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A brief introduction to a two-layer morphological associative memory based on fuzzy operations

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1. Introduction

Morphological neural networks (MNNs) are a class of artificial neural networks that perform one of the elementary operations of mathematical morphology at every node. Morphological associative memories (MAMs) are among the types of MNNs that have emerged in recent years. Unlike many other models of neural associative memory, MAMs have been proposed from the outset for the storage and recall of real-valued patterns. Nevertheless, our focus in this paper is on the binary case.

In this paper, we present a new two-layer MAM model. The first layer executes fuzzy operations that incorporate information on the kernel vectors corresponding to the fundamental memories. The second layer uses this information on the kernel vectors in order to recall the desired output pattern. This approach can be applied to the auto-associative as well as to the hetero-associative case. Our new MAM model outperformed several well-known neural associative memory models in experiments concerning the error correction capability.

2. Matrix operations in Minimax Algebra

The theories of *minimax algebra* and *mathematical morphology* are closely related although they were developed for completely different purposes.

One of the basic algebraic structures occurring in minimax algebra is called *blog* (bounded lattice ordered group). The set $\mathbb{R}_{\pm\infty} = \mathbb{R} \cup \{+\infty, -\infty\}$ together with the operations "maximum" ($\vee$), "minimum" ($\wedge$), "addition" ($+$), and "dual addition" ($+'$), provides the canonical example of a blog. For the purposes of this paper, it often suffices to consider $\mathbb{R}$, the set of finite elements of $\mathbb{R}_{\pm\infty}$. Two types of matrix products exist in minimax algebra [1]. For $A \in \mathbb{R}^{m\times p}$ and $B \in \mathbb{R}^{p\times n}_{\pm\infty}$, the matrix $C = A \boxdot\!\!\!\vee\, B$, also called the *max product* of A and B, and the matrix $D = A \boxdot\!\!\!\wedge\, B$, also called the *min product* of A and B, are defined by

$$c_{ij} = \bigvee_{k=1}^{p} (a_{ik} + b_{kj}) \,, \quad d_{ij} = \bigwedge_{k=1}^{p} (a_{ik} + b_{kj}) \,. \qquad (1)$$

Let $A \in \mathbb{R}^{m\times n}$. Let ε_A and δ_A be such that $\varepsilon_A(\mathbf{x}) = A \boxdot\!\!\!\wedge\, \mathbf{x}$ and $\delta_A(\mathbf{x}) = A \boxdot\!\!\!\vee\, \mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n_{\pm\infty}$. Obviously, ε_A represents an *erosion* and δ_A represents a *dilation* from the *complete lattice* $\mathbb{R}^n_{\pm\infty}$ into the *complete lattice* $\mathbb{R}^m_{\pm\infty}$.

3. Basic concepts of morphological associative memorie

Suppose that we wish to record k vector pairs $(\mathbf{x}^1, \mathbf{y}^1), \ldots, (\mathbf{x}^k, \mathbf{y}^k)$ using a *morphological associative memory* (MAM). Let $X = [\mathbf{x}^1, \ldots, \mathbf{x}^k]$ denote the matrix whose columns are the input patterns $\mathbf{x}^1, \ldots, \mathbf{x}^k$. Similarly, let $Y = [\mathbf{y}^1, \ldots, \mathbf{y}^k]$. We introduced two basic morphological memory models. The first approach consists of constructing an $m \times n$ matrix W_{XY} as follows:

$$W_{XY} = Y \boxdot\!\!\!\wedge\, (-X)^t = \bigwedge_{\xi=1}^{k} \mathbf{y}^\xi \boxdot\!\!\!\wedge -(\mathbf{x}^\xi)^t \,. \qquad (2)$$

The second, dual approach consists of constructing an $m \times n$ matrix M_{XY} of the form

$$M_{XY} = Y \boxdot\!\!\!\vee\, (-X)^t = \bigvee_{\xi=1}^{k} \mathbf{y}^\xi \boxdot\!\!\!\vee\, (-\mathbf{x}^\xi)^t \,. \qquad (3)$$

If the matrix W_{XY} receives a vector $\mathbf{x}$ as input, the product $W_{XY} \boxdot\!\!\!\vee\, \mathbf{x}$ is formed. Dually, if the matrix M_{XY} receives a vector $\mathbf{x}$ as input, the product $M_{XY} \boxdot\!\!\!\wedge\, \mathbf{x}$ is formed.

If $X = Y$, we obtain the *autoassociative morphological memories* (AMMs) W_{XX} and M_{XX}. The properties of AMMs include an optimal absolute storage capacity and one-step convergence.

Example.

