AN H-P ADAPTIVE FINITE ELEMENT METHOD FOR THE NUMERICAL SIMULATION OF COMPRESSIBLE FLOW

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In this paper, we present a combined h-p-adaptive strategy for accurately modeling the Navier Stokes equations of compressible fluid flow in two dimensions. The strategy used in the adaptive h-p procedure is similar to the one used for adaptive h-refinement procedures. First, the mesh is refined in an h-fashion up to a maximum level. After an optimal h-mesh has been obtained, the principle of equidistribution of error is used to locally increase the degree of polynomial. In the present study, an upwinded-diffusion Petrov Galerkin scheme is used to produce a relaxed (or regularized) model of the compressible Navier Stokes equations for two dimensional domains.

We shall be concerned with the numerical analysis of high speed, viscous, compressible flow in arbitrary two-dimensional domains. An acceptable mathematical model for such flow phenomena is embodied in the Navier Stokes equations, which can be written in the compact conservation form as:

$$\frac{\partial U}{\partial t} + \text{div } F = 0 \quad \forall (x,t) \in \Omega \times (0,t_F) \tag{1}$$

with appropriate initial conditions and where the boundary are an appropriate combination of Dirichlet conditions (supersonic inflow), Neumann conditions (no-flow condition) and nonlinear constraints (no-slip condition). Ω = the flow domain, $(0,t_F)$ = a time interval of interest and U = vector of conservation variables = $[\rho, \rho u, \rho v, \rho e]^T$.

The above equations will be approximated by a version of the SUPG algorithm developed by the authors. The implementation of SUPG follows closely earlier work by Mallet [4]. Concisely, the SUPG approximation can be reduced to a Galerkin approximation of the following set of relaxed equations:

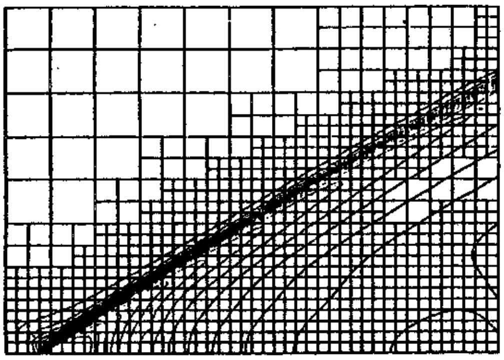
$$\text{div } F - \frac{CFL_{\Delta x}}{2}\left(\Delta x_e A \frac{1}{\sqrt{A^2+B^2}} \text{ div } F\right)_x - \frac{CFL_{\Delta x}}{2}\left(\Delta x_e B \frac{1}{\sqrt{A^2+B^2}} \text{ div } F\right)_y = 0 \tag{2}$$

+ appropriate boundary conditions

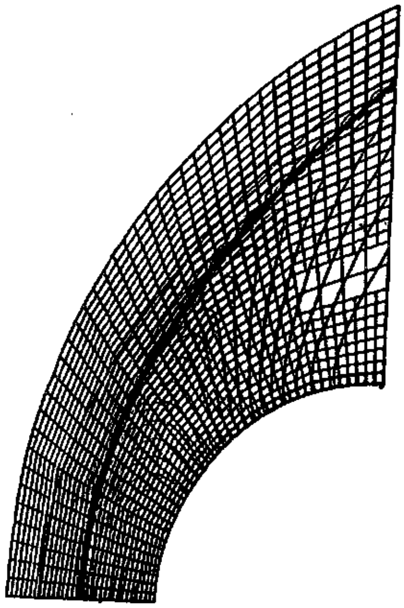
where A and B are the tangent matrices to x and y components of the flux tensor F, $CFL_{\Delta x}$ is an upwind parameter (spatial CFL-number) and Δx_e, is a measure of the element size. $\frac{1}{\sqrt{A^2+B^2}}$ is the matrix whose inverse square is $A^2 + B^2$.

The numerical viscosity terms in 2 (those involving the parameter $CFL_{\Delta x}$) provide an indication of the local approximation error. These terms, which are similar to a truncation error in the Petrov Galerkin approximation, provide a conveniently calculated local error indicator and are used as a basis for mesh refinement in this study. The underlying idea is, that as the mesh size is decreased, the magnitude of this viscous term will also decrease. Roughly, this gives an estimate of second derivatives at the nodes.

Two graphs, representing pressure contours in the domain are shown to demonstrate the resolution that can be obtained and the type of problems that can be resolved by the method described above. The first graph shows pressure contours of a supersonic flow impinging on a flat plate. The inflow mach number is 3, while on the plate a no-slip condition is imposed. The second graph shows pressure contours of a supersonic flow around a spherical object. The inflow mach number is 6. A curved bow shock can be clearly distinguished.



Numerical Simulation of Carter's Flat Plate Problem



Pressure Contours, Blunt Body, Quadratic Elements

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