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Rotation-based sampling Multi-Particle Collision Algorithm with Hooke-Jeeves for Vibration-based Damage Identification

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Abstract

The structural vibration-based damage identification can be formulated as an optimization problem. The objective functional is expressed by a least square difference between measured and computed forward model displacements. The latter functional is minimized by using a hybrid scheme combining Rotation-based sampling Multi-Particle Collision Algorithm with Hooke-Jeeves heuristic (RMPCA-HJ).

Multi-Particle Collision Algorithm (MPCA) is a stochastic optimization method inspired by the physics in the nuclear reactor, where absorption and scattering phenomena are represented. In the MPCA algorithm, a set of particles (solutions) travels in the search space. After a certain number of function evaluations, they share the best particle solution found. MPCA, working together with the Rotation-Based Learning (RBL), is used as a first stage of the hybrid method performing a global exploratory search. RBL is a novel extension of Opposition-based Learning (OBL). In RBL, a rotated solution is calculated by applying a specific rotation angle to the original solution. Here, the new Rotation-Based Sampling (RBS) solution projects a point between the original solution and its rotated solution. RBS could be more flexible than RBL, and also OBL, to find the promising candidate solutions. The intensification search stage of the hybrid metaheuristic is addressed by the direct search Hooke-Jeeves (HJ) method. HJ consists of the repeated application of exploratory searches for all dimensions around a base point. If the exploration has success finding a better solution, a pattern move is performed.

The hybrid algorithm is tested to identify damages on a truss structure. Experimental data was generated in silico, using time-invariant damages. Experiments with noiseless and noisy data, under several levels of noise, were carried out. Good estimations of damage location and severity are achieved.

Keywords: Vibration-based damage identification; hybrid metaheuristic algorithm; inverse problem;

1 Introduction

The field of System Identification includes important applications such as Structural Health Monitoring (SHM). SHM performs a global damage identification for aerospace, civil and mechanical engineering infrastructures, and it can be operated off-line as well as on-line. The capacity of early detection of possible damages allows to repair or rehabilitate a structure before it has major damages. The identification should be independent of changes in the operational and environmental conditions. The methods used in damage identification should also be well suited to automation, and it should be independent of human judgment and ability [1].

To have the capability of identifying damages in an accurate and a safe way is essential in critical systems, such as aerospace structures. Damages which are not detected and, therefore, not repaired, could produce catastrophic consequences, with human and economic losses.

The damage identification becomes a task computationally expensive when the structures are complex, with a high number of degrees of freedom. In the inverse solution, it is necessary to develop a method that could solve the problem with a low number of objective function evaluations.

MPCA is a stochastic algorithm based on phenomena occurring inside a nuclear reactor. Particles travel within the reactor and collide between them, being absorbed or scattered. In MPCA, some solutions (called particles) travel in the search space and cooperate sharing the best candidate overall, after reaching a specific number of function objective evaluations [2].

Recently, a hybrid algorithm using MPCA was used in the identification of damages in some simple structures: a 10-DOF mass-spring-damper system, a beam and a three-bay truss [3, 4]. In this work, new variants of this hybrid algorithm will be used to accelerate the convergence speed, reaching the same results in less time. More details about the hybrid algorithm are found on Section 4.

The inverse solution is evaluated over a three-bay truss structure. Subsection 2.1 describes this case study. Some results comparing the canonical version MPCA-HJ [3, 4] with the new variants are presented in Section 5.

2 Equations of motion for modeling a forward problem of forced vibration

The dynamic response of motion of a structure is given by a second order, non-homogeneous ordinary differential equation, shown in Eq. (1). In the equation, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ represent the $d \times d$ mass, damping and stiffness matrices, respectively; d is the number of degrees of freedom of the structure. $\mathbf{F}$ and $\mathbf{u}$ are the external force and the displacement vectors, respectively. The initial conditions for the model are given by Eq. (2).