We used the ten pattern vectors $\mathbf{x}^1, \ldots, \mathbf{x}^{10} \in \{0,1\}^{49}$ corresponding to the images in Figure 1 in constructing the morphological memories W_{XX} and M_{XX}. As expected each individual pattern vector $\mathbf{x}^\xi$ was perfectly recalled in a single application of either W_{XX} or M_{XX} and remains stable under renewed applications of either W_{XX} or M_{XX}.

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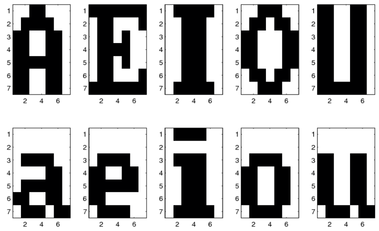

Figure 1. Ten binary patterns.

4. A new MAM based on fuzzy operations and the kernel method

The AMMs W_{XX} e M_{XX} suffer from a large number of spurious memories and limited error correction capability [5]. Recently, some modified MAM models have been proposed in order to overcome these difficulties. For example, Sussner introduced a two-layer binary MAM that yields a greatly reduced number of spurious memories and an improved error correction capability [5]. Another approach is based on either one of the following input-output schemes [6]:

$$\mathbf{x} \longrightarrow W_{XX} \mathbin{\tilde{\boxed{\vee}}} \mathbf{x} \longrightarrow \text{Defuzz.} \longrightarrow \mathbf{y}\,, \qquad (4)$$

$$\mathbf{x} \longrightarrow M_{XX} \mathbin{\tilde{\boxed{\wedge}}} \mathbf{x} \longrightarrow \text{Defuzz.} \longrightarrow \mathbf{y}\,. \qquad (5)$$

Equations 4 and 5 involve the fuzzy max product $\tilde{\boxed{\vee}}$ and the fuzzy min product $\tilde{\boxed{\wedge}}$ [6].

The new two-layer MAM model that we introduce in this paper is based on a combination of the two approaches mentioned above [5,6].

Let $\{(\mathbf{x}^\xi, \mathbf{y}^\xi) : \xi = 1, \ldots, k\}$ be the set of fundamental memories, where $\mathbf{x}^\xi \in \{0,1\}^n$ and $\mathbf{y}^\xi \in \{0,1\}^m$. Let Z be a matrix of the form $[\mathbf{z}^1, \mathbf{z}^2, \ldots, \mathbf{z}^k] \in \{0,1\}^{p\times k}$ such that the columns $\mathbf{z}^\xi$ satisfy the conditions $\mathbf{z}^\xi \not\leq \mathbf{z}^\gamma$ and $\mathbf{z}^\xi \wedge \mathbf{z}^\gamma = \mathbf{0}$ for all $\gamma \neq \xi$.

The following equations determine the recording phase of the proposed two-layer MAM. Recall that the symbol M_X^{XZ} denotes $M_{XZ} \mathbin{\boxed{\wedge}} M_{XX}$ [5].

$$\mathbf{w} = h(M_X^{XZ} \mathbin{\tilde{\boxed{\wedge}}} \mathbf{x}) \qquad (6)$$

$$\mathbf{y} = W_{ZY} \mathbin{\boxed{\vee}} \mathbf{w}\,. \qquad (7)$$

Here, we employed $h_i(\mathbf{x}) = 1 \Leftrightarrow x_i \geq \bigvee_{j=1}^n x_j$.

5. Experimental results

We used an experiment from the literature to test our new model [3, 6]. Consider the ten

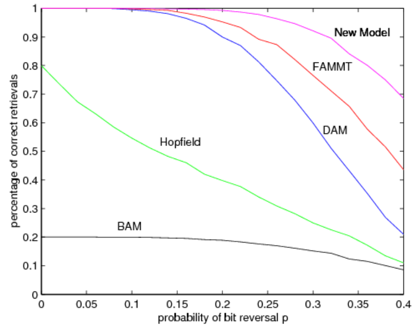

Figure 2. Percentage of Perfect Recall.

images $\mathbf{x}^1, \ldots, \mathbf{x}^{10}$ corresponding to the images of Figure 1. We stored the five associations $(\mathbf{x}^1, \mathbf{x}^1), \ldots, (\mathbf{x}^5, \mathbf{x}^5)$ using W_{XX}, a discrete Hopfield net, and a projection-recorded DAM [2]. Moreover, we stored $(\mathbf{x}^1, \mathbf{x}^6), \ldots, (\mathbf{x}^5, \mathbf{x}^{10})$ using a correlation-recorded BAM [2], the morphological model given by Equation 4 (FAMMT), and our new MAM model given by Equations 6 and 7 for the special case where $Z = I \in \{0,1\}^{k\times k}$.

We introduced random noise into each of the uppercase vowels by randomly reversing each pixel with probability p. Figure 2 shows the mean percentage of perfect recalls of $\mathbf{x}^\xi$ for each probability p in 1000 experiments for each ξ.

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