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# SEARCHING FOR ORBITS THAT CAN BE CONTROLLED BY NATURAL FORCES

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This paper proposes a procedure to map orbits with respect to the perturbation forces with the goal of finding orbits that may use the solar radiation pressure for station-keeping maneuvers. The calculations are made based on the measurement of the total effects of the disturbing forces obtained by using the integrals of those forces over the time. The paper shows different types of integrals of the perturbing forces over the time. These integrals represent the total change of velocity per orbit that the satellite receives from the perturbing forces. Solar radiation pressure, J2 to J4 zonal harmonics terms of the geopotential, and lunisolar perturbations are considered. The results provide the magnitude of each perturbing forces, so it is possible to see if the radiation pressure can be used to control the effect of other forces or at least to help in reducing the cost of the control.

## INTRODUCTION

Orbital maneuvers are essential for space missions, whether to keep the satellite in its nominal orbit or to achieve the final orbit planned for the mission. It can be an extensive topic and, for each mission, there are specific requirements to be achieved by it. Usually, orbital maneuvers are studied in order to reach the final orbital constraints and to reduce the fuel consumption. This paper aims to evaluate the total effects that the perturbation forces cause in the spacecraft, by evaluating the integral over the time of those forces. These integrals measure the amount of variation of velocity caused by the perturbation forces, so it is a form of quantifying the participation of each individual force in the motion of the spacecraft. By considering different orbits under this criterion, it is possible to map orbits in order to find the less perturbed ones that have a good potential to have lower cost for the station keeping.

A station-keeping maneuver is a procedure that is performed in order to keep the satellite in the nominal orbit planned for its mission; more information about station-keeping maneuvers can be found in the references [1-6]. These maneuvers are needed because the perturbation forces

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acting on the satellite deviate it from the final planned orbit. The perturbation forces can be the third body perturbation from the Sun and the Moon, the solar radiation pressure, the oblatness of the Earth, etc. After mapping these orbits with respect to the most important perturbations, it is possible to verify which orbits have a good potential to use the radiation pressure to control the effects of the other perturbations, by comparing the results of the integrals over the time. This last activity is the main goal of the present paper. This approach of using integrals of the perturbing forces was first used in Prado [7], which studied orbits of satellites travelling around the Earth perturbed by the Sun and the Moon.

The third-body perturbation, which is one the perturbing forces considered in the present paper, has an extensive literature. It can be caused by the Sun, the Moon or any other body. Kozai [8,9] studied the influence of the Sun and Moon in a satellite using the Lagrangian equations. Blitzer [10] studied the equations of motion of a satellite under the influence of the third body perturbation with the help of classical mechanics. Musen [11] studied this perturbation and its periodic effects with two different approaches, one using the equation of Gauss and the second one with the help of numerical methods.

The $J_n$ perturbation (in the present paper it means $J_2$, $J_3$ and $J_4$) has also been studied widely in the literature, like in Hau-Hi [12], that changed the semi-major axis of the orbit in order to reduce the $J_n$ perturbation. Wang [13] studied frozen orbits and considered the $J_n$ perturbation as an essential perturbation to keep the argument of the perigee and the eccentricity of the orbit constant.

The solar radiation pressure perturbations have also been deeply studied extensively in the literature. The references used in the present paper to study the solar radiation pressure perturbation are Carrara [14], Sidi [15] and Tewari [16]. Some ideas on the use of those forces to control a satellite can be found in references [17-21].

## MATHEMATICAL MODEL

### The Third Body Perturbation

Concerning the third body perturbation, this paper examines the possibility of analyzing the perturbations of the Sun and the Moon for a satellite orbiting the Earth using a set of integrals defined later. The orbits of the Sun and the Moon are considered to be circular. The inclination of the orbit of the Sun is 23 degrees, while the orbit of the Moon varies from 18 to 28 degrees. In this paper, the inclination of the Moon was fixed to 18 degrees, because some preliminary studies showed a maximum variation of 2% in the integral values along all the different possible inclinations of the Moon and this variation was assumed to be small enough to be neglected. The mean motion $n'$ of the disturbing body is given by:

$$n' = \sqrt{\frac{G[m_0+m\prime]}{a\prime^3}} \tag{1}$$

where $m'$ is the mass of the third body, which can be the mass of the Sun or the mass of the Moon, $m_0$ is the mass of the Earth and $a'$ is the semi-major axis of the third body. The potential due to the perturbation of the Sun and the Moon is given by Equation (2) [7][22]:

$$U = -G\left[m_M\left(\frac{xx_M+yy_M+zz_M}{r_{EM}^3} - \frac{1}{r_M}\right) + m_S\left(\frac{xx_S+yy_S+zz_S}{r_{ES}^3} - \frac{1}{r_S}\right)\right] \tag{2}$$

where the coordinates of the Sun are $x_s$, $y_s$ and $z_s$; the coordinates of the Moon are $x_m$, $y_m$ and $z_m$; the coordinates of the spacecraft are $x$, $y$ and $z$; $G$ is the gravitational constant; $m_S$ is the mass of the Sun; $m_M$ is the mass of the Moon and $r_{EM}$, $r_{ES}$, $r_M$ and $r_S$ are the distances from the Moon to the Earth; from the Sun to the Earth; from the spacecraft to the Moon and from the spacecraft to the Sun, respectively. The reference system considered in this paper is the inertial one with the $X$ axis pointed at the vernal point, the $XY$ plane is the equatorial plane of the Earth and the center of the reference system is the Earth. The distances related to the Sun, Moon, spacecraft and the Earth are given by:

$$r^2 = x^2 + y^2 + z^2 \tag{3}$$

$$r_{ES}^2 = x_S^2 + y_S^2 + z_S^2 \tag{4}$$

$$r_{EM}^2 = x_M^2 + y_M^2 + z_M^2 \tag{5}$$

$$r_M^2 = (x_M - x)^2 + (y_M - x)^2 + (z_M - x)^2 \tag{6}$$

$$r_S^2 = (x_S - x)^2 + (y_S - x)^2 + (z_S - x)^2 \tag{7}$$

The goal of the present paper is to integrate the perturbative forces over the time in order to find the amount of velocity change that each perturbation delivers to the satellite. Concerning the perturbing force due the third body, the gradient of the potential, that gives the perturbing force, is given by:

$$\nabla U = f_x + f_y + f_z \tag{8}$$

where $f_x$, $f_y$ and $f_z$ are [7] [14]:

$$f_x = -G\left(m_M\left(\frac{x_M}{r_{EM}{}^3} - \frac{x_M - x}{r_M{}^3}\right) + m_S\left(\frac{x_S}{r_{ES}{}^3} - \frac{x_S - x}{r_S{}^3}\right)\right) \tag{9}$$

$$f_x = -G\left(m_M\left(\frac{y_M}{r_{EM}{}^3} - \frac{y_M - y}{r_M{}^3}\right) + m_S\left(\frac{y_S}{r_{ES}{}^3} - \frac{y_S - y}{r_S{}^3}\right)\right) \tag{10}$$

$$f_z = -G\left(m_M\left(\frac{z_M}{r_{EM}{}^3} - \frac{z_M - z}{r_M{}^3}\right) + m_S\left(\frac{z_S}{r_{ES}{}^3} - \frac{z_S - z}{r_S{}^3}\right)\right) \tag{11}$$