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t)\,; \tag{1}$$

$$\mathbf{u}(0) = \mathbf{u}_0\ ,\ \dot{\mathbf{u}}(0) = \dot{\mathbf{u}}_0 \tag{2}$$

The numerical solution for this model is obtained using the Newmark method, since no analytical solution exists for any arbitrary functions of $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{F}$ [5].

2.1 Case Study: Three-bay Truss Structure

The structure for our study is the same we used in previous works [3, 4]: a three-bay truss structure modeled with 12 aluminum bars and 12 degrees of freedom, shown in Figure 1, with properties and dimensions shown in Table 1.

Table 1: Truss Model Properties

Property	Value
Element type	Bars
Material	Aluminium
Youngs modulus (E)	70 GPa
Material density (ρ)	2700 kg/m^3
Square cross section area (A)	2.5×10^{-5} m^2
Non-diagonal elements length ($l_{non-diagonal}$)	1.0 m
Diagonal elements length ($l_{diagonal}$)	1.414 m

The damping in the structure is assumed proportional to the stiffness ($\mathbf{C}_i = 10^{-5}\mathbf{K}_i$). External forces in the positive diagonal direction over the nodes A and B are imposed.

Initial conditions for displacement and velocity are equal to zero ($\mathbf{u}(0) = 0\,, \dot{\mathbf{u}}(0) = 0$).

3 Vibration-based damage identification problem as an optimization problem

In this work, we formulate the inverse problem of localizing and quantifying damages on the structure as an optimization problem. An optimization algorithm will minimize the squared error between the

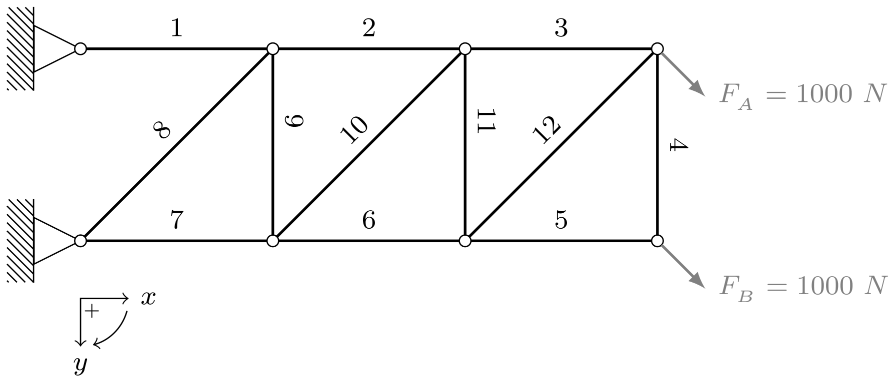


Figure 1: Three-bay truss structure

computed displacements $\mathbf{u}^{mod}$ (obtained after running the structural model with a stiffness vector k) and the measured displacements $\mathbf{u}^{obs}$ (acquired from the vibration experiments), as follows:

$$J(k) = \sum_{i=0}^{d_m} \left[\mathbf{u}_i^{obs}(t) - \mathbf{u}_i^{mod}(k,t)\right]^2 \,, \tag{3}$$

where t represent the time, d_m is the number of measured displacements, and $k = (k_1, k_2, \cdots, k_n)$ contains the values of the stiffness for each element, with n elements in total.

Figure 2 shows a graphical representation of the inverse solution for a generic problem.

Estimated damages Θ^{d} (in percent) are represented by the loss of stiffness:

$$\Theta^{\mathrm{d}} = \left(1 - \frac{k^{\mathrm{d}}}{k^{\mathrm{u}}}\right) \times 100\% \,, \tag{4}$$

where $k^{\mathrm{d}} = \left(k_1^{\mathrm{d}}, k_2^{\mathrm{d}}, \cdots, k_n^{\mathrm{d}}\right)$ and $k^{\mathrm{u}} = (k_1^{\mathrm{u}}, k_2^{\mathrm{u}}, \cdots, k_n^{\mathrm{u}})$ are the estimated stiffness vector for the damaged system, and the stiffness vector of the undamaged system, respectively.

