flight.

Table 1– Keplerian Elements

Name	2002TC70	1989UQ
Epoch	54200	54200
a	1.369831	0.915249
e	0.19691574	0.264832
i	2.13932	1.29152
Ω	161.89427	178.2965
ω	134.84892	15.0241
M	351.6336031	189.172
r_a	1.639572	1.157636
r_p	1.10009	0.672861

4.1.1 Simulation without Flyby

Using the optimization procedure we can find optimal trajectories, with the maximization of the spacecraft final mass (i.é., minimum fuel consumption). These trajectories depend on the mission objectives, for example, the performance depends on the mission time length. It is possible to reduce the time with some more spend of propellant.

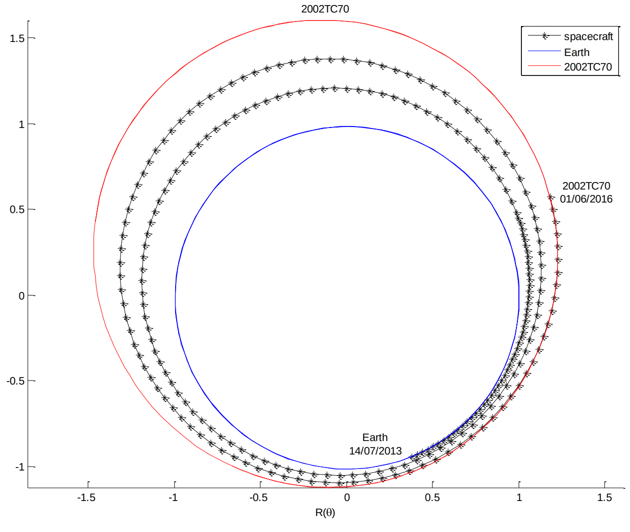

Figure 1 - Trajectory direct (without flyby) for the asteroid 2002TC70.

The structure of the switching function that shows the thrust arc (red) and coast arc (blue), i.é., shows the alternation between the propulsion arcs and the arcs without propulsion (Eq. 14, Figure 2); During the transfer maneuver happen variations in the eccentricity (e), semi-major axis (a) and hamiltonian and energy orbits.

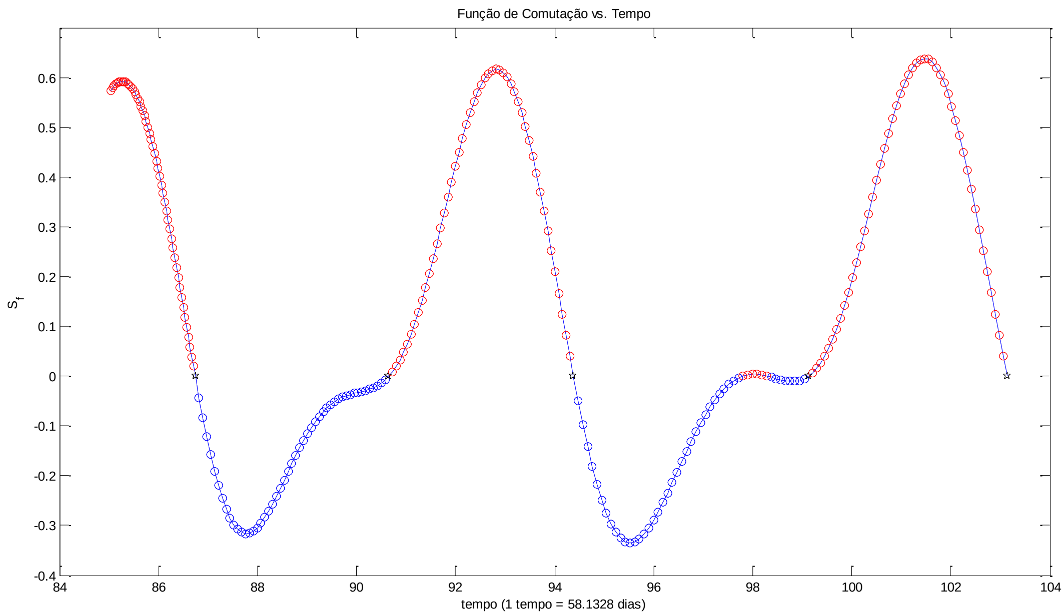

Figure 2 - The Switching Function in Trajectory direct for the asteroid 2002TC70.