### The $J_2$, $J_3$ and $J_4$ perturbations

It is known that the Earth is not symmetric with homogeneous distribution of mass. Moreover, the Earth is not perfectly spherical and the mass distribution has a displacement, resulting in a non Keplerian gravitational force. The biggest influence of this non-symmetry on the gravitational force is caused by the oblatness of the Earth, which can be described by the $J_2$ coefficient [16]. The $J_2$ coefficient along with the $J_3$, $J_4$ and others, called spherical harmonics, are used to describe the non-symmetry of the Earth by the superposition of the contribution of each angular portion considered [16]. The $J_2$, $J_3$ and $J_4$ are dimensionless constants and they are the most common spherical harmonics used to describe the gravity model of the Earth. Hence, the present paper only considers these three spherical harmonics as the perturbative gravitational force. The values of these spherical harmonics considered here are $J_2 = 1.08263\text{x}10^{-3}$, $J_3 = -2.54\ \text{x}10^{-6}$ and $J_4 = -1.61\ \text{x}10^{-6}$ [16]. Before describing the gravitational potential due to the non-spherical Earth, it is necessary to define the position vector used. It is given by [16]:

$$\mathbf{r} = r\boldsymbol{i}_r + r\emptyset\boldsymbol{i}_\emptyset \tag{12}$$

where the unit vector $\boldsymbol{i}_r$ denotes the radial direction and the unit vector $\boldsymbol{i}_\emptyset$ denotes the southward direction in the local horizon frame.

Subsequently, the gravitational potential can be described with the help of the position vector, as follows [16]:

$$U(r,\emptyset) = \frac{GM_0}{r}\left\{1 - \sum_{n=2}^{4}\left(\frac{Re}{r}\right)^n J_n P_n(\cos\emptyset)\right\} \tag{13}$$

where $Re$ is the equatorial radius of the Earth, $M_0$ is the mass of the Earth, $P_n$ is the Legendre polynomial of $n^{th}$ degree, $J_n$ is the spherical harmonic coefficient of $n^{th}$ degree and $\emptyset$ is the co-latitude of the position of the sub-satellite point. Equation (13) only describes the gravitational potential until de fourth order of the spherical harmonics, but it can be extended to the $n^{th}$ order if more accuracy is required by the problem.

The gravity is a conservative force, as well as the third body perturbation, so the gravity force can be expressed by the gradient of the potential. The gradient of the potential in Equation (13) with respect to the position vector of Equation (12) is given by [16]:

$$\mathbf{g} = g_r\boldsymbol{i}_r + g_\emptyset\boldsymbol{i}_\emptyset \tag{14}$$

where

$$\mathrm{g_r} = -\frac{GM}{r^2}\left[1 - 3J_2\left(\frac{Re}{r}\right)^2 P_2(\cos\emptyset) - 4J_3\left(\frac{Re}{r}\right)^3 P_3(\cos\emptyset) - 5J_4\left(\frac{Re}{r}\right)^4 P_4(\cos\emptyset)\right] \tag{15}$$

and

$$\mathrm{g}_\emptyset = \frac{3GM}{r^2}\left(\frac{Re}{r}\right)^2 \sin\emptyset\cos\emptyset\left[J_2 + \frac{1}{2}J_3\left(\frac{Re}{r}\right)\sec\emptyset\,(5\cos^2\emptyset - 1) + \frac{5}{6}J_4\left(\frac{Re}{r}\right)^2(7\cos^2\emptyset - 3)\right] \tag{16}$$

**The Solar Radiation Pressure**

The solar radiation pressure is caused by the electromagnetic radiation that comes from the Sun and collides with the surface of the satellite, resulting in a pressure exerted on it. The magnitude of this perturbation depends on the momentum of each photon that collides with the satellite; therefore, the force applied to the satellite depends on the energy emitted by the Sun. For a satellite orbiting the Earth, this energy, $W$, emitted by the Sun is nearly constant and equals to 1350 W/m$^2$ [14]. The acceleration caused by the radiation pressure in the orbit of the satellite acts on the opposite direction of the unit vector Satellite-Sun, $\hat{r}_s$, and, in this paper, it is given by [14] [16]:

$$\overrightarrow{A_{PR}} = -\gamma C_R S P_s \hat{r}_s, \tag{17}$$

where $\gamma$ is the eclipse factor, which is 0 if the satellite is on the shadow of the Earth and is 1 if the satellite is illuminated by the Sun; $C_R$ is the reflectivity factor, usually known as the radiation pressure coefficient; S is ratio of cross-section area exposed to the incidence of the solar rays with the mass of the satellite; $P_s$ is the radiation pressure in a orbit around the Earth, with an approximated value of 4.55x10$^{-6}$ N/m$^2$. The eclipse factor considered here is related to Equation (18) and it depends on the position of the satellite relative to the position of the Earth and the Sun [14]. If the following equation:

$$h = \vec{r} \cdot \vec{r_s} \geq 0 \tag{18}$$

is true, then the satellite is illuminated by the Sun and $\gamma = 1$. Furthermore, if Equation (18) does not hold and if the following equation:

$$d = |\vec{r} \times \vec{r_s}| \tag{19}$$

is bigger or equal to the radius of the Earth, then $\gamma = 1$ [14]. On the other hand, if $d$ inEquation (19) is smaller than the radius of the Earth and Equation (18) does not hold, then $\gamma = 0$ and the satellite is in the shadow of the Earth.

The radiation pressure, $P_s \left[\frac{W}{m^2}\right]$, is directly related to the intensity of the radiation of the Sun. The value of $P_s$ is defined as:

$$P_s = \frac{W}{c} \tag{20}$$

where c is the velocity of the light and W is the radiation intensity (energy flux) emitted by the Sun.