4 Hybrid metaheuristic for the solution of the inverse problem

In this work, we present an extension of the hybrid algorithm MPCA-HJ [3, 4], using the Rotation-based Learning (RBL) mechanism to improve the exploration of solution space. The algorithm acts as a multi-stage structure [6]: a global search or exploration stage (performed by the variant of MPCA) followed by a local search or intensification stage (using HJ).

Different mechanisms based on the Opposition-based Learning (OBL) are added to MPCA for increasing the capacity of exploration. Specifically, in this work, the Center-based Sampling (CBS) will be compared with the Rotation-based Sampling (RBS), described in the next subsections.

4.1 Multi-Particle Collision Algorithm

The creation of MPCA was inspired by the physics of nuclear particle collision reactions [7]. In the nuclear reactor, some phenomena occur, including scattering (an incident particle is scattered by a target nucleus) and absorption (an incident particle is absorbed by the target nucleus).

MPCA can be loosely described as an algorithm consisting of a set of particles traveling inside a nuclear reactor. New particles are generated, and they can be absorbed or scattered, depending on their fitness, and if the fitness is better, they will substitute the *old* particles.

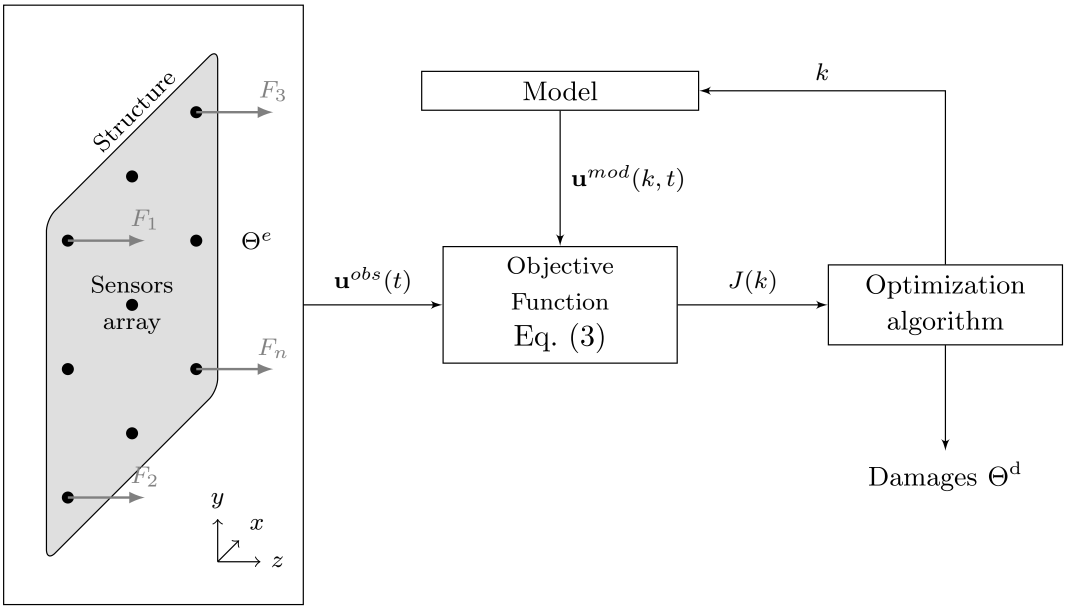


Figure 2: Vibration-Based Damage Identification as optimization problem

A parallel version of the MPCA is used, taking advantages of a high-performance environment, and its pseudocode is shown in Algorithm 1. In each processor (of a total of $N_{processors}$), $N_{particles}$ candidate solutions are set. This partition leads to a considerable reduction of computing time [2].

MPCA starts with initial particles randomly created spread all across the search space (Algorithm 1 – lines 3 and 4). After creating the initial set, a blackboard strategy is used for sharing the best particle among all the particles (Algorithm 1 – lines 6 and 7). Later, the traveling process of particles is started, involving three main functions: Perturbation, Exploitation, and Scattering [2, 7].