4.1.2 Earth Gravity Assisted - EGA mission

An Earth *flyby* can be used to vary the semi-major axis (a) and the eccentricity (e) in order to increase the apoapsis (r_a) (or reduce the periapsis r_p) or to vary the inclination (i) of the orbit.

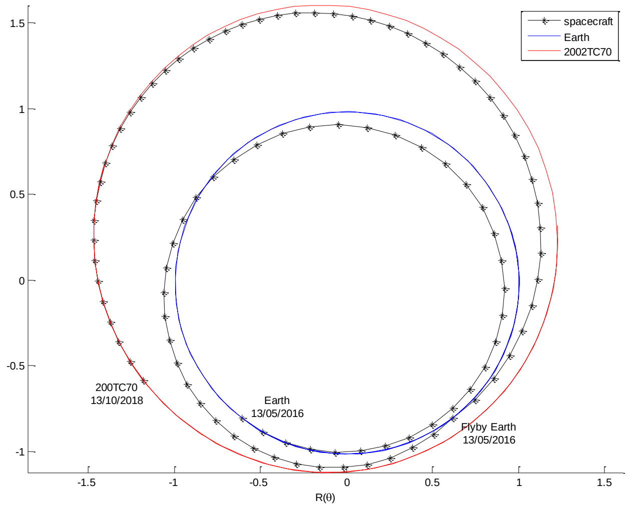

Figure 3 - Trajectory leaving the Earth and arriving to the asteroid, using EGA maneuvers.

Both the effects can be achieved if the line of nodes and the line of apsides are aligned (i.e. the argument of periapsis is close to $\omega \approx 0$, 180 or 360 degrees).

4.1.3 Earth Mars Gravity Assisted - EMGA mission

The formulation allows the use of multiple flybies searching for a better performance. The criterion of the choice of *flyby* in Mars or Venus is the semi-major axis (a) of the asteroid:

$$\begin{cases} a > 1 \rightarrow Mars\ Flyby \\ a < 1 \rightarrow Venus\ Flyby \end{cases} \quad (19)$$

Mars flyby can be used to increase the periapsis (r_p) and in Venus to reduce the apoapsis (r_a).

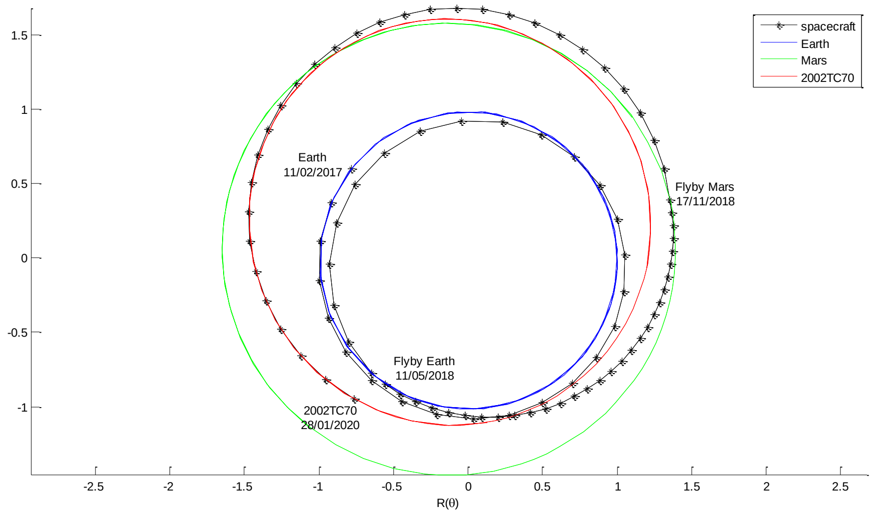

Figure 4 - Trajectory leaving the Earth and arriving to the asteroid, using EMGA maneuvers.

The Table 2 show the parameters of a mission leaving the Earth, making one flyby in the Earth and other in Mars (EMGA), with the objective of intercepting the asteroid 2002TC70.
Table 2 exhibits a comparison of time and final mass of the vehicle with the use of the optimized maneuver without flyby e with flyby at the Earth, and, Earth and Mars.