The radiation pressure coefficient $C_R$ considered in this paper, along with the acceleration equation due to the radiation pressure shown here, is a very simple model, although it is a good approximation for a first study. Usually, the values of $C_R$ can vary from 1 to 3. It is assumed here that this coefficient is constant, but a more complex model can be used if required by the mission [14].

**PERTURBATION INTEGRALS**

In this section several options of perturbation integrals are presented that can be used to evaluate the magnitude of the effects of the perturbing forces, to search for the potential orbits that can use natural forces to control some of the other perturbation forces.

The integrals do not provide the fuel consumption for the station-keeping maneuver, because it depends on the constraints imposed by each mission, but they quantify the amount of variation of velocity caused by the perturbation forces, that is a form of representing the magnitude of the perturbation forces acting on the satellite. Those values can be used to map orbits with respect to the perturbation effects, so helping to find which ones can be controlled by the solar radiation pressure.

Before introducing these integrals, some averaged techniques used in the present study to eliminate the dependence of some initial parameters of the system need to be explained. There are several types of integrals as defined later in this paper. For each type of integral, the averaging technique applied is the same. In this way, before the introduction of specific types of integrals, they will be called PI (Perturbation Integral) in order to explain the averaging technique. Note also that the PI integrals are defined as integrals over the time, nevertheless, for the numerical solutions, the results were obtained by integration carried on with respect to the eccentric anomaly instead of the time. The change of the integration variable is possible since the time is strictly related to the eccentric anomaly by Equations (21-23) shown below:

$$M = M_0 + n(t - t_0), \tag{21}$$

$$M = E - esin(E), \tag{22}$$

$$dM = (1 - ecos(E))dE \tag{23}$$

where $M$ is the mean anomaly, $M_0$ is the initial mean anomaly, $t$ is the time and $E$ is the eccentric anomaly.

For the third body perturbation, the initial position of the third body influences the value of the integral, since we are evaluating the integral only for one period of the orbit of the satellite. In order to have results that do not depend on the initial position of the Sun or the Moon, an average technique is applied to the integral. This average technique provides a value for the integral that does not depend on the initial position of the third body. This average is important in order to consider the mean value of the integral for a large number of orbits of the satellite. This average technique, for the perturbation of the Sun and the Moon, is given by:

$$PI_{MS} = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}(PI)\, df_{0Moon} df_{0Sun} \tag{24}$$

where $f_{0Sun}$ is the initial true anomaly of the Sun and $f_{0Moon}$ is the initial true anomaly of the Moon. Similarly, this average can be simplified to consider only the Sun ($PI_S$) or the Moon perturbation ( $PI_M$) as shown below:

$$PI_S = \frac{1}{2\pi}\int_0^{2\pi}(PI) df_{0Sun} \tag{25}$$

$$PI_M = \frac{1}{2\pi}\int_0^{2\pi}(PI) df_{0Moon} \tag{26}$$

The solar radiation pressure is another perturbation force that depends on the position of the Sun, so it also considers the average technique given by Equation (25).

For the $J_n$ perturbation, an average technique is also applied in order to analyze the mean value of the integral. This average is necessary because the rotation of the Earth is considered and then the value of the integral depends on the initial position of the satellite. The average technique for the $J_n$ perturbation is given by:

$$PI_{SAT} = \frac{1}{2\pi}\int_0^{2\pi}(PI)\, dE_{0Satellite} \tag{27}$$

where $E_{0Satellite}$ is the initial eccentric anomaly of the orbit of the satellite.

Equations (25) to (27) describe the average integrals for one specific perturbation force only. Evidently, more than one force needs to be considered at the same time. This paper considers all the average integrals from (25) to (27) and also the integral with all the perturbations as defined by Eq. (28):

$$PI_{all} = \frac{1}{8\pi^3}\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi}(PI)\, df_{0Moon} df_{0Sun}\, dE_{0Satellite} \tag{28}$$

**The Keplerian Perturbation Integral**

The Keplerian perturbation integral (KPI) is the integral of the magnitude of the perturbing forces that acts on the satellite for one period of the orbit. This integral is given by [7]:

$$KPI = \int_0^T \left|\vec{\mathrm{F}}\right| dt \tag{29}$$

where T is the period of the orbit and $\vec{\mathrm{F}}$ is the acceleration of the perturbation force or forces. Equation (29) and the following equations related to the perturbation integrals are based on the time as the variable of integration. Nevertheless, as mentioned before, to make the numerical simulations easier, the variable of integration is changed to the eccentric anomaly.

The integral shown in Equation (29) gives the total amount of velocity change the satellite receives from the perturbation forces. It would be the variation of velocity required to be applied by the control system to keep the satellite in the Keplerian orbit all the time. The $KPI$ considers that the propulsion system is able to correct the shifts caused by the perturbation forces at every instant of time, by applying a force that has exactly the same magnitude of the perturbation force, but at the opposite direction. Although the $KPI$ can be considered a non-realistic approach to evaluate the costs of the station keeping maneuvers, because the constraint of keeping the orbit Keplerian all the time is very demanding in terms of fuel consumption, it can be useful to map the orbits that are less perturbed.

**The Perturbation Integral of the magnitude of the components of the perturbing forces evaluated in a system of reference fixed on the satellite**

The Perturbation Integral of the magnitude of the components of the perturbing forces evaluated in a system of reference fixed on the satellite (or PIMFS) is similar to the KPI integral. The main difference of the PIMFS with respect to the KPI integral, is that the perturbation forces are not evaluated in the inertial reference frame. The PIMFS is evaluated in a reference system fixed on the satellite and they are shown expressed in three components. It is obtained based on the scalar product of the perturbation force and the three unit vectors that define the reference system. These unit vectors are: the one that is in the direction of motion of the satellite; the one that is perpendicular to the velocity of the satellite, directed towards the inside of elliptical orbit; and the last one is the unit vector perpendicular to the orbital plane of the satellite, pointing at the opposite direction of the angular momentum. The main reason for study of these integrals is that each component has a different effects in the orbit of the spacecraft. The component in the direction of the motion of the satellite is the one that changes the kinetic energy of the spacecraft, by increasing or decreasing the magnitude of its velocity. The other component in the orbital plane is responsible for changing the direction of the velocity, keeping its magnitude constant and so not changing the kinetic energy of the spacecraft. The out of plane component of the force is responsible for modifying the plane of the orbit of the spacecraft, also not increasing or decreasing the kinetic energy, because this component is also perpendicular to the motion of the satellite. Note that these integrals are made over the magnitude of the forces, not considering their positive and negative signs. The reason for that is the existence of situations where compensations occur between positive and negative values in the same orbital period of the satellite, resulting in total values near zero, but leaving effects in the trajectory of the satellite. A common situation occurs when considering the effects non-zero of the solar radiation pressure. Assuming that the Sun does not move during one orbital period of the satellite and that the initial orbit of the satellite is circular, in half of the time the satellite is travelling directed to the Sun, so the solar radiation pressure is slowing down the satellite, so reducing its energy. On the other hand, in the second half of its orbital period, the satellite is travelling in such way that it is moving away from the Sun. During that time, the solar radiation pressure is working in the direction of the motion, so accelerating the satellite and increasing its energy. The result of the integration over the complete period of time of this force is zero. Since the Sun is not fixed, but it moves in its orbit during the integration time, the final result is not exactly zero, but it is very small. On the other side, thinking about the physical effects of this perturbation, it is not zero, since the net result of those forces acting in different directions with respect to the motion of the satellite is to increase the eccentricity of its orbit. So, to avoid mathematical results that do not agree with the physical effects, and to keep the idea of obtaining the results of integration of each component of the forces, the integrals are evaluated considering their magnitude. The reference system fixed in the satellite used here is based on three axis named $I,\ J\ and\ K$ axis, which are shown in Figure 1. The K axis is perpendicular to both I and J axis and it points in the direction of the cross product of $J$ and $I$, into the paper. The orbit is on the XY plane of the inertial reference system (the same as mentioned on the third body perturbation).