In the ending of each iteration, if a specified number of function evaluations ($NFE_{blackboard}$) was reached after the last blackboard updating, (computed as the difference between the Number of Function Evaluations (NFE_i) and the last Number of Function Evaluations when the blackboard was updated ($lastUpdate_i$), as seen in Algorithm 1 – line 19), then the mechanism of cooperation is triggered (Algorithm 1 – lines 20-22). Again, the best particle is shared among all the particles in the set.

As stopping criterion, a maximum number of function evaluations (NFE_{mpca}) is defined.

The current version of MPCA was implemented in FORTRAN 90, using the OpenMPI library, in a multiprocessor architecture with distributed memory machine.

4.2 Opposition-Based Learning and variants

The OBL concept was introduced in 2005 by Tizhoosh[8]. The idea of OBL is to evaluate the opposite of the candidate solution, that have a certain probability of being better. Then, a simple choose of the better solution among them is done.

OBL and their extensions have been applied to improve the performance of various computational intelligence methods, such as artificial neural networks, fuzzy logic, metaheuristic algorithms, and miscellaneous applications [9].

Mathematically, the opposite number z_o of a real number $z \in [a, b]$ is defined by:

$$z_o = a + b - z \ . \tag{1}$$

The opposite point $Z_o(z_{o_1}, z_{o_2}, \cdots, z_{o_D})$ of a point $Z(z_1, z_2, \cdots, z_D)$, with D dimensions, is completely defined by its coordinates as show in equation (2).

$$z_{o_d} = a_d + b_d - z_d \tag{2}$$

Algorithm 1 Multi-Particle Collision Algorithm

```
for i ← 1 to N_processors do                                    ▷ Initial set of particles
    NFE_i = 0, lastUpdate_i = 0
    for j ← 1 to N_particles do
        newP_{i,j} = RandomSolution
        NFE_i = NFE_i + 1
for i ← 1 to N_processors do                                    ▷ Initial blackboard
    bestP_i = UpdateBlackboard
while NFE < NFE_max do                                          ▷ Stopping criteria
    for i ← 1 to N_processors do
        for j ← 1 to N_particles do
            newP_{i,j} = Perturbation(newP_{i,j})
            if f(newP_{i,j}) < f(newP_{i,j}) then
                newP_{i,j} = newP_{i,j}
                newP_{i,j} = Exploration(newP_{i,j})
            else
                newP_{i,j} = Scattering(newP_{i,j}, newP_{i,j}, bestP_i)
            if f(newP_{i,j}) < f(bestP_i) then
                bestP_i = newP_{i,j}
    if NFE_i - lastUpdate_i > NFE_blackboard then
        for i ← 1 to N_processors do                            ▷ Blackboard
            bestP_i = UpdateBlackboard
            lastUpdate_i = NFE_i
for i ← 1 to N_processors do                                    ▷ Final blackboard
    bestP_i = UpdateBlackboard
return bestP_1
```

where $z_d \in \mathbb{R}$, with $a_d \leq z_d \leq b_d \; \forall d \in \{1, 2, \cdots, D\}$, $A = (a_1, a_2, \cdots, a_D)$ and $B = (b_1, b_2, \cdots, b_D)$ are the lower and upper boundaries of the search space, respectively. The center of the search space in each dimension is denoted by $C = (c_1, c_2, \cdots, c_D)$ and $c_d = (a_d + b_d)/2$.

