SATELLITE ATTITUDE CONTROL SYSTEM DESIGN USING MULTIVARIABLE CONTROL METHOD CONSIDERING FLEXIBILITY AND FUEL SLOSH DYNAMICS

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Abstract: *The design of the satellite Attitude Control System (ACS) becomes more complex when the satellite structure has different type of components like, flexible solar panels, antennas, mechanical manipulators and tanks with fuel, since the ACS performance and robustness will depend if the dynamics interaction effects between these components are considered in the satellite controller design. A crucial interaction can occur between the fuel slosh motion and the satellite rigid motion during translational and/or rotational manoeuvre since these interactions can change the satellite center of mass position damaging the ACS pointing accuracy. Although, a well-designed controller can suppress such disturbances quickly, the controller error pointing may be limited by the minimum time necessary to suppress such disturbances affecting thus the satellite attitude acquisition. It is known that one way to minimize such problems is to design controllers with a bandwidth below the lowest slosh and/or vibration mode which can result in slow maneuvers inconsistent with the space mission requirements. As a result, the design of the satellite controller needs to explore the limits between the conflicting requirements of performance and robustness. This paper investigates the effects of the interaction between the liquid motion (slosh) and the flexibility in the satellite dynamics in order to predict what the damage to the LQR and LQG controller performance and robustness.*

Keywords: *slosh and flexibility, LQR, LQG, attitude control system*

1 Introduction

The problem of interaction between fluid and structure is important when one needs to study the dynamic behavior of offshore and marine structures, road and railroad containers partially filled with a fluid, spinning spacecraft with liquid fuel, damping devices involving fluid as the damping material, robots carrying nuclear waste, floating bodies, ship motion and oil tankers. As for space missions the interaction between fluid and structure is important for a rigid or flexible structure interacting with a fluid under the sloshing effects. An interesting approach to analyze a rigid container mounted on flexible springs interacting with a perfect fluid including sloshing effects has been done by Lui and Lou (1990). Space structures, like rockets, geosynchronous satellites and the space station usually contains liquid in tanks that can represent more than 40% of the initial mass of the system, creating the need for more detailed dynamics studies and for the ACS design. When the fuel tanks are only partially filled and suffer a transversal acceleration, large quantities of fuel moves uncontrollably inside the tanks and generate the sloshing effects. The dynamics of the motion of the fuel interacts with the rigid body dynamics producing attitude instability. For minimizing these effects control methods are employed in the system so as to have an adequate solution, to assure stability, and to achieve good attitude control which softens the slosh effects (Sidi, 1997).

The dynamics of rigid-flexible satellite with fuel tanks when subject to large angle maneuver is only captured by complex non-linear mathematical model. Besides, the remaining flexible and/or liquid vibration can introduce a tracking error resulting in a minimum attitude acquisition time. Souza (1996) has done a detailed investigation of the influence of the non-linearities introduced by the panel's flexibility into the ACS design. It was shown that system parameters variation can degrade the control system performance, indicating the necessity to improve the ACS robustness. An experimental controller robustness and performance investigation was done by Conti and Souza (2008), where the estimation of the platform inertia parameters was introduced as part of the platform ACS design. The problem of designing satellite nonlinear controller for rigid satellite has been done by Gonzales and Souza (2011) using the State Dependent Riccati Equation (SDRE) method which is able to deal with high nonlinear plants. Due to the complexity of modeling the fluid and/or flexible dynamic of the system it is common to use mechanical systems analogies that describe this dynamic.

2 Satellite with slosh and flexible model

The phenomenon of sloshing is given by the movement of a free surface of a liquid that partially fills a compartment. The movement caused by the liquid is an oscillating movement, which depends on shape of the tank, the acceleration of gravity, or axial acceleration of the tank. As representative of the behavior of the total weight of the system it is accepted that when the mass of the liquid oscillates the mass center of the rigid body also oscillates, thereby disturbing the rigid-flexible part of the vehicle under consideration.