4.2 NUMERICAL ANALYSIS WITH PHALL 1 (UNB)

The researchers of the Plasma Laboratory of the Physics Institute of the Brasilia University (UNB), since 2002, pledge in the study and development of a propellant that uses a plasma propulsion system produced by current Hall, based on Stationary Plasma Thrusters (SPT). In this project uses permanent magnets with generating the magnetic field, reducing the electricity consumption.
The characteristics of the spacecraft propulsion system are:

1. the mass of the spacecraft with an altitude of 200 km in circular LEO is 2133.3 Kg;
2. specific impulse I_s = 1607s , specific energy $\varepsilon = 0.06$;
3. $T = 2 \cdot 126$mN (thruster Phall 1 - UNB);
4. The time: *time* = 0 corresponds to the date 01/01/2000.

Diverse missions can be implemented with the optimization algorithm used in this work, of which the main ones are: transference with free time (to change to space vehicle orbit without restrictions to the necessary time the execution maneuver); “Rendezvous” (one desires that the space vehicle if finds and remains to the side of as a space vehicle); “Flyby” (desires to intercept one another celestial body, however without the objective to remain next); “Swing-By” (is used of a next ticket to a celestial body to gain or to lose energy, speed and angular moment), etc.

4.2.1 - Simulation without flyby

In this section seen direct orbits with the Thruster Phall 1. In the Figure 5, the space vehicle trajectory is observed that leaves the Earth and arrives to the asteroid. The mission duration time (ΔT) is approximately 748 days and the final mass of the vehicle (Δm) is of 1673,7 Kg.

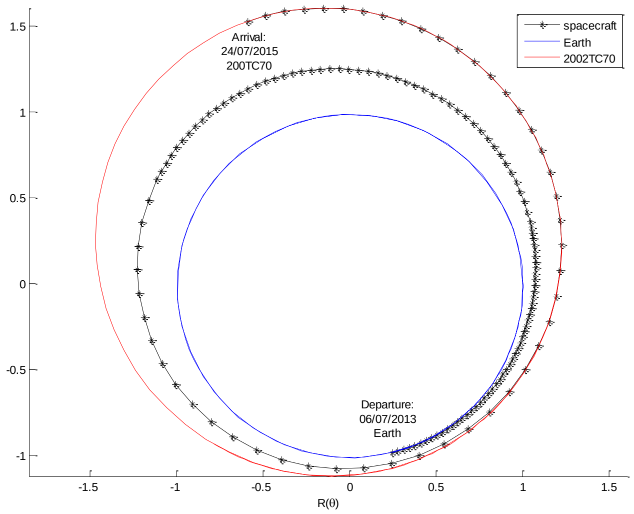

Figure 5 - Trajectory direct (without flyby) for the asteroid 2002TC70, using Phall 1 .

4.2.2 – Earth Gravity Assisted - EGA mission

The transference orbit leaving Earth, make Earth flyby approximately one year and 3 months later, and, arriving at the asteroid 2002TC70 with the mission total time (ΔT) of 845.23 days after the departure and final mass of the vehicle (Δm) is of 1875 kg.

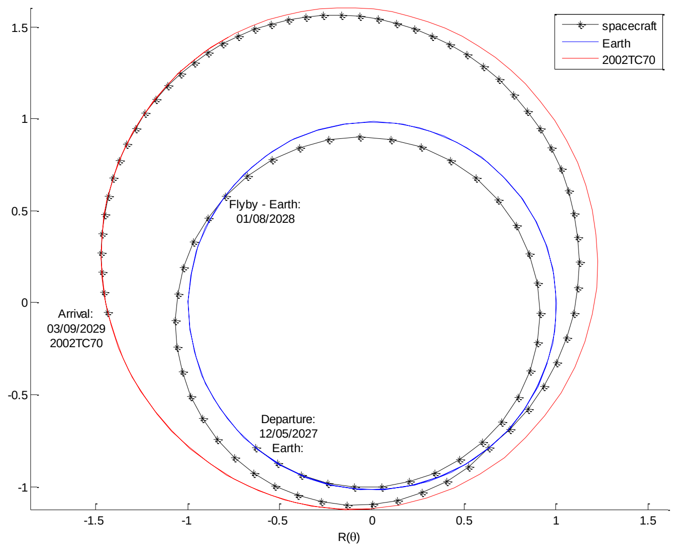

Figure 6 - Trajectory leaving the Earth and arriving to the asteroid, using EGA maneuvers, with Phall 1.

