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DETECTING PHASE SYNCHRONIZATION IN COUPLED CHAOTIC NONCOHERENT OSCILLATORS BY USING COMPLEX WAVELET TRANSFORM

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Abstract. *Phase synchronization in nonidentical coupled chaotic systems appears for some conditions in which weak coupling causes the systems evolution on time to lock in phase to one another, while their amplitudes may remain chaotic and are, in general, uncorrelated. To identify this phenomenon, given a signal it is necessary to measure properly its phase. If a system has a dominant peak in the power spectrum, there are several methods to define the phase. However, if the signal has a broad-band spectrum, which is typical for non-coherent signal, then the measurement of the phase may be a challenge. Phase is defined as an increasing function of time. The standard method for measuring phase does no complain with this requirement. In this work we present an innovative method for measuring phase that complain with the increasing function of time requirement. This method is based on Dual Tree Complex Wavelet Transform, which is a form of discrete wavelet transform that generates complex coefficient by using a dual tree wavelet filters to obtain their real and imaginary parts. The proposed approach is robust and computationally efficient. Furthermore, this approach shows flexibility and in principle is applicable to any experimental time series.*

1 INTRODUCTION

The phenomenon of synchronization has been observed in various natural systems, such as in heart cells, applause, flashing of the Southeast Asian fireflies, chirping of crickets, and more [1, 2, 3]. Basically, synchronization is understood as the mutual adjustment of the oscillations of periodic oscillators due to some sort of interaction between them [4].

Over the last decade, considerable progress has been made toward generalizing the concept of synchronization, allowing it to encompass chaotic oscillators [4, 5, 6, 7, 8, 9]. In this work our interest is phase synchronization, which occurs mainly for weak coupling with the amplitudes of the oscillators remaining uncorrelated while their phases evolve in step with each other over time [8, 9].

Detecting phase synchronization of chaotic systems requires a clear and unambiguous definition of phase for the oscillators in order to test the condition $\Delta\phi(t) = \mid \phi_1(t) - \phi_2(t) \mid < const < 2\pi$, where $\phi_1(t)$ and $\phi_2(t)$ are the phases of systems 1 and 2, respectively. As far as obtaining the phase of the oscillators is concerned, one can use direct measurements of the phase angle on a projection of the attractor, to know: Hilbert transform, Poincaré surface of section [7], curvature and recurrence plots [10, 11], localized sets [12], short-time Fourier transforms and the continuous complex wavelet transform methodologies [13, 14, 15, 16]. In particular, the methodologies that use continuous complex wavelet transform are based on the complex Morlet wavelet to obtain the phase of a chaotic series yielding good results for coherent systems [14, 16]. In addition, it has a high computational cost and may produce some dificulties to interprete the results, due to its redundancy framework, when applied to large time series.

To overcame these difficulties, we propose an approach to extract the phase of chaotic systems based on a efficient wavelet transform, the Dual-Tree Complex Wavelet Transform (DT-CWT).

2 METHODOLOGY

The Wavelet Transform (WT) is a linear transform that can be used in the analysis of non-stationary signals in order to extract information of the variations in this frequency signals and to detect their structure temporally and/or spatially localized [17]. Therefore, the WT provides a time-frequency representation of the signal using the multiresolution technique by which different frequencies are analyzed with different resolutions [18].

In theory there are two forms of WT, a continuous form and a discrete form. When the parameters of scale and translation are continuous we have the called Continuous Wavelet Transform (CWT), which transforms a one-dimensional signal (time) in a two-dimensional representation (time, scale) that is highly redundant.

The CWT has been used in previous work in order to calculate the phase of chaotic systems and evaluate phase synchronization in coupled Rössler systems [13, 14, 15, 16].

In this paper we will use the dual-tree approach, described in [19], which is relatively recent enhancement to the Discrete Wavelet Transfom (DWT), with important additional properties: it is nearly shift invariant and directionally selective in two and higher dimensions.

