Modeling and Simulation of Launch Vehicles Using Object-Oriented Programming: Impact of the Engine Parameters on the Launcher Performance

F. A. S. Mota^{*}, J. N. Hinckel^{*}, E. M. Rocco^{*} and H. Schlingloff^{**} *National Institute for Space Research (INPE) Av dos Astronautas, 1758, 12227-010, São José dos Campos - SP, biomota@yahoo.com.br, hinckeljn@gmail.com and evandro_mr@yahoo.com.br **Ostbayerische Technische Hochschule Regensburg (OTH) Seybothstrasse 2, 93053 Regensburg, hanfried@drschlingloff.de

[†]Hinckel, J.N.

Abstract

This paper proposes the development of a tool that facilitates the modeling and the analysis of the interaction between subsystems and components as well as the influence of changes in the design parameters of the propulsion system in the trajectory and payload mass. In other words the coupling among the propulsion system, the trajectory, and the mass of the launch vehicle is studied. To allow for greater reusability and extensibility of code it is used a modular structure using object-oriented programming. This software helps to identify the relevant structures to the problem and the relationships between them. As a case study, it is considered the future brazilian launch vehicle VLS-alfa, which will use the future L75 engine as the upper stage. The engine is a gas generator cycle and has ethanol and LOX as working fluids. The influence of parameters of the L75 engine on the trajectory and payload mass are evaluated.

1. Introduction

Due to the inherent complexity of a launch vehicle its design is usually divided into multiple disciplines, such as trajectory, propulsion, mass and geometry. Propulsion systems are composed of a large number of components grouped into subsystems hierarchy. The performance of the vehicle depends on the individual performance of each of the subsystems which in turn depends on material properties and design parameters. Changes in design parameters are propagated throughout the cluster hierarchy of subsystems and components, flight trajectory and payload mass. One of the most important and robust tools for vehicle/propulsion analysis were developed when DLR and NASA combined computer codes to provide a capability to optimize rocket engines cycles and its parameters as well as launch vehicles considering the coupling between them (see References [⁹], [¹⁰] and [⁸]). This work aims to deal with the problem of studying the coupling between propulsion and vehicle using object-oriented programming.

2. Mathematical Modeling

The modeling of a launch vehicle is based on employing a set of performance codes, which are based on physical models for propulsion, mass properties, aerodynamics, and flight dynamics of the vehicle.

2.1 Component Modeling

The common components for all configurations are pumps, turbine(s), valves, pipes and thrust chamber. Depending on the the configuration we can also find a gas generator (for gas generator cycle), a pre-burner (for staged combustion cycle) and boost-pumps.

Copyright © 2015 by [Mota, F.A.S; Hinckel, J.N.; Rocco, E.M.; Schlingloff, H.]. Published by the EUCASS association with permission.

2.1.1 Turbopump

The turbopump is required when we want a higher pressure in the combustion chamber. The required pump power is given by

$$P_P = \frac{(p_d - p_i)\dot{m}}{\eta_P \rho} \tag{1}$$

where p_d is the discharge pressure, p_i is the inlet pressure, η_P is the pump efficiency, ρ is the propellant density and \dot{m} is the mass flow rate. For preliminary analysis, Humble⁶ gives a design efficiency of 0.75 for LH2 and 0.80 for all other types of propellants. The energy required for the pumps comes from the turbine. Ideally there are two types of turbines of interest to rocket pump drives: impulse turbines and reaction turbines. The turbine pressure ratio is defined as

$$p_{Tr} = \frac{p_{Ti}}{p_{Td}} \tag{2}$$

where p_{Tr} is the turbine pressure ratio and the indexes Ti, Td refer to turbine inlet and turbine discharge respectively. Thus if we define the turbine efficiency η_T , the mass flow m_T , the specific heat c_p , the inlet temperature T_i and the heat capacity ratio γ then the power of the turbine can be given as

$$P_{T} = \eta_{T} \dot{m}_{T} c_{p} T_{i} \left[1 - \left(\frac{1}{p_{Tr}} \right)^{(\gamma-1)/\gamma} \right]$$
(3)