The sloshing dynamics is usually represented by mechanical equivalents that describes a similar and reproduce faithfully the actions and reactions due to forces and torques acting on the system. The main advantage of replacing the fluid model with an equivalent oscillating model (Ibrahim, 2005) is simplifying the analysis of motion in the rigid body dynamics, compared to the fluid dynamics equations.

Due to the complexity of establishing an analytical model for the fluid moving freely within a closed tank, it is used a simplified system, taking into account the following criteria (Sidi, 1997): a) Small displacements, b) A rigid tank and c) No viscous, incompressible and homogeneous liquid. Under these conditions the dynamics of the sloshing can be approximated by mechanical system consisting of a mass-spring or pendulum.

The flexible dynamics can be modeled by a single discrete mass, positioned at a distance ℓ from the center of mass of the rigid body (Sidi, 1997). A torque is applied about the axis of rotation, thus exciting only the anti-symmetric elastic mode. Since the panel is not rigid, a deformation δ with respect to the rigid body axis is to be anticipate. The mass m_p will experience two motions (Sidi, 1997):

(1) a motion θ with the rigid body, with linear velocity $\dot{\theta}$

(2) a deformation δ from the rigid-body axis, with velocity $\dot{\delta}$

For the study model, consider a rigid spacecraft moving in a fixed plane, with a spherical fuel tank and including the lowest frequency slosh mode and a flexible panel. Based on the Lagrange equation and the Rayleigh dissipation function one can model systems using the mechanical mass-spring and pendulum type system, respectively.

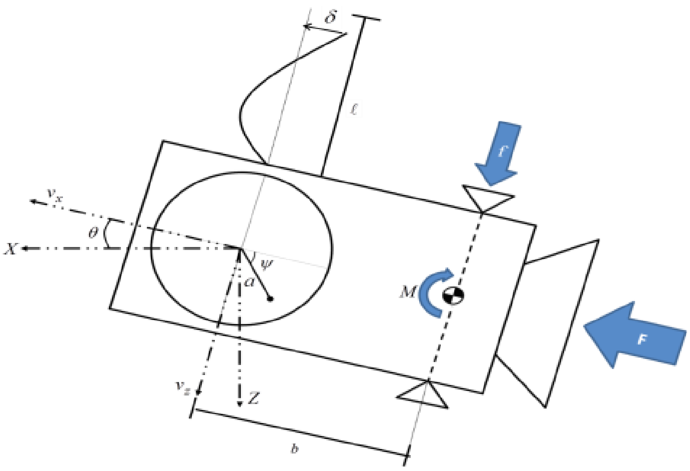

Figure 1. Satellite model with flexibility and fuel slosh

Figure1 shows a satellite model where slosh dynamics is represented by its pendulum analogous mechanical system.

Where the mass of the satellite and the moment of inertia, regardless of the fuel, are given by m and I respectively, the mass equivalent of fuel and its inertia moment is given by m_f and I_f respectively and the mass of the panel is m_p, the inertia moment I_p and the panel length ℓ. It is assumed a transverse force f a pitching moment M and the force F is assumed to act on the spacecraft longitudinal axis.

Also it is given the velocity of the center of the fuel tank υ_x, υ_v and the attitude angle θ of the spacecraft with respect to a fixed reference $(X, \mathrm{Y}, \mathrm{Z})$. Besides, one assumes as generalized coordinates: representing the linear velocity, ω representing the angular velocity of the rigid body, a is the length of the pendulum rod, b is the distance from satellite center of mass to the pendulum connected point, ψ is the angle of the pendulum with respect to the spacecraft longitudinal axis, which is assumed in the equilibrium position $\psi = 0$ about the reference axis, δ is the elastic displacement of the panel. The parameters m_f, I_f and a depend on the shape of the tank, chemical-physical characteristics of the fuel and the fill ratio of the fuel tank.