4.2.2 EMGA mission

Been verified resulted better in comparison to the results gotten with PPS 1350 (Table 2), therefore, Phall 1 possesss a bigger specific thrust and the truster (t) is bigger in magnitude.

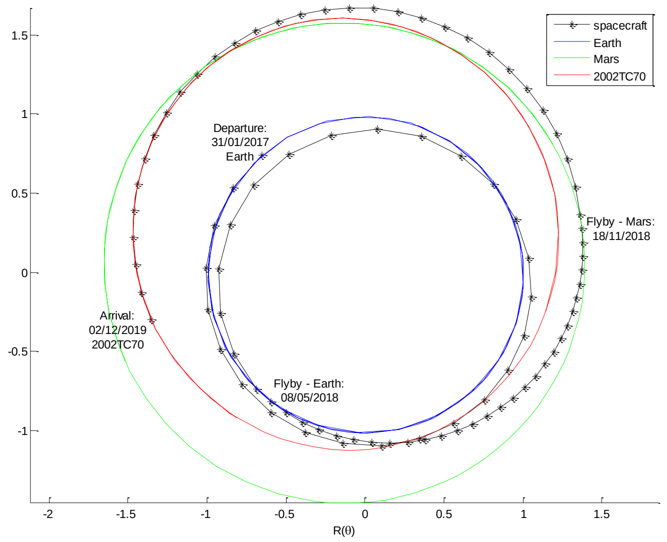

Figure 7 - Trajectory leaving the Earth and arriving to the asteroid, using EMGA maneuvers, with Phall 1 .

Table 2 - Comparison table between the PPS 1350 e Phall 1, he use of the flyby for asteroids 2002TC70 using two thrusters, where are evidenced several missions without flyby, EGA and EMGA for asteroid missions, involving propellant consumption optimization (Δm) and gains in velocity because the maneuvers and possible dates for the missions.

Asteroid 2002TC70									
PPS 1350 (I_s= 1550s) (2 x 70mN)					**Hall (UNB) (Is= 1607s) (2 x 126mN)**				
Duration (days)	m_f	***Data***	**ΔV_{el} (electric) (km/s)**	**V_∞ (km/s)**	***Duration (days)***	m_f	***Data***	**ΔV_{el} (electric) (km/s)**	**V_∞ (km/s)**
Flyby: 0					***Flyby: 0***				
1052.85	0.7768	Departure: 14/07/2013 Arrival: 01/06/2016	3.83917757	1.25579628	748.24	0.7846	Departure: 06/07/2013 Arrival: 24/07/2015	3.820898907	1.117814179
***Flyby: 1* – Earth (EGA)**					***Flyby: 1* – Earth (EGA)**				
883.40	0.8684	Departure: 13/05/2016 *Flyby* - Earth: 30/07/2017 Arrival: 13/10/2018	2.14480657	0.8376562	845.23	0.8789	Departure: 12/05/2027 *Flyby* - Earth: 01/08/2028 Arrival: 03/09/2029	2.034274163	0.870011798
***Flyby*: 2 - Earth – Mars (EMGA)**					***Flyby: 2* - Earth – Mars (EMGA)**				
1081.15	0.8739	Departure: 11/02/2017 *Flyby* - Earth: 11/05/2018 *Flyby* - Mars: 17/11/2018 Arrival: 28/01/2020	2.04883921	0.58081769	1035.51	0.8886	Departure: 31/01/2017 *Flyby* - Earth: 08/05/2018 *Flyby* - Mars: 18/11/2018 Arrival: 02/12/2019	1.86129921	0.647333385

In the table 2 is evidenced that in all the simulated missions, that the direct missions (without flyby) possess a bigger consumption of the propellant (Δm), and, as consequence missions that use gravitational maneuvers reduce the consumption.