Other variants of this mechanism have been developed, giving more success in the exploration/exploitation of the search space and improving the convergence [10]. Figure 3a shows a graphical representation of these mechanisms, that are defined as follows:

- *Quasi-opposition* reflects a point to a random point between the center of the domain and the opposite point $(Z_{qo}(z_{qo_1}, z_{qo_2}, \cdots, z_{qo_D}) \mid z_{qo_d} = rand(c_d, z_{o_d}))$
- *Quasi-reflection* projects the point to a random point between the center of the domain and itself $(Z_{qr}(z_{qr_1}, z_{qr_2}, \cdots, z_{qr_D}) \mid z_{qr_d} = rand(c_d, z_d))$
- *Center-based sampling* projects the point between itself and its opposite $(Z_{cb}(z_{cb_1}, z_{cb_2}, \cdots, z_{cb_D}) \mid z_{cb_d} = rand(z_d, z_{o_d}))$

4.2.1 Rotated-Based Learning and Rotated-Based Sampling

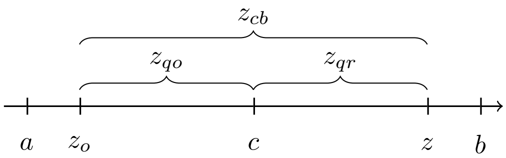
The RBL mechanism is another extension of the OBL [10]. In this strategy, each coordinate of the rotation point $Z_r = (z_{r_1}, z_{r_2}, \cdots, z_{r_D})$ can be calculated in two dimensions (2D), as represented in Figure 3b.

Defining the quantity from the original point z_i to the center $(u_i = z_i - c_i)$, and the lenght from the original number to the corresponding intersection point l_i on the circle $(v_i = \sqrt{(z_i - a_i)(b_i - z_i)})$, each i-th rotation number can be calculated as:

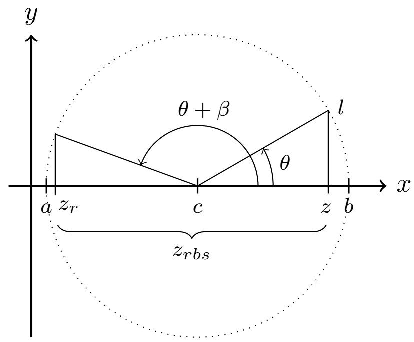
$$z_{r_i} = c_i + u_i \times \cos\beta - v_i \times \sin\beta\,, \tag{3}$$

The deflection angle β is a random value generated by a Gaussian distribution of mean equal β_0 and standard deviation δ, and it is defined in the equation (4).

$$\beta = \beta_0 \cdot \mathcal{N}(1, \delta)\ . \tag{4}$$



(a) Opposite (z_o), Quasi-Opposite (z_{qo}), Quasi-Reflected (z_{qr}) and Center-based sampled (z_{cb}) points



(b) Geometric interpretation of the Rotation (z_r) and the Rotation-based Sampling (z_{rbs}) numbers in 2D space

Figure 3: Concepts derivated from Opposition-based Learning

The Rotation-based Sampling (RBS) is a variant derived from the CBL and RBL mechanisms. As shown in Figure 3b, a rotation-based sampled number z_{rbs} is found with the projection of the number z between itself and a rotation number found z_r. Then, the Rotation-Based Sampling point is defined as

$$Z_{rbs}(z_{rbs_1}, z_{rbs_2}, \cdots, z_{rbs_D}) \mid z_{rbs_d} = rand(z_d, z_{r_d}). \tag{5}$$

4.3 Hooke-Jeeves Direct Search Method

The direct search method of Hooke-Jeeves [11] consists of the repeated application of exploratory moves about a base point which, if successful, is followed by pattern moves. Details about the algorithm of HJ can be found in the literature [11].

5 Experimental results

The RMPCA-HJ is compared with results obtained using the hybrid algorithms MPCA-HJ and CBMPCA-HJ. For the experiments, synthetic data were created running the forward model with simulated damages, reducing the stiffness value of some elements. Table 2 shows the stifness values of all elements, and how much they were reduced. Damaged elements are represented on Figure 4 with dashed lines.