3 Satellite equations of motion

The satellite equations of motion can be derived using the Quasi-Lagrange equations (Souza, 2008) given by

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial V}\right)+\omega^{\times}\frac{\partial L}{\partial V}=\tau_t \\
&\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \omega}\right)+\omega^{\times}\frac{\partial L}{\partial \omega}+V^{\times}\frac{\partial L}{\partial V}=\tau_r \\
&\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{\psi}}\right)-\frac{\partial L}{\partial \psi}+\frac{\partial R}{\partial \dot{\psi}}=0 \\
&\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\delta}}\right)-\frac{\partial L}{\partial \delta}+\frac{\partial D}{\partial \dot{\delta}}=0
\end{aligned} \tag{1}$$

where L is the Lagrangian of the system, R, D are the Rayleigh dissipation function, τ_r is the internal torque and τ_t is the external torque. Assuming that $R, D, \omega, V, \tau_r, \tau_t$ are given by

$$R=\frac{1}{2}\varepsilon\dot{\psi}^2;\ D=\frac{1}{2}k_d\dot{\delta}^2;\ V=\begin{bmatrix} v_x \\ 0 \\ v_z \end{bmatrix};\ \omega=\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix};\ \tau_t=\begin{bmatrix} F \\ 0 \\ f \end{bmatrix};\ \tau_r=\begin{bmatrix} 0 \\ M+fb \\ 0 \end{bmatrix} \tag{2}$$

The position vector of the satellite mass center with respect to the inertial coordination system (x,y,z) is

$$\vec{r}=(x-b)i+z\hat{k} \tag{3}$$

The position of the mass of fuel is given by

$$\vec{r}_f=(x-a\cos(\psi))\hat{i}+(z+a\,\mathrm{sen}(\psi))\hat{k} \tag{4}$$

Considering little deformations the equation of the velocity of the panel can write as

$$v_p=\dot{\delta}+\ell\dot{\theta} \tag{5}$$

And the Lagrangian of this model is given by

$$L=\frac{1}{2}m\dot{\vec{r}}^2+\frac{1}{2}m_f\dot{\vec{r}}_f^2+\frac{1}{2}m_p{v_p}^2+\frac{1}{2}I_f(\dot{\theta}+\dot{\psi})^2+\frac{1}{2}I\dot{\theta}^2-\frac{1}{2}\delta^2k \tag{6}$$

Substituting the Equations (3), (4) and (5) into Eq. (6), using the relations given by Eq. (2) and performing the derivations of Eq. (1), one obtains the satellite equations of motion given by

$$\left(m+m_f\right)a_x+mb\dot{\theta}^2+m_fa\left(\ddot{\psi}+\ddot{\theta}\right)sen\left(\psi\right)+m_fa\left(\dot{\theta}+\dot{\psi}\right)^2\cos\left(\psi\right)=F \tag{7}$$

$$\left(m+m_f\right)a_z+m_fa\left(\ddot{\psi}+\ddot{\theta}\right)\cos\left(\psi\right)-m_fa\left(\dot{\theta}+\dot{\psi}\right)^2sen\left(\psi\right)+mb\ddot{\theta}=f \tag{8}$$

$$\left(mb^2 + I_f + m_p\ell^2\right)\ddot{\theta} + \ddot{\delta}m_p\ell + mba_z - \varepsilon\dot{\psi} = M + fb \tag{9}$$

$$\left(m_f a^2 + I_f\right)\left(\ddot{\psi} + \ddot{\theta}\right) + m_f a\left(sen\left(\psi\right)a_x + \cos\left(\psi\right)a_z\right) + \varepsilon\dot{\psi} = 0 \tag{10}$$

$$\ddot{\delta}m_p + \ddot{\theta}m_p\ell + \dot{\delta}k_d + \delta k = 0 \tag{11}$$