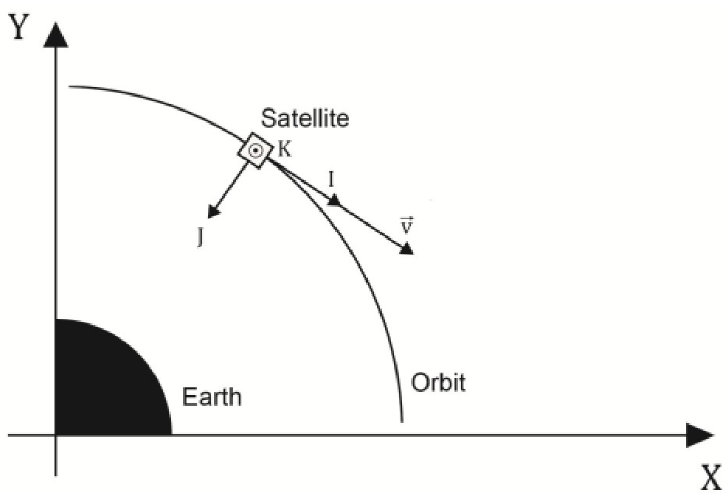


**Figure 1. The reference system fixed on the satellite.**

The PIMFS integrals provide the effects of the projections of the perturbing forces on the three directions explained above. Consequently, it is possible to evaluate the solar radiation pressure as possible forces that can help in maneuvering the vehicle. The PIMFS integrals are given by:

$$PIMFS_I = \int_0^T \left| \vec{F} \cdot \frac{\vec{v}}{|\vec{v}|} \right| dt \tag{30}$$

$$PIMFS_K = \int_0^T \left|\vec{\mathrm{F}} \cdot \left(\frac{\vec{r}}{|\vec{r}|} \times \frac{\vec{v}}{|\vec{v}|}\right)\right| dt \tag{31}$$

$$PIMFS_J = \int_0^T \left|\vec{\mathrm{F}} \cdot \left(\frac{\vec{v}}{|\vec{v}|} \times K\right)\right| dt \tag{32}$$

where $\vec{v}$ *and* $\vec{r}$ are the velocity and the radius vectors of the satellite in the inertial reference system.

The PIMFS integrals, similarly to the KPI integral, require that the satellite stays in a keplerian orbit all the time. Nevertheless, the results of those two types of integrals are different, since the PIMFS evaluates the total effects of each component of the force, while the KPI considers the total magnitude of the force.

**Using the Solar Radiation Pressure Perturbation to Maneuver a Satellite**

This integral is used to evaluate the potential use of the solar radiation pressure to maneuver a satellite. The idea of this integral is to use a solar sail in order to amplify the effects of the solar radiation pressure when this force is acting in the desired direction and to reduce it to near zero when this force is acting in the undesired direction. It is a mathematical formulation to represent an engineering solution of having a device in the satellite that can extend a solar sail when the satellite is in a position where the solar radiation pressure is acting in the direction that it is necessary to apply the thrust to the satellite, but that can close this solar sail when the direction of the force is not in the desired direction. In this way, the satellite can perform orbital maneuvers with this perturbation force and avoid the compensations of losing energy when the geometry is not favorable. The integral is the same of the PIMFS or the KPI integrals, with some changes on the parameters of the satellite and on the integral itself.

The idea here is very simple. If the scalar product of the direction of motion of the satellite with the solar radiation force is positive ($\vec{F} \cdot \vec{v} > 0$), than the solar sail is opened to amplify the radiation effects. The solar sail can be included in the perturbation force by increasing the effective ratio area/mass of the satellite in Equation (17). If this scalar product is negative, than the solar sail is not active and it is necessary to stop the integration during those times. This approach can be used to help the study of searching for orbits that can perform maneuvers using the radiation perturbation force instead of the propulsion systems.

**The Work Integral**

This integral is used to analyze the work realized by the perturbation forces in the satellite. The physical definition of the work realized by a force when moving a particle from the points $a$ to $b$ is given by:

$$KW = \int_a^b \vec{F} \cdot d\vec{r}\,, \tag{33}$$

where $\vec{F}$ is the force and $d\vec{r}$ is the infinitesimal displacement of the particle. Taking into account that $\vec{v} = \frac{d\vec{r}}{dt}$, and using the magnitude of the integrand to avoid undesired compensations among positive and negative values, it is possible to reach the result given:

$$KW = \int_0^T \left|\vec{F} \cdot \vec{v}\right| dt \tag{34}$$

assuming that the instant of time is zero when that particle is at the point $a$ and that it is $T$ when it is at the point $b$. This equation provides the total variation of the kinetic energy of the satellite due the force represented by the potential U. The reason to use the magnitude of the integrand is that forces that have zero work realized do not change the energy of the satellite (so do not change the semi-major axis of the orbit), but can cause perturbations in eccentricity, inclination, argument of periapsis or argument of the ascending node.

## RESULTS

The results presented here are obtained based on several orbits. The Keplerian elements of those orbits are specified and then the results presented are the integral value vs. the variation of the semi-major axis.

The standard parameter of the effective area per mass of the satellite without panels was considered to be $S = 0.05\ m^2/kg$ for all figures that do not specify a different value of $S$. For the results that specify a different value of S, panels were used in order to study possible maneuvers with the use of the radiation pressure perturbation. The panels are active when the dot product of the radiation pressure force and the velocity of the satellite is greater than zero. Whenever this dot product is negative, the constant value of S is the standard one mentioned before.