2.1 Dual-Tree Complex Wavelet Transform

Mathematically, any finite energy signal $x(t)$ can be decomposed in terms of basis functions, this study multiscale are the scale functions $\phi_{j,n}(t)$ and their respective wavelet functions associated $\psi_{j,n}(t)$ via the equation

$$x(t) = \sum_{n \in Z} c(j,n)\ \phi_{j,n}(t) + \sum_{j,n \in Z} d(j,n)\ \psi_{j,n}(t), \tag{1}$$

wherein $c(j,n)$ are scale coefficients and $d(j,n)$ are wavelet coefficients

$$c(j,n) = \langle x, \phi_{j,n} \rangle = \int x(t)\ \phi_{j,n}(t)\ dt \tag{2}$$

$$d(j,n) = \langle x, \psi_{j,n} \rangle = \int x(t)\ \psi_{j,n}(t)\ dt. \tag{3}$$

The coefficients $c(j,n)$ and $d(j,n)$ are calculated by using a very efficient, linear time complexity algorithm based in convolutions in the analyzed signal $x(n)$ with a discrete-time low-pass filter $h_0(n)$, a high-pass filter $h_1(n)$ and downsampling operations. Two channels there are in implementations by filter bank, one being associated with the filter $h_0(n)$ and their scaling functions $c(j,n)$ and other associated with filter $h_1(n)$ and their wavelet coefficient $d(j,n)$. This is called the Mallat algorithm or Mallat-tree decomposition [18].

Despite its applicability and its efficient computational algorithm, the DWT has four fundamental deficiencies and interlaced, which are: oscillations, shift variance, aliasing and lack of directionality. In [19] a possible solution to resolve these deficiencies is presented by making use of complex wavelets $\psi_c(t) = \psi_h(t) + \imath\ \psi_g(t)$, wherein $\psi_h(t)$ denotes the real part and $\psi_g(t)$ the imaginary part.

The DT-CWT employs two real DWT's, the first DWT (upper filter bank or upper tree) consists of low-pass filters $h_0(n)$ and high-pass filters $h_1(n)$ and the second DWT (lower filter bank or lower tree) is composed by low-pass filters $g_0(n)$ and high-pass filters $g_1(n)$. Each DWT is composed by two different filter sets, which satisfying the conditions of perfect reconstruction and constructed jointly so that the overall transformed is approximately analytical.

Considering a real signal $x(n)$, is obtained as output of the first DWT the real part $\psi_h(t)$ and $d_h(j,n)$ of the complex wavelets and of the complex wavelets coefficients, respectively. In the second DWT is obtained as output the imaginary part $\psi_g(t)$ and $d_g(j,n)$ of the complex wavelets and of the complex wavelets coefficients, respectively (see Figure 1(a)). Thus, we obtain the complex wavelets $\psi_c(t) = \psi_h(t) + \imath\ \psi_g(t)$ and the complex wavelets coefficients $d_c(j,n) = d_h(j,n) + \imath\ d_g(j,n)$.

The Figure (1) shows a schematic representation of the decomposition of the real signal $x(n)$ using DT-CWT. In Figure (1)(a) the signal is decomposed in one scale ($j = 1$) and (b) in three scales ($j = 1, 2, 3$).

In order to satisfy the conditions of perfect reconstruction, the filters are designed such that the complex wavelets $\psi_c(t) := \psi_h(t) + \imath\ \psi_g(t)$ are approximately analytic. To do this, they are designed so that $\psi_g(t)$ is approximately the Hilbert pair of $\psi_h(t)$, in other words $\psi_g(t) \approx \mathcal{H}\{\psi_h(t)\}$, where $\mathcal{H}$ denotes the Hilbert transform [20, 21, 22].