All equations derived previously are no linear. However, in order to design a LQR and LQG controllers and estimate the sloshing parameters using the Kaman filter technique one has to get the linear set of equations of motion, which is obtained assuming that the system makes small movements around the point of equilibrium very close to zero (Reyanoglu et al, 2011). Now, substituting the Equations (9) and (10) into Equations (7) and (8) and assuming the linearization conditions, one has the satellite equation of motion given by

$$\begin{aligned}
&\ddot{\theta}\left(I + m_p\ell^2 + m^*(b^2 - ba)\cos\left(\psi\right)\right) + m_p\ell\ddot{\delta} - m^*ab\ddot{\psi}\cos\left(\psi\right) + m^*ab\left(\dot{\theta} + \dot{\psi}\right)^2 sen\left(\psi\right) - \varepsilon\dot{\psi} = M + b^* f \\
&\ddot{\theta}\left(I_f + m^*\left(a^2 - bacos\left(\psi\right)\right)\right) + \ddot{\psi}\left(I_f + m^*a^2\right) + \left(a^*F - m^*ab\dot{\theta}^2\right)sen\left(\psi\right) + \varepsilon\dot{\psi} = -a^* f\cos\left(\psi\right) \\
&\ddot{\delta}m_p + \ddot{\theta}m_p\ell + \dot{\delta}k_d + \delta k = 0
\end{aligned} \tag{12}$$

where $m^* = \dfrac{m_f m}{m + m_f}$, $a^* = \dfrac{m_f a}{m + m_f}$, $b^* = \dfrac{m_f b}{m + m_f}$

These equations describe the angular displacement of the spacecraft, the angular displacement of the pendulum and the elastic displacement of the panel.

4 Linear quadratic Regulator (LQR)

Assume a plant described by the linear state equations:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{13}$$

where $\mathbf{x}$ represent the state vector, the matrix $\mathbf{A}$ is called state matrix, $\mathbf{B}$ the input matrix and $\mathbf{u}$ is the control law, given by (Ogata, 1997):

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t) \tag{14}$$

where the variable $\mathbf{K}$ is a gain matrix.

The LQR is an optimal control than consists of minimizing the function cost Eq. (3) and solving the Riccati equation Eq. (4) (Kirk, 1998).

$$J = \frac{1}{2}\mathbf{x}^t(t_f)\mathbf{H}\mathbf{x}(t_f) + \frac{1}{2}\int_{t_0}^{t_f}[\mathbf{x}^t(t)\mathbf{Q}(t)\mathbf{x}(t) + \mathbf{u}^t(t)\mathbf{R}(t)\mathbf{u}(t)]dt \tag{15}$$

The final time t_f is fixed, $\mathbf{Q}$ and $\mathbf{H}$ are real positive semi-definite matrices, and $\mathbf{R}$ is real symmetric positive definite matrix. The Riccati equation is given by:

$$\dot{\mathbf{P}}(t) = -\mathbf{P}(t)\mathbf{A} - \mathbf{A}^t\mathbf{P}(t) - \mathbf{Q} + \mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^t\mathbf{P}(t) \tag{16}$$

where $\mathbf{P}$ is the symmetrical solution matrix of the well-known differential Riccati equation. The optimal control signal can also be written as

$$\mathbf{u}(t) = -\mathbf{R}^{-1}\mathbf{B}^t\mathbf{P}(t)\mathbf{x}(t) \tag{17}$$

Substituting the Eq. (5) in Eq. (2),

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^{t}\mathbf{P}(t) \tag{18}$$

so, it is possible to determine the optimal gain $\mathbf{K}$.

5 Linear quadratic Gaussian (LQG)

This method is the union with the LQR and the Kalman filter. We kwon that the LQR admits all variable available to feedback but in reality that is not true, because is impossible measure all variable of the system then we need estimate this variables for the feedback. The separation principle ensures that each step of this process can made independently of each other, or may first solve the problem of LQR and then design the optimal estimator (Kalman filter), or vice versa, so that the global solution is always the same.