Table 2: Stiffness values and assumed damage percentage

Element	k^{u} $[N/m]$	k^{d} $[N/m]$	Θ^{d}
1, 3, 5, 6, 9, 11	1750000	1750000	0
2	1750000	1487500	15
4	1750000	1662500	5
7	1750000	1225000	30
8	1237440	1237440	0
10	1237440	1113696	10
12	1237440	989952	20

The final time for all the numerical simulations was assumed as $t_f = 5 \times 10^{-2}$ s, with a time step of 5×10^{-4} s. Table 3 shows the stopping criteria and the control parameters of the hybrid algorithm. For the HJ method, the iterative procedure stops if either the minimum step ($h_{\min}$) or the maximum number of function evaluations (NFE_{hj}) are reached.

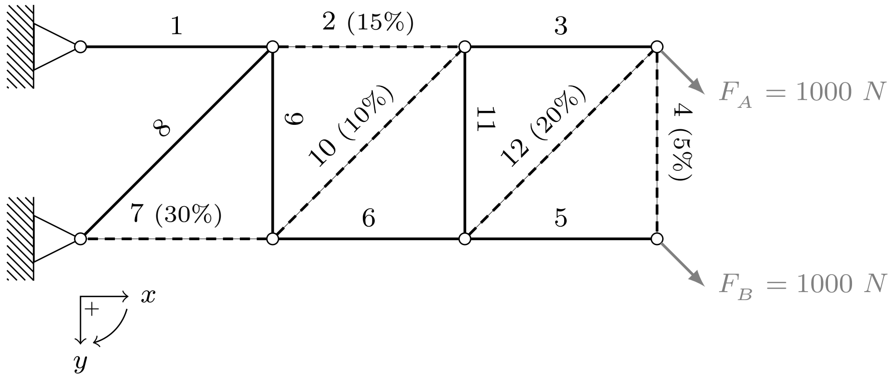


Figure 4: Three-bay truss structure with damages represented with dashed lines. Numbers in the parenthesis represent the damage percentage.

Table 3: Search space, control parameters and stopping criteria for the hybrid algorithm

Algorithm	Parameter	Value
MPCA	$N_{\text{particles}}$	10
	IL	0.7
	SL	1.1
	$NFE_{\text{blackboard}}$	1000
	NFE_{mpca}	100000
RBL	β_0	3.14 rad
	δ	0.25
HJ	ρ	0.8
	h_{min}	1×10^{-7}
	NFE_{hj}	100000
Search space		$[0.5, 1.05]K^{\text{u}}$

Figure 5 shows the results of the damage identification with noiseless data graphically. The upper graph shows the results with the algorithms without the intensification stage (MPCA, CBMPCA or RMPCA), while the graph at the bottom represents the achieved results with the hybrid algorithms (MPCA-HJ, CBMPCA-HJ or RMPCA-HJ). Each column represents an element; the white bar represents the value of the damage to be estimated, and each colored bar the result with a specific algorithm. Negative damages, i.e. stiffness values greater than integral value, are not supported physically and are represented as (-1). Without the HJ method, the results are far from the desired ones. When the intensification is applied after the exploration performed by MPCA and their variants, the values obtained are almost perfect for the data noise-free.

Figure 6 represents the bar graph for the results with noisy data. The noise was simulated mathematically using a random error added to the node displacements along all the coordinates, as follows:

$$\hat{\mathbf{u}}(t) = \mathbf{u}(t) + \mathcal{N}(0, \sigma), \tag{6}$$

where $\hat{\mathbf{u}}(t)$ represents the noisy data to be obtained, and $\mathcal{N}(0, \sigma)$ is a normal distribution with mean equal zero and standard deviation σ. In the experiments, three cases of noise are tested, with $\sigma = 0.02$, $\sigma = 0.05$ and $\sigma = 0.10$.

When assumed noisy experimental data with $\sigma = 0.02$, good results were obtained. Almost all damaged elements were well identified with a difference less than 2% of damage. The fourth element had an error of 4% of damage. In the 6^{th} element, a false damage appeared with a value of almost 4%. The 8^{th}, 9^{th} and 11^{th} elements presented a stiffness greater than the integral value.