2.2 Phase Computation

In order to calculate the phase of a chaotic system using the approach of DT-CWT, the series in the variable x of the system 1, i.e, $x_1(n)$, is analyzed by DT-CWT multiscale. From this analysis we obtain the complex wavelet coefficients $d_c(j,n)$ at each scale j. After obtaining the coefficients, the energy $E(j,n)$ at each scale j is calculated as the square root of the modulus of complex wavelet coefficients, i.e, $E(j,n) = \mid d_c(j,n) \mid^2$. After the calculate the energy in each scale, the maximum energy $max_j E(j,n) = E(J,n)$ is found in order to localize the scale

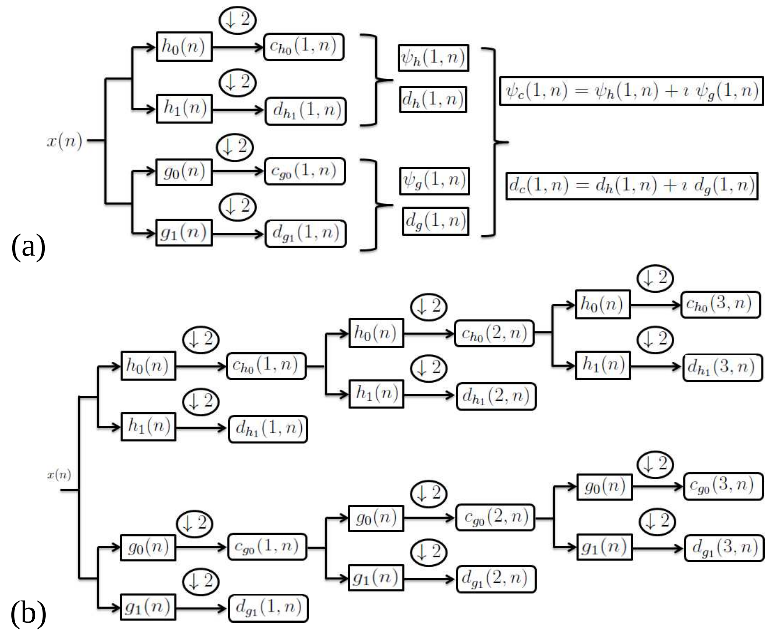

Figure 1: Schematic representation of the decomposition of the signal $x(n)$ in: (a) one scale ($j = 1$) and (b) three scales ($j = 1, 2, 3$), using the DT-CWT.

J which has the maximum energy. The scale J associate to the maximum energy are used to compute the phase $\phi(J, n) = \arctan\left(\frac{d_g(J,n)}{d_h(J,n)}\right)$ in a specific time (see Figure 2(a)). Subsequently, the same procedure are used to the series in the variable x of the system 2, i.e, $x_2(n)$.

Then we calculated the phase of each system, $\phi_1(J, n)$ and $\phi_2(J, n)$. Next, the phase difference between the systems is calculated as $\Delta\phi(J, n) = \phi_1(J, n) - \phi_2(J, n)$ and phase synchronization condition is evaluated (see Figure 2(b)).

The Figure 2(a) illustrates a schematic representation of the method for calculating the phase of a chaotic system by using the approach of DT-CWT. In Figure 2(b) illustrates a schematic representation of the application of the method in two series $x_1(n)$ and $x_2(n)$, which are the series in the variable x of the system 1 and 2, respectively.