Assume a plant described by the linear state equations:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \Gamma w \\ \mathbf{y} &= \mathbf{C}\mathbf{x}(t) + \upsilon\end{aligned} \tag{19}$$

where $\mathbf{x}$ represent the state vector, the matrix $\mathbf{A}$ is called state matrix, $\mathbf{B}$ the input matrix, $\mathbf{y}$ is the output vector, $\mathbf{C}$ is the output matrix, υ and w are white noise and $\mathbf{u}$ is the control law (Ogata, 1997).

a) For the controller:

$$\mathbf{K}_c = \mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P}_c \tag{20}$$

where $\mathbf{R}$ is real symmetric positive definite matrix and $\mathbf{P}_c$ is the symmetrical solution matrix of the well-known differential Riccati equation.

$$\mathbf{A}^{T}\mathbf{P}_c + \mathbf{P}_c\mathbf{A} - \mathbf{P}_c\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P}_c + \mathbf{M}^{T}\mathbf{Q}\mathbf{M} = \mathbf{0} \tag{21}$$

b) For the filter:

$$\mathbf{K}_f = \mathbf{P}_f\mathbf{C}^{T}\mathbf{V}^{-1} \tag{22}$$

where $\mathbf{V}$ is real symmetric positive definite matrix and $\mathbf{P}_f$ is the symmetrical solution matrix of the well-known differential Riccati equation.

$$\mathbf{P}_f\mathbf{A}^{T} + \mathbf{A}\mathbf{P}_f - \mathbf{P}_f\mathbf{C}^{T}\mathbf{V}^{-1}\mathbf{C}\mathbf{P}_f + \mathbf{\Gamma}^{T}\mathbf{W}\mathbf{\Gamma} = 0 \tag{23}$$

Where $\mathbf{P}_c = \mathbf{P}_c^{T} \geq 0$ and $\mathbf{P}_f = \mathbf{P}_f^{T} \geq 0$ are weights matrices $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{V}$ e $\mathbf{W}$ can be regarded as setting parameters ("tuning") that must be manipulated until they find one acceptable response to the system. A necessary and sufficient condition to guarantee the existence of the $\mathbf{K}_c$ and $\mathbf{K}_f$ is guarantee if the system is completely controllable and observable.

We can regard this method as realistic as compared with the LQR, since this estimated with a Kalman filter states are not available for feedback, or non-measurable states with the aid of sensors, and accepts the insertion of noise representing modeling imperfections of the system under study.

6 Simulations and results

The simulations is the design of the control law using the LQR and LQG control theory, for the spacecraft with a partially filled tank, to account for the sloshing dynamics by the mechanically analog pendulum type and the flexible panel. The physical parameters used in the simulations are (Reyanoglu et al, 2011):
$m = 600kg$, $m_f = 100kg$, $I = 720kg/m^2$, $I_f = 90kg/m^2$, $a = 0.3m$, $b = 0.3m$, $F = 500N$, $\varepsilon = 0.19kgm^2/s$,
$\ell = 1{,}5\ m$, $m_p = 10\ kg$, $k = 320\ kg\ rad^2/s^2$ e $k_d = 0{,}48kg\ rad^2/s$.

The initial conditions used are:

$\theta = 2rad, \dot{\theta} = 0,57\, rad/s, \psi = 15rad, \dot{\psi} = 0\, rad/s, \delta = 0rad$ and $\dot{\delta} = 0rad$.

Figure 2 and 3 show the LQR and LQG control law performance.