5. Mission Asteroid 1989UQ

The following types of missions had been simulated: without flyby; Earth Gravity Assisted - EGA mission; Earth and Venus Gravity Assisted - EVGA mission.
Using the optimization procedure we can find optimal trajectories, with the maximization of the spacecraft final mass (i.é., minimum fuel consumption). These trajectories depend on the mission objectives, for example, the performance depends on the mission time length. It is possible to reduce the time with some more spend of propellant.

5.1 Analysis with PHALL 1 (UNB)

The researchers of the Plasma Laboratory of the Physics Institute of the Brasilia University (UNB), since 2002, pledge in the study and development of a propellant that uses a plasma propulsion system produced by current Hall, based on Stationary Plasma Thrusters (SPT). Been verified resulted better in comparison to the results gotten with PPS 1350 (Table 3), therefore, Phall 1 possess a bigger specific thrust and the thruster (T) is bigger in magnitude.

Table 3 - Comparison table between the PPS 1350 e Phall 1, he use of the flyby for asteroids 1989UQ using two thrusters, where are evidenced several missions without flyby, EGA and EMVA for asteroid missions, involving propellant consumption optimization (Δm) and gains in velocity because the maneuvers and possible dates for the missions.

Asteroid 1989UQ									
PPS 1350 (I_s= 1550s) (2 x 70mN)					**Hall (UNB) (Is= 1607s) (2 x 126mN)**				
Duration ΔT (days)	***Δm***	***Data***	***ΔV_{el} (Electrical) (km/s)***	***V_∞ (km/s)***	***Duration ΔT(days)***	***Δm***	***Data***	***ΔV_{el} (Electrical) (km/s)***	***V_∞ (km/s)***
Flyby: 0					***Flyby: 0***				
893.48	0.7652	Departure: 01/10/2017 Arrival: 13/03/2020	4.06805051	1.25579628	1305.5	0.7822	Departure: 01/10/2017 Arrival: 29/04/2021	3.870516065	0.986023774
***Flyby: 1* – Earth (EGA)**					***Flyby: 1* – Earth (EGA)**				
920.09	0.8494	Departure: 11/07/2025 *Flyby* - Earth: 30/09/2026 Arrival: 17/01/2028	2.481256546	0.800492443	567.62	0.8244	Departure: 11/08/2024 *Flyby* - Earth: 10/11/2025 Arrival: 02/03/2026	3.04349697	0.844407717
***Flyby*: 2 - Earth – Venus (EVGA)**					***Flyby: 2* - Earth – Venus (EVGA)**				
1164.92	0.8792	Departure: 25/06/2017 *Flyby* - Earth: 19/09/2018 *Flyby* - Venus: 05/03/2019 Arrival: 02/09/2020	1.956977088	0.694198512	1096.3	0.9177	Departure: 16/06/2017 *Flyby* - Earth: 20/09/2018 *Flyby* - Venus: 06/03/2019 Arrival: 16/06/2020	1.353277282	0.748891517

The present analysis favors a guess at the tentative solution as the Earth's positions as departure and flyby are a priori known. The ideal asteroid has perihelion radius which is close to 1AU, a low-energy orbit and low inclination with relation to the ecliptic plane.

6. Conclusion

The search for the best initial parameters for a mission is facilitated, if the transfer orbit with free time is optimized first. The ideal asteroids for EGA missions should possess low orbit energy, perihelion close to 1 UA, low inclination per EGA.
Indirect optimization methods based on optimal control theory supply accurate solutions. The use of Gravity Assisted Maneuver (EGA, EMGA or EVGA) in this mission reduces the fuel consumption and the time of the maneuver, demonstrating that this important formulation is viable and useful.
Orbits with Phall 1 had been analyzed using gravity assisted maneuvers and verified resulted optimistical for the implantation of probes using this technology, also being able to use this formularization in the future missions that use launch vehicle that is in development/improvement (VLS-2, Brazil), which can inject in LEO (low earth orbit) a satellite medium sized, thereafter, use the solar electric propulsion (SEP) or nuclear (NEP) to dislocate the vehicle for desired orbits, maximizing them with the maneuver that use assisted gravity.
The present analysis favor a guess at the tentative solution as the Earth's positions as departure and flyby are a priori known. The ideal asteroid has perihelion radius which is close to 1AU, a low-energy orbit and low inclination with relation to the ecliptical axis.
The performance parameters of Phall are competitive with known electromagnet Hall thrusters found on the literature.

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