The constant $S$ was the only parameter varied in the simulations shown here, since the constant $C_R$ constant was not changed. Both parameters were considered to be constants in the present paper, so they can be considered a multiplying factor of the integrals. In this way, the constant $C_R$ was considered to be 3 in all the results.

### Equatorial and Planar Orbits near GEO

The first family of orbits studied in the present study is the equatorial and planar orbits. One of the most important orbits that belong to this category is the geostationary orbit, which has Keplerian elements: $a = 42164000\ m,\ e = 0,\ i = 0,\ \omega = 0$ and $\Omega = 0$. The study presented here covers equatorial and planar orbits with semi-major axis in the region 30000 km to 50000 km, so it includes the geostationary orbit.

The results presented in Figures 2 to 11 are the variation of the PI as a function of the semi-major axis for the equatorial and planar orbits. It is necessary to take into account that, as the semi-major axis varies, the orbital period of the satellite also varies. So, in order to keep the integral value independent of the period of the orbit, the PI value is multiplied by the period of the geostationary orbit and divided by the period of the current orbit, so the values of the PI are always calculated for the same amount of time. Figures 2 to 4 show the results for the KPI integral.

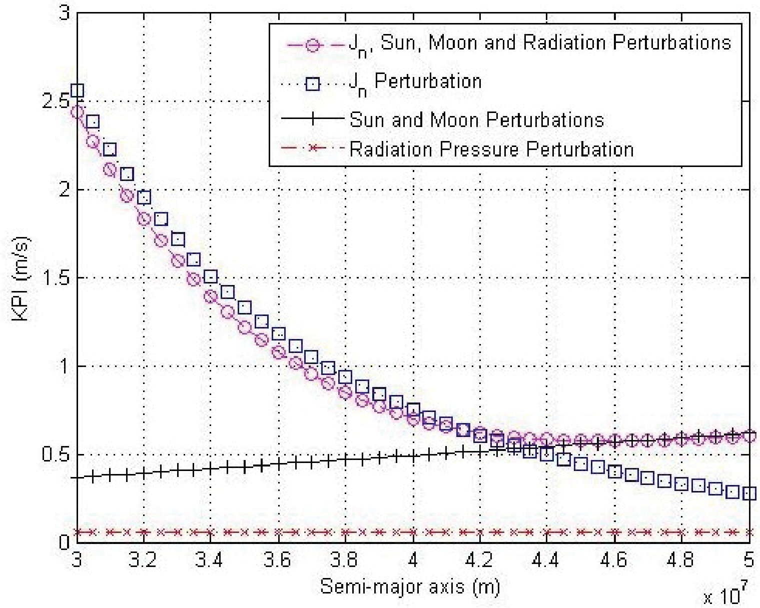


**Figure 2. KPI value for all perturbations vs. semi-major axis of the orbit of the satellite.**

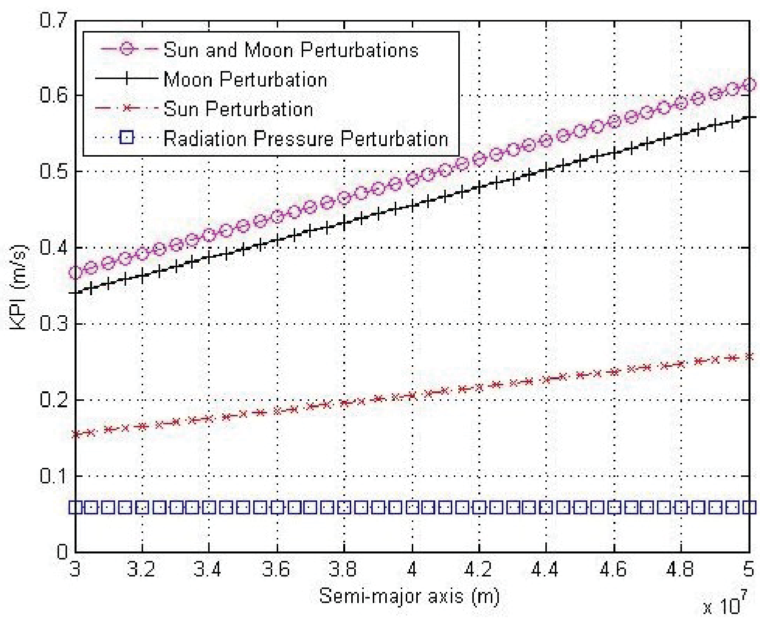


**Figure 3. KPI value for the Sun, Moon and Radiation Pressure perturbations vs. semi-major axis of the orbit of the satellite.**