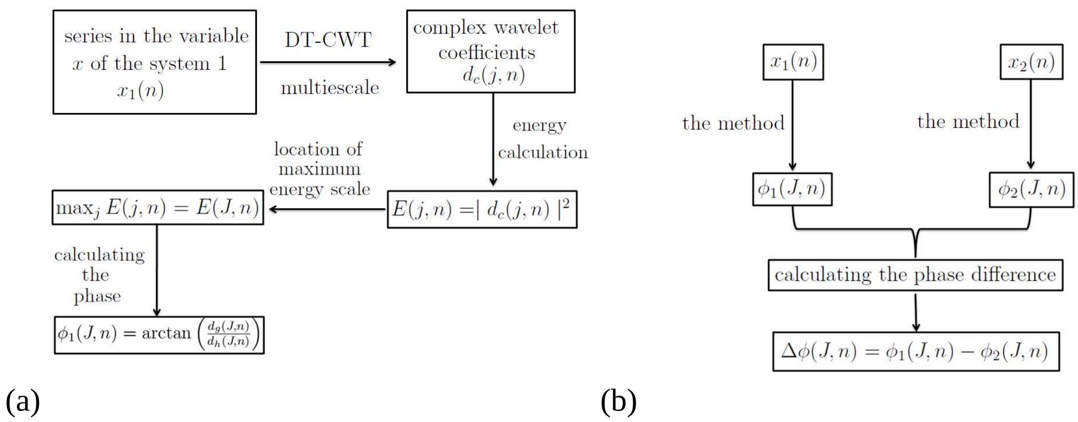

Figure 2: Schematic representation in (a) of the proposed method for calculating the phase using the x variable of a chaotic system and (b) of the application of the method in two series $x_1(n)$ and $x_2(n)$.

3 APPLICATION AND RESULTS

In our applications, the time series data used are generated by two non-identical Rössler systems [23] in non-phase-coherent regime and coupled bidirectionally by variable x. We use the Runge-Kutta 4th order method with integration time step equal to 0.01. The system is given by equations (4) and the parameters considered were based in the study in [4].

$$\begin{aligned} \dot{x}_{1,2} &= -\omega_{1,2}\, y_{1,2} - z_{1,2} + \eta\,(x_{2,1} - x_{1,2}) \\ \dot{y}_{1,2} &= \omega_{1,2}\, x_{1,2} + 0.2925\, y_{1,2} \\ \dot{z}_{1,2} &= 0.4 + z_{1,2}\,(x_{1,2} - 8.5) \end{aligned} \tag{4}$$

where $\omega_1 = 0.98$ and $\omega_2 = 1.02$.

The number of points in the time series analyzed is 2^{23}. For each system, using the x variable, the DT-CWT method was applied considering 23 scales of decomposition. Then, the maximum energy scale was localized and the phase was calculated on this scale. After the application of the method at the two systems under study, we have the phase of each system, so that, phase difference between the systems is calculated in order to check the condition of phase synchronization.

The Fig. 3 illustrates in (a) the projection of the attractor in the xy plane; scale versus the maximum energy in each scale considering a coupling strength of (b) 0.05, (c) 0.15, (d) 0.2; phase at $t = [1000, 2000]$ in the scale of maximum energy considering a coupling strength of (e) 0.05 and (f) 0.15 and (g) phase at $t = [1000, 1050]$ in the scale of maximum energy considering a coupling strength of 0.2.

The Fig 4 illustrates the phase difference using in (a) the wavelet method; (b) the traditional method for calculate the phase [8] and (c) is shown a zoom, $t = [0, 600]$ of the phase difference between systems considering $\eta = 0.15$ using the traditional method and the wavelet method proposed. For small intensity of coupling $\eta = 0.05$, the phase difference increases with time characterizing no phase synchronization. By increasing more the coupling ($\eta = 0.15$), some plateaus of phase synchronization appear and when $\eta = 0.2$ the phase synchronization occurs. Note in Fig 4(c) that the traditional method can not find the plateaus of phase synchronization rightly. It is worth mentioning that the plateaus of the phase synchronization were properly verified on their associated series.