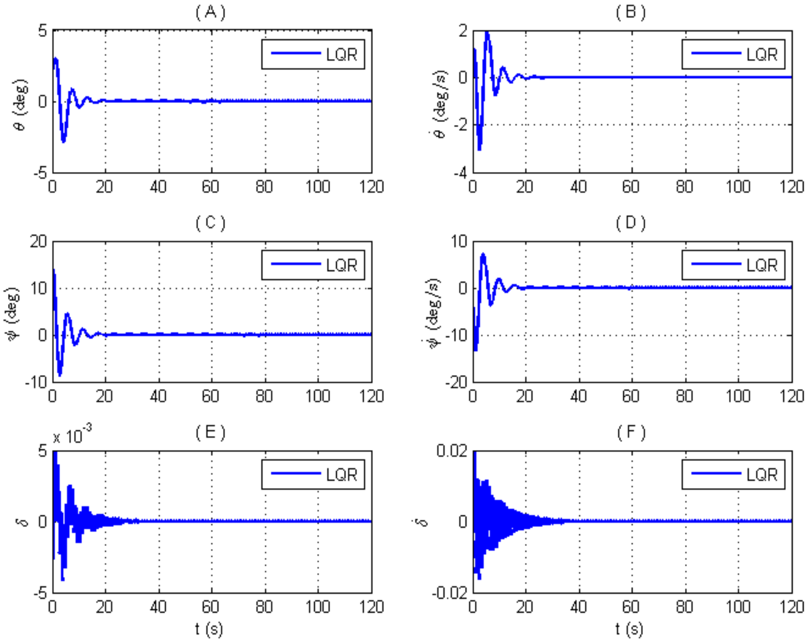

Figure 2. Performance of the LQR control law.

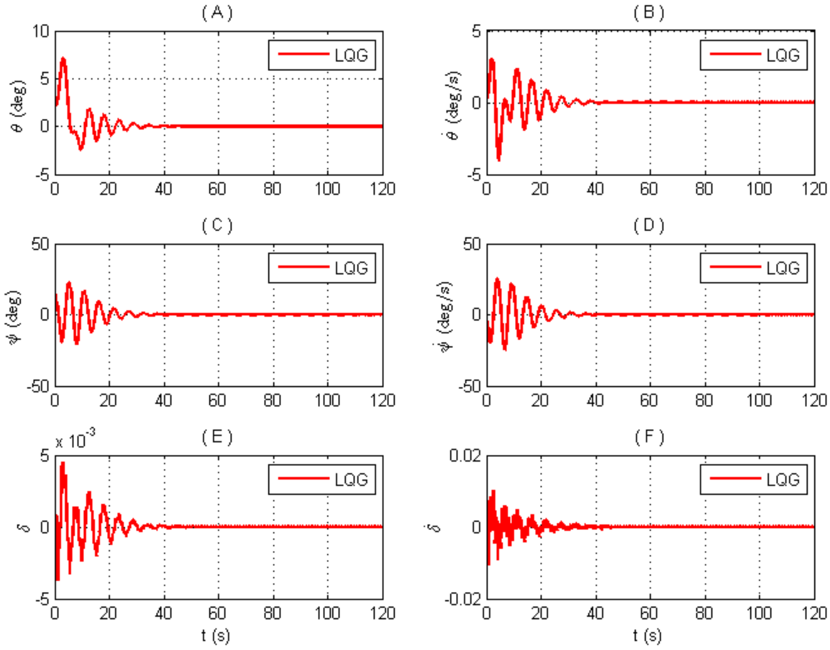

Figure 3. Performance of the LQG control law.

The LQG method is more realistic as compared with the LQR, since this estimated with a Kalman filter states are not available for feedback, or non-measurable states with the aid of sensors, and accepts the insertion of noise representing modeling imperfections of the system under study.

The performance of the LQG control law is damage because the sloshing and the flexible motion is controlled indirectly and the states variables of the slosh and flexible motion are not available to be feedback and they need to be estimated by the Kalman filter, or in the case of the flexible motion, include to the panel a piezoelectric sensor.

7 Conclusions

In this paper one described briefly the concepts of the sloshing phenomenon which is associated with the dynamics of a liquid moving into at partially fills reservoir. To derive the equation of motion of a spacecraft with slosh and a flexiblity it was used the Lagrangian approaches. The sloshing phenomenon is represented by its mechanical analog of a pendulum type. The control laws used LQR and LQG methods were able to control the system under study. As discussed in Chapter 5 the LQG method is more realistic than the LQR since no feeds the system with values that are not measured by sensors, since they are estimated by the filter, and we still have the possibility to insert noises representing the imperfections of the models, so to compare the two laws applied to these systems we see that LQG is more slower than the system controlled purely with LQR. The reason the LQG control degraded is because the sloshing states need to be estimated by the filter, besides that there is noises representing the imperfections of the models acting over the system.

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