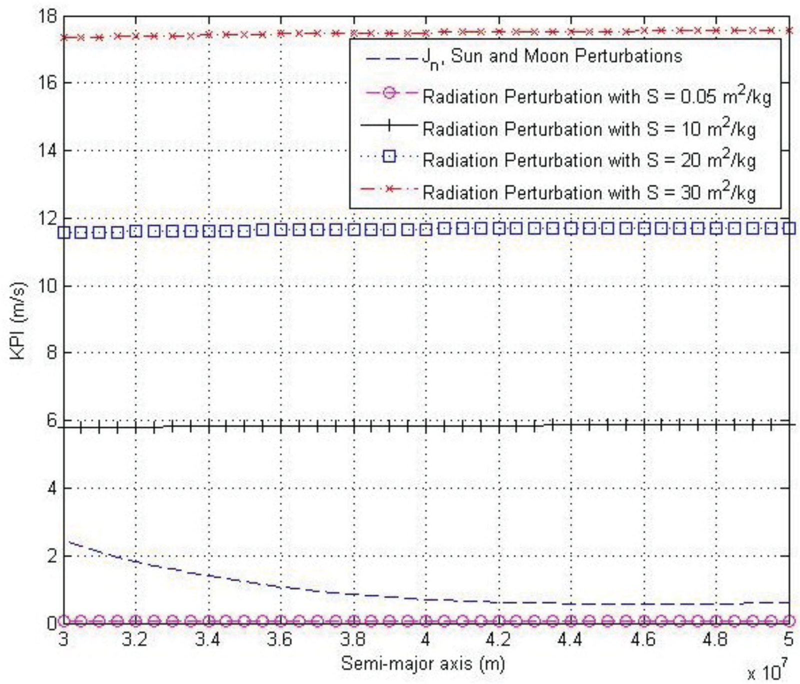


**Figure 4. KPI value for all perturbations vs. semi-major axis of the orbit of the satellite.**

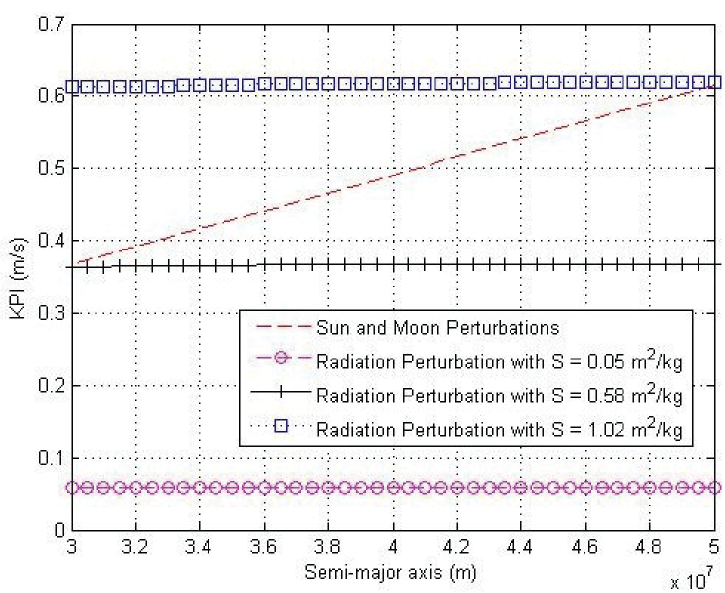


**Figure 5. KPI value for the Sun, Moon and Radiation Pressure perturbations vs. semi-major axis of the orbit of the satellite.**

From Figure 2, it is clear that the effect of the $J_n$ perturbation decreases as the semi-major axis increases. This occurs because the distance from the satellite to the Earth increases as the semi-major axis increases, so the $J_n$ perturbation decreases.

The third body increases the KPI value as the semi-major axis increases, because the distance from the satellite to the third body decreases. This is valid for the sum of the Sun and Moon perturbations and for each perturbation individually.

The solar radiation pressure perturbation has the lowest KPI value when compared to the other perturbations for the orbits shown in Figure 1. The KPI value for the solar radiation pressure perturbation is almost constant with a value close to 0.058 m/s at $S = 0.05\, m^2/kg$, which represents the satellite without panels. In this way, for the approach of the KPI integral that is based on a Keplerian satellite's orbit all the time, for the orbits shown in Figure 2, the radiation pressure perturbation cannot control the other perturbation forces alone if panels are not used.

From Figure 2 it is also possible to note that the sum of all perturbations results in a curve that has a minimum value for the KPI at the semi-major axis value of 46000 km. This is a result of the equilibrium of the perturbing forces, with the ones deriving from the geopotential decreasing and the one from the third body increasing. It is also visible that the effects of the total perturbation is smaller than the effects of $J_n$ perturbations, so there is a compensation and the third body perturbation is helping to control the orbit of the satellite. This is valid for semi-major axis in the range from 30000 km until the GEO. After that the total effects follow the third-body perturbation, since the terms coming from the geopotential becomes negligible.

From Figure 3, it is possible to see that the sum of the effects of the Sun and the Moon, the respective individual values, and the solar radiation pressure perturbations. It is an amplification of Figure 2, which shows the variations of the forces of lower magnitude. The perturbation of the Moon is always larger than the perturbation of the Sun and the combined effects of both third body perturbations are smaller than the sum of the individual contributions, so there are compensations in the effects and one force helps to control the other. The total perturbations always increase, since the spacecraft is getting closer to the perturbing body.

Figure 4 uses the panels when the dot product of the radiation perturbation force and velocity is larger than zero. It is shown several values for the KPI integral as the ratio of the effective area by mass is varied, to represent different sizes of the panels. It is possible to note that, as the value of S increases, the radiation pressure increases proportionally. This occurs because S is a constant in this paper, then this constant is a multiplying factor for the KPI integral. The results were omitted in this paper, but the same pattern obtained for the S constant was also obtained for the $C_R$ constant. The $C_R$ parameter is also a constant at the KPI integral and its value can be also considered a multiplying constant.

The solar radiation pressure is capable of controlling the other perturbation forces with the help of the panels, as shown in Figure 4.

Figure 5 shows the perturbation of the radiation pressure without the panels, with the standard value of S. Additionally, the panels are applied for two different values of S. The panel with $S = 0.58\, m^2/kg$ is not able to control the third-body perturbation, while the panel with $S = 1.02\, m^2/kg$ has a larger KPI value than the third-body perturbation for the semi-major axis range studied. The value of S that generates an integral for the solar radiation pressure with the same value of the total perturbations is in the range $S = 0.98\, m^2/kg$ to $S = 4.18\, m^2/kg$, depending on the semi-major axis of the orbit.

Next, Figures 6 and 7 show the results of the $PIMFS_I$ integral, the Perturbation Integral of the magnitude of the components of the perturbing forces evaluated in a system of reference fixed in the satellite and in the direction of motion of the satellite.