2.1.2 Thrust Chamber

The thrust chamber consists of injectors, combustion chamber, nozzle, igniter and cooling system. In the thrust chamber the propellants that come from the feed system, are injected, atomized, mixed and burned to turn into hot gases that are ejected at high speeds. The thrust force can be calculated as

$$F_c = \lambda(\eta_c c^* C_f) \dot{m}_c \tag{4}$$

where

 \boldsymbol{c}^* - characteristic velocity

 $\lambda = 1/2(1 + \theta)$ - nozzle divergence factor

 η_c - combustion efficiency

 C_f - thrust coefficient

$$C_{f} = \sqrt{\frac{2\gamma_{c}^{2}}{\gamma_{c} - 1} \left(\frac{2}{\gamma_{c} + 1}\right)^{(\gamma_{c} + 1)/(\gamma_{c} - 1)} \left[1 - \left(\frac{p_{e}}{p_{c}}\right)^{(\gamma_{c} - 1)/\gamma_{c}}\right] + \left(\frac{p_{e} - p_{a}}{p_{c}}\right) \frac{A_{e}}{A_{t}}$$
(5)

$$c^* = \frac{p_1 A_t}{\dot{m}} = \frac{\eta_c^* \sqrt{\gamma_c R T_c}}{\left(\frac{2\gamma_c}{\gamma_c + 1}\right)^{(\gamma_c + 1)/(2\gamma_c - 2)}}$$
(6)

where γ_c , *M*, p_e and T_c are respectively, the heat capacity ratio, the molar mass, the chamber pressure and the combustion temperature and can be calculated using the well known software CEA.¹¹ For LOX/LH2, in Schmucker¹⁴ is presented closed forms for these parameters, however these equations are derived from CEA and thus, are less accurate, but for preliminary design it is a good option for simplifying calculations.

2.1.3 Gas Generator or Pre-burner

The gas generator or the pre-burner operates exactly the same way, they are responsible to drive the turbine(s) by means of gases from combustion. To assess the parameters of the combustion gases, as for the combustion chamber, the software CEA can be used.

2.1.4 Feed Lines

The required pump discharge pressure is determined from the chamber pressure and the hydraulic losses in valves, lines, cooling jacket (for the fuel), and injector head. To obtain the rated flow at the rated pressure, an additional adjustable pressure drop for a flow orifice is usually included which permits a calibration adjustment or change in the required feed pressure. For a gas generator cycle, the stagnation pressure drop of the propellants between the pump discharge and the combustion chamber can be given as

$$p_{pump,f} - p_c = \Delta p_{f,pipe} + \Delta p_{f,valve} + \Delta p_{f,cooling} + \Delta p_{f,injector}$$
(7)

$$p_{pump,o} - p_c = \Delta p_{o,pipe} + \Delta p_{o,valve} + \Delta p_{o,injector}$$
(8)

These pressure drop can be estimated by relations that are function of chamber pressure.

2.2 Mass Modeling

There are numerous relations for estimating engine and stage mass in the literature and most of them are based in historical and empirical date as we can see in the following references Felber,⁵ Schlingloff¹³ and Ernst⁴ in turn taken from Zandbergen.¹⁷

2.2.1 Estimate Engine Mass

The mass and dimensions of existing and historical liquid rocket systems (excluding tanks) correlate well with thrust magnitude. From mission-level analysis, we know how much thrust we need, so we can easily estimate system mass Humble.⁶ Using a data base with 51 LRE, linear, quadratic, power law and logarithmic curves were analyzed in Castellini.³ The best resulting regression in terms of quadratic fit error for each technology were implemented within the propulsion models. Another way to estimate the engine mass is calculating the mass of all components of the engine and then summing them. But this can cause large dispersion to the real value, so we can adjust a curve with some knowing engines. Using SSME, J-2, HM-60, HM-7A, HM-7B, LE-7 and LE-5 Silva¹⁵ developed the following equation valid to thrust varying between 60 and 2300 kN. The following equation represents a so-called analytical/statistical model, which means it considers not only statistical data but also physical relationships.

$$m_{eng} = 1.76459(m_{tp} + m_{valve} + m_{inj} + m_{cc} + m_{ne})^{0.98636}$$
⁽⁹⁾

where

 m_{tp} - turbo-pump mass (kg)

 m_{valve} - mass of all valves (kg)

mini - injector mass (kg)

 m_{cc} - combustion chamber and gas generator mass (kg)

 m_{ne} - nozzle extension mass (kg)

The equations for components mass are function of engine parameters as chamber pressure and mixture ratio and were taken from Felber.⁵ The turbopump mass, for example, is calculated as

$$m_{tp} = \frac{0.178}{k_{TB}} \rho^{0.148} P_T^{0.73} \tag{10}$$

where

 $k_{TB} = \begin{cases} 1, & \text{if no boost-pumps,} \\ 2, & \text{if with boost-pumps.} \end{cases}$