For the cases with $\sigma = 0.05$ and $\sigma = 0.10$, worse results were obtained. All the estimated values of damage were incremented, appearing false damages in the 1^{st} and the 3^{rd} elements, and becoming worse in the 6^{th} element. All the existing damages were well identified although their estimated values are different from the real ones. Again, the 8^{th}, 9^{th} and 11^{th} elements presented negative damages, and for $\sigma = 0.10$ the 5^{th} element also presented an increased stiffness.

Table 4 shows the mean of the NFE spent to reach the best solution found in all cases tested. In the noiseless case, and noisy cases with $\sigma = 0.02$ and $\sigma = 0.05$, the RMPCA-HJ and the CBMPCA-HJ arrived at the best value faster than the canonical MPCA-HJ, with a difference of more than 10,000 evaluations. In the noisy case with $\sigma = 0.10$, the difference in the convergence of the compared algorithms is lesser.

Table 4: Mean of Number of Function Evaluations spent to reach the best solution found

Noise (σ)	Variant	NFE with MPCA	NFE with HJ	Total NFE
0.00	-	101417	73536	174953
	CB	101024	40638	141662
	RB	100658	37747	138405
0.02	-	100930	50516	151446
	CB	100934	30111	131045
	RB	101233	34742	135975
0.05	-	100716	39101	139817
	CB	101301	20709	122010
	RB	100921	27827	128748
0.10	-	101109	27225	128334
	CB	101187	21043	122230
	RB	101263	28804	130067

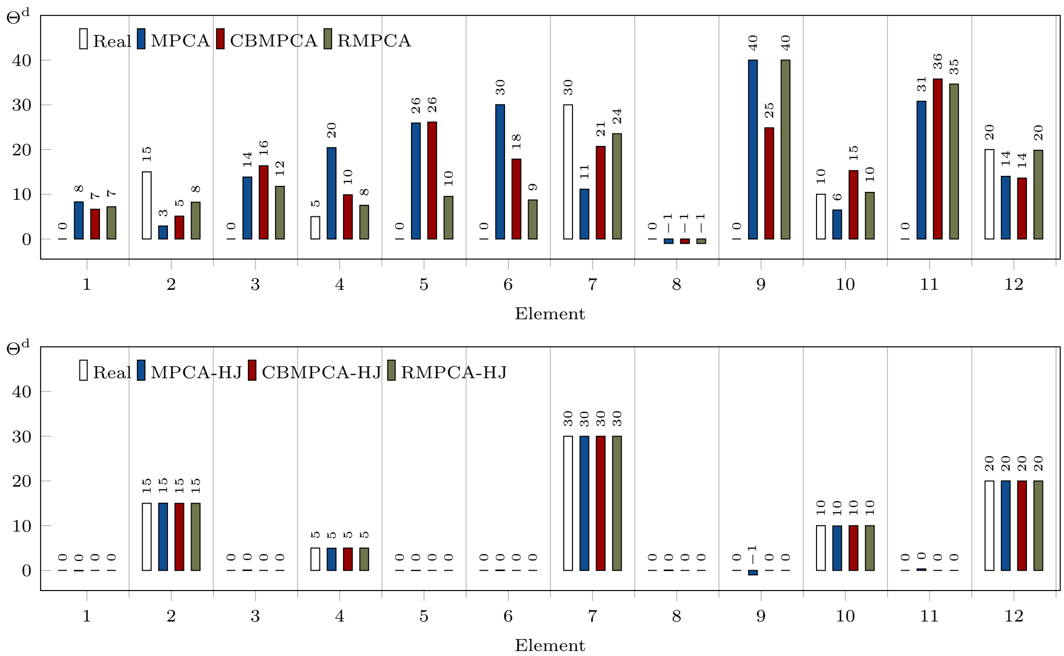


Figure 5: Damage identification with noiseless data

6 Final Remarks

In this work, the inverse problem of structural damage identification was solved by using some variants of the hybrid method MPCA-HJ. The addition of mechanisms based on the Opposition-based Learning allowed converging to good damage estimates in a lower number of function evaluations, meaning that it improves the speed of convergence.