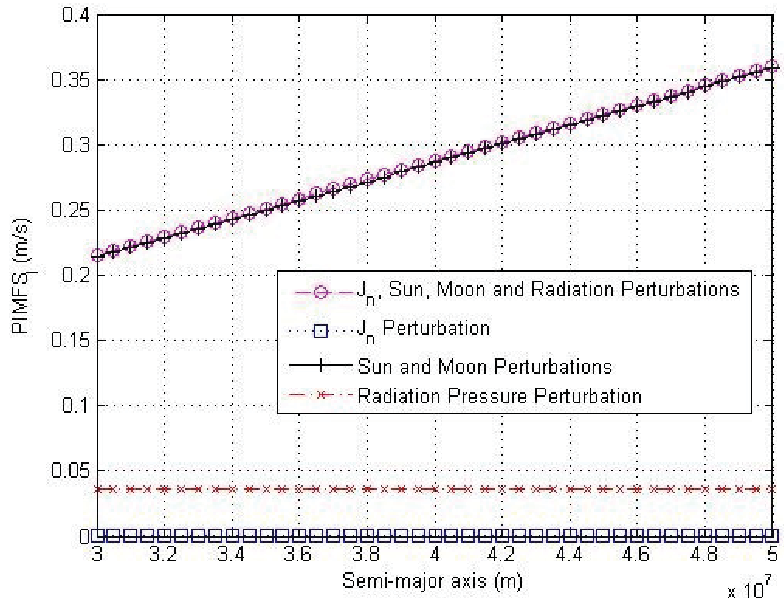


**Figure 6. $PIMFS_I$ value for all perturbations vs. semi-major axis of the orbit of the satellite.**

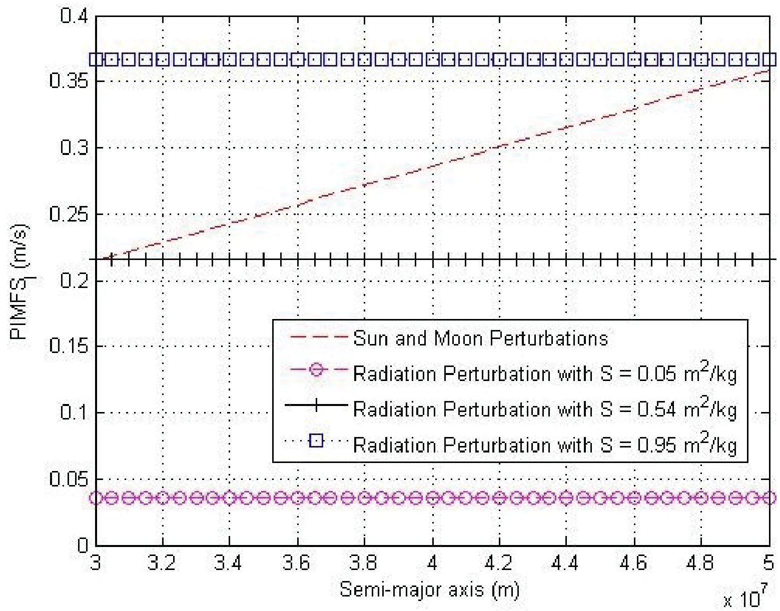


**Figure 7. PIMFS$_I$ value for all perturbations vs. semi-major axis of the orbit of the satellite.**

From Figure 6, it is clear that the third body perturbation is predominant on the direction of motion of the satellite. The radiation pressure perturbation for a satellite without panels is the second perturbation factor, been in the order of 1/5 to 1/8 of the total perturbation, depending on the semi-major axis. The $J_n$ perturbation has almost no affect, independent of the semi-major axis. In this way, the radiation pressure perturbation has a considerable value and can be used to help the control system, even without the help of the panels.

Figure 7 shows the third-body and the radiation pressure perturbations for a satellite with and without panels. When the panel is active, for this semi-major axis range, the value of $S = 0.54\ m^2/kg$ has a lower PIMFS$_I$ value than any value of the third-body perturbation and the value of $S = 0.95\ m^2/kg$ has the biggest value of any perturbation shown in Figure 7.

Figure 8 shows the equivalent results when the direction perpendicular to the direction of motion is considered, the J axis.

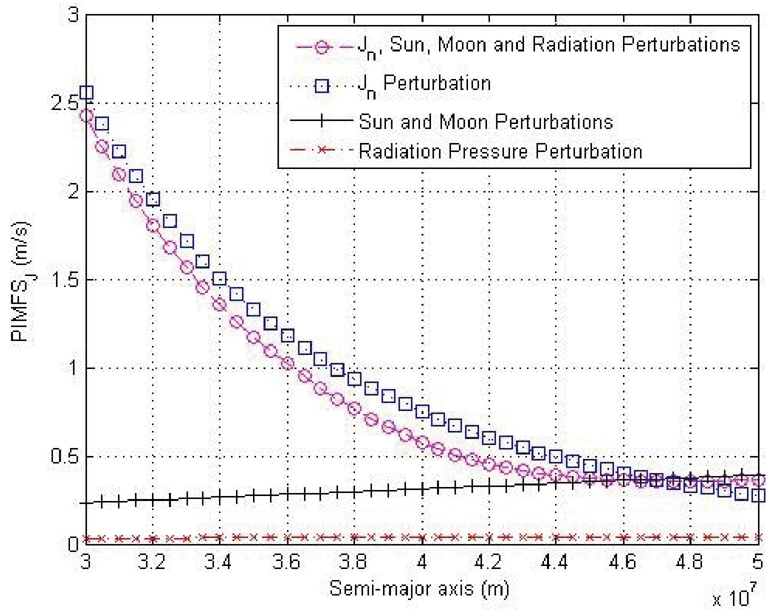


**Figure 8. $PIMFS_J$ value for all perturbations vs. semi-major axis of the orbit of the satellite..**

Figure 8 shows the $PIMFS_J$ value for all perturbations. It is the value of the integral of the magnitude of the perturbation forces in the $y$-axis (perpendicular to the direction of motion of the spacecraft, but in the orbital plane) of the reference frame fixed in the satellite. This figure shows that the $J_n$ perturbation has the biggest influence. This result was expected, since the y axis has a direction close to the unit radius vector $\vec{r}$ most of the time, then close to the $J_n$ perturbation direction. The third body perturbation has also a strong influence. For the $J_n$ perturbation, it decreases as the semi-major axis increases, and, on the contrary, for the third body perturbation, the $PIMFS_J$ value increases as the semi-major axis increases. The explanation for these results is the same presented for Figure 2. Figure 8 has a minimum value when considering all the perturbations at approximately 46000 km. The same effects noted before are present, with compensations among the forces.

The radiation pressure perturbation is almost constant for the $PIMFS_J$ and its value is approximately 0.035 m/s when the satellite have no panels. According to the same approach used before, if the panel is active, the value of $S = 0.62\, m^2/kg$ represents the minimum of the $PIMFS_J$ value for the third body perturbation and the value of $S = 1.1\, m^2/kg$ is enough to match with the maximum value of the third body perturbation.

Figure 9 shows the equivalent results when the direction perpendicular to the orbital plane is considered.