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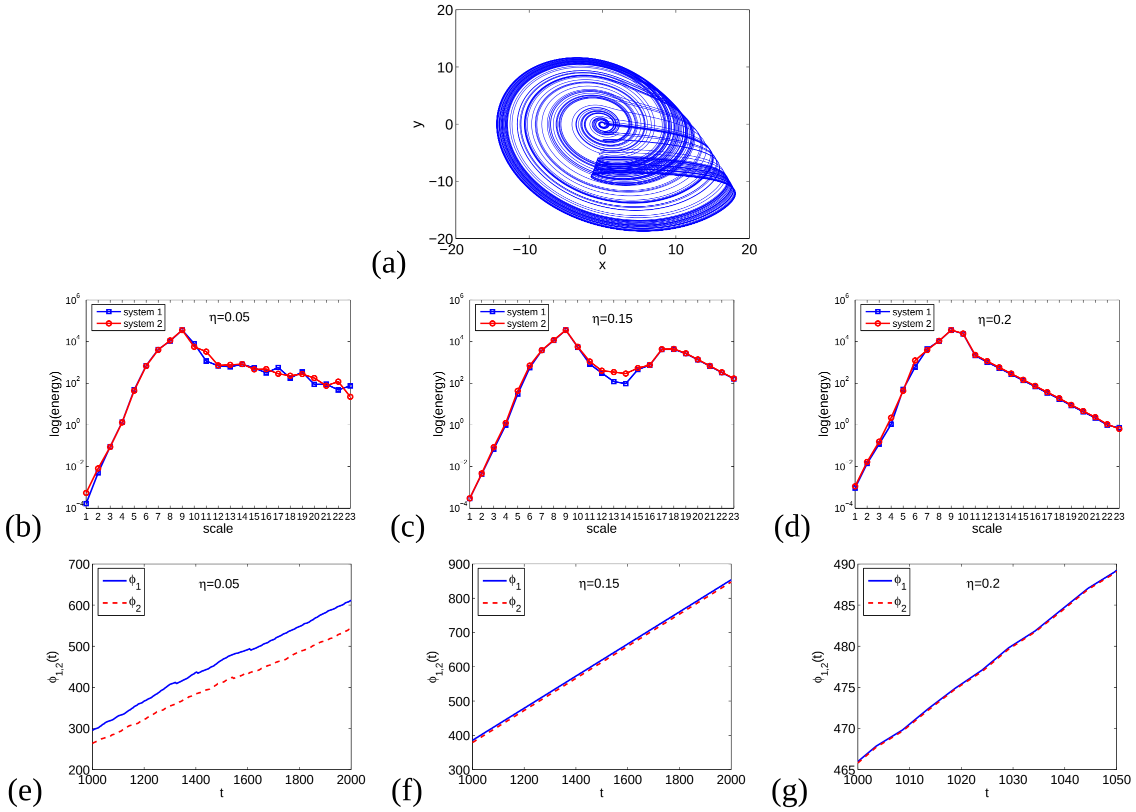

Figure 3: In (a) the projection of the attractor in the xy plane; scale versus the maximum energy in each scale considering a coupling strength of (b) 0.05, (c) 0.15, (d) 0.2; phase at $t = [1000, 2000]$ in the scale of maximum energy considering a coupling strength of (e) 0.05 and (f) 0.15 and (g) phase at $t = [1000, 1050]$ in the scale of maximum energy considering a coupling strength of 0.2.

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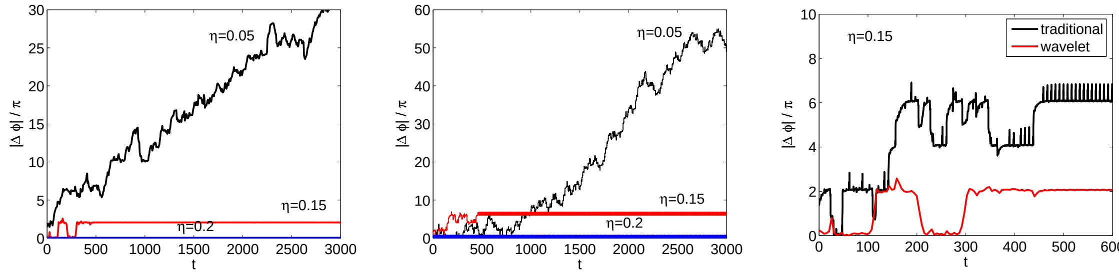

Figure 4: In (a) the wavelet method using the x variable; (b) the traditional method and (c) is shown a zoom, $t = [0, 600]$ of the phase difference between systems considering $\eta = 0.15$ using the traditional method for calculate the phase [8], and the wavelet method proposed.

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