The inverse solution using hybrid algorithms for identifying structural damages has achieved good results.

6.1 Further works

In future works, the proposed hybrid method will be applied to more complex structures, with a high number of degrees of freedom, such as a simplify model of the International Space Station [12]. Also, some experiments with real data acquired in a laboratory will be performed. Another issue to be developed is to couple the inverse solution with commercial CAEs, such as ANSYS or NASTRAN.

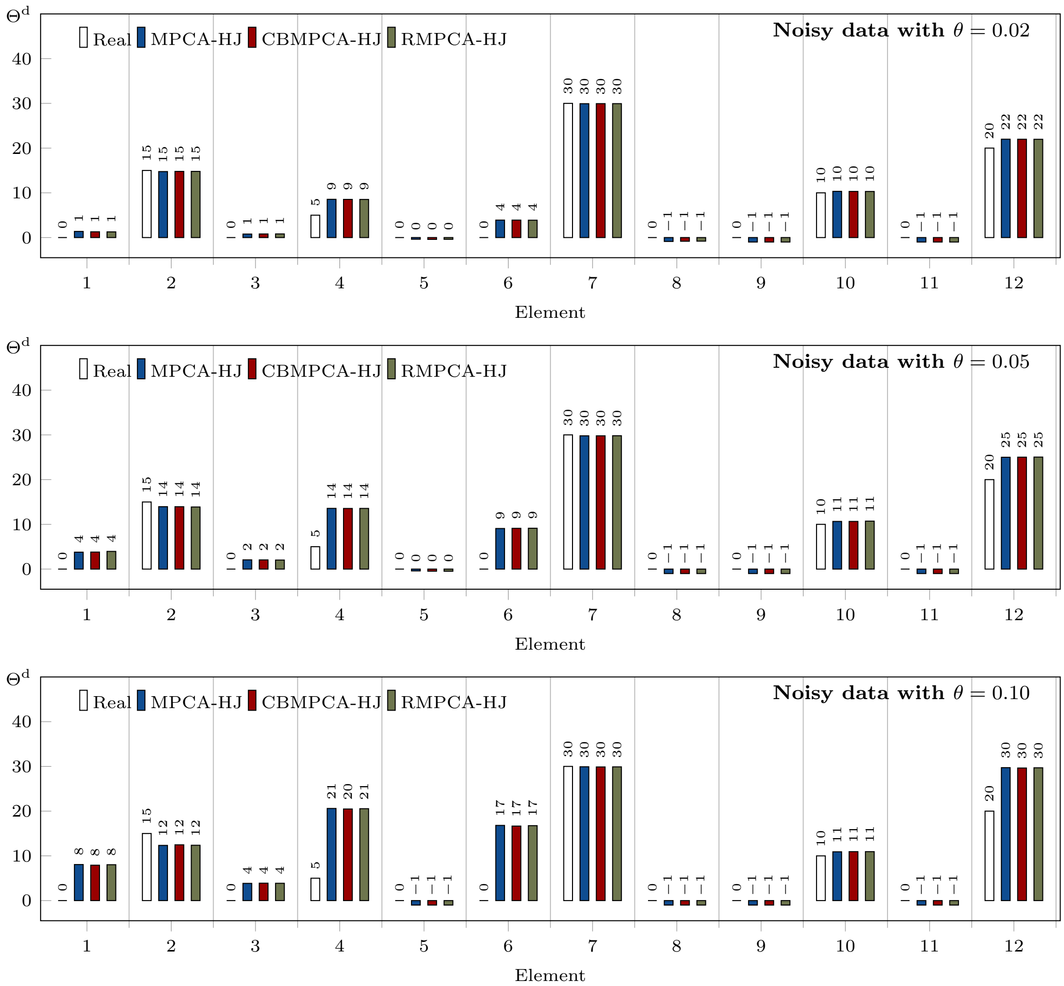


Figure 6: Damage identification with noisy data

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