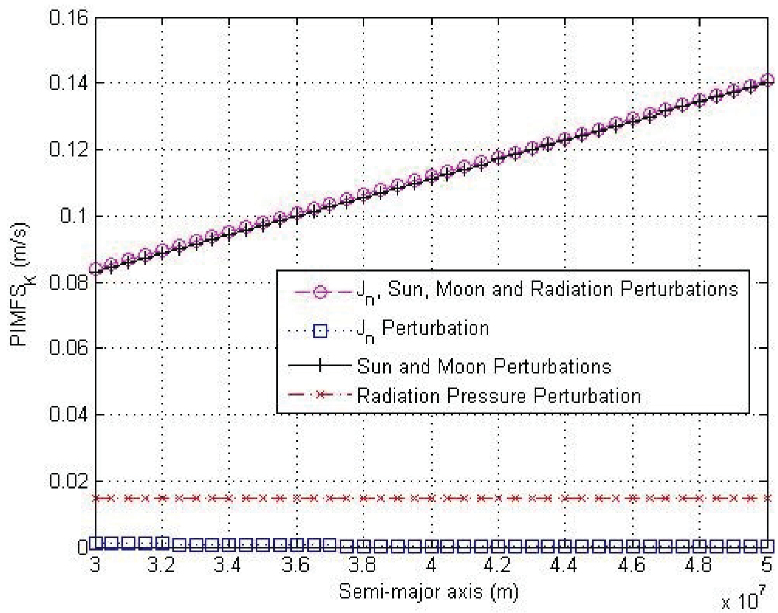


**Figure 9. $PIMFS_K$ value for all perturbations vs. semi-major axis of the orbit of the satellite.**

For the value of the same integral in the direction of the perpendicular to the orbital plane, shown in Figure 9, the $J_n$ perturbation has almost no effect, because the orbits are equatorial and circular, so the symmetry of the geometry reduces the perturbations coming from the geopotential. The third-body perturbation has the largest influence and the radiation pressure is the second largest, ranging from about 1/5 and 1/8 of the total value, depending on the value of the semi-major axis, so it can help to control the spacecraft, even without the help of panels.

The radiation pressure perturbation is almost constant for the $PIMFS_J$ value and it's approximately 0.0145 m/s if panels are not used. The value $S = 0.52\ m^2/kg$ is enough to match the minimum of the third body perturbation and $S = 0.92\ m^2/kg$ the maximum.

One can now study the work integral. Figures 10 and 11 show the results for the integral of the magnitude of the scalar product of the force by the increment of displacement of the satellite.

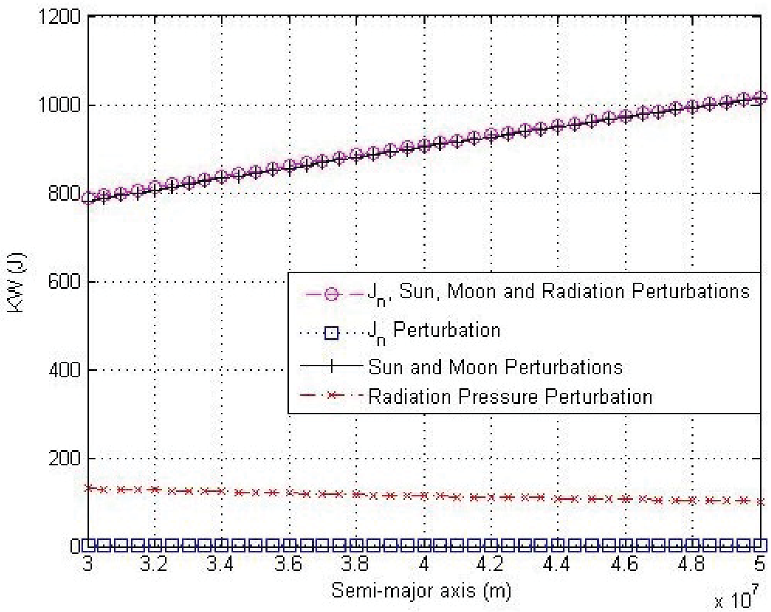


**Figure 10. KW value for all perturbations vs. semi-major axis of the orbit of the satellite.**

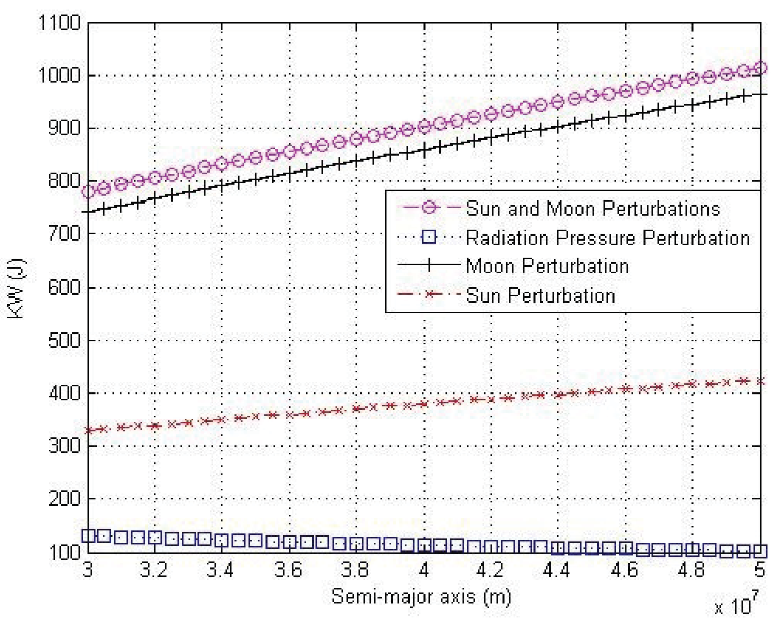


**Figure 11. KW value for the Sun, the Moon and radiation pressure perturbations vs. semi-major axis of the orbit of the satellite.**

The KW is the work realized by the perturbing forces. The third body perturbation has the most important effect on this work and the radiation pressure perturbation is the second one. The Jn perturbation realizes almost no work, as shown in Figure 10. The pattern of Figure 10 is the same as the one in Figure 6, since the integral formulation of both figures are similar. The work is calculated by the scalar product of the perturbation force and the velocity of the satellite, so the Jn perturbation barely affects the KW value. Figure 11 shows the KW values for the Moon and Sun, individually. It is noticed the same compensation effects that happened before, and the effects of the Sun and the Moon combined are smaller than addition of the individual participations.

## CONCLUSION

This paper formulated a procedure to map orbits of a given satellite with respect to the perturbations experienced by this satellite from the third-body (Sun and Moon), the geopotential ($J_2$ to $J_4$) and the solar radiation pressure. The criterion for those mappings is based on the integral of those perturbing forces over one period of the orbit. Based on those mappings, it is possible to choose orbits for a space mission, focused on orbits that are less perturbed.

Since this index depends on the initial relative geometry of the bodies, the procedure used average techniques over the initial positions of the bodies and the satellite to remove this dependence and to generate results that do not depend on the initial configurations.

The results show the importance of each force in the motion of the satellite, considering the total magnitude as well as each component of the forces: in the direction of the motion of the satellite, perpendicular to this direction in the plane of the orbit and in the direction normal the orbital plane of the satellite. The work integral is also examined.

In particular, it is shown under which conditions the integral related to the solar radiation pressure is larger than the corresponding integrals of the other perturbations, with the goal of verifying necessary conditions to control the satellite using natural force. The value of the area/mass ratio that allows this situation is calculated for several conditions.

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