This equation is valid for power varying between 300 to $6 \times 10^4 kW$. This model is sufficiently detailed when the influence of the engine parameters on the engine mass or payload mass are aim of study. However for rapid estimation of the performance of a launch vehicle, simpler relations functions of the thrust force are sufficient as the following equation taken from Schlingloff¹³

$$m_{eng} = 4.74(T)^{0.75} \tag{11}$$

2.2.2 Stage Dry Mass

The sum of the component masses does not simply yield the total propulsion system mass as some major components such as the electrical system, the hydraulic control system and the flight instrumentation system and minor components such as ignitor and starter are not taken into account. In Schlingloff¹³ the dry mass of the stage can be calculated by the sum of engine mass m_{eng} (Eq. 11) and structure mass m_{str}

$$m_{str} = C \left(\frac{m_{prop}}{\rho}\right)^{0.666} \tag{12}$$

It was used existing hardware to find the constants in the model above: the constant *C* assumes approximately the value 225 for high energetic propellant (or $\rho \approx 280 kg/m^3$) and 350 for low energetic propellant (or $\rho \approx 1220 kg/m^3$).

2.3 Cycle Modeling and Simulation

To simulate a LRE cycle we need to make use of the a mass and energy conservation laws. Components mass balance, a energy balance in the turbomachinery, pressure balance and a thrust force balance or a global mass balance can define a nonlinear set of equations. For each type of cycle or engine a different set of equations can be stated. Here it will be presented a modeling for a gas generator cycle with direct drive turbopump and with dump and regenerative cooling systems, which represents the L75 rocket engine. Thus if we consider the vector of variables (see Figure 1)



Figure 1: Gas Generator Cycle



so we can define the nonlinear system of equations as follows

$$\eta_m P_T(\dot{m}_T) = P_{P,o}(p_{o2}, \dot{m}_{o,g}, \dot{m}_{o,c}) + P_{P,f}(p_{f2}, \dot{m}_{f,g}, \dot{m}_{f,c})$$
(13)

$$\dot{m}_T = \dot{m}_{f,g} + \dot{m}_{o,g} \tag{14}$$

$$c_c = \frac{\dot{m}_{o,c}}{\dot{m}_{f,c}} \tag{15}$$

$$r_g = \frac{\dot{m}_{o,g}}{\dot{m}_{f,g}} \tag{16}$$

$$p_{f2} - p_c = \Delta p_{pipe}^{fuel} + \Delta p_{valve}^{fuel} + \Delta p_{cooling}^{fuel} + \Delta p_{inj}^{fuel}$$
(17)

$$p_{o2} - p_c = \Delta p_{pipe}^{ox} + \Delta p_{valve}^{ox} + \Delta p_{inj}^{ox}$$
⁽¹⁸⁾

$$F = F_c + F_T + F_{dc} \tag{19}$$

Then we have the same numbers of unknowns and equations and this problem can be solved by numerical methods. In these equations p_c , F, r_c and r_g are given. In the above set of equations the overall thrust remains constant, however if we want to simulate the cycle for a constant mass flow in the engine, then the Eq. 19 can be replaced by

$$\dot{m}_{stage} = \dot{m}_g + \dot{m}_c + \dot{m}_{dc} \tag{20}$$

2.4 Trajectory Modeling and Optimization

There are numerous ways to represent the translational motion of a launch vehicle. Depend on the goal of a given project we can choose the more appropriate reference frames and set of state variables. In this work will be considered a modeling which was derived in Tewari¹⁶ and it is used in most of the recent publications. The reference frames adopted in this modeling are the planet-fixed reference (SXYZ) frame and the local horizontal frame (oxyz), both are non-inertial (Figure 2a). Here, some steps in the derivation of the modeling equation will be omitted, but as aforementioned a detailed derivation can be found in Tewari.¹⁶ From Figure 2a, the relative velocity **v** and the local velocity of the local horizontal frame (oxyz) relative to the planet-centered rotating frame (SXYZ) can be expressed as

$$\mathbf{v}(v, \gamma, \zeta) = v(\sin \gamma \mathbf{i} + \cos \gamma \sin \zeta \mathbf{j} + \cos \gamma \cos \zeta \mathbf{k})$$
(21)

$$\mathbf{\Omega} = \dot{\boldsymbol{\xi}} \mathbf{K} - \dot{\boldsymbol{\phi}} \mathbf{j} \tag{22}$$

with a convenient rotation matrix, Eq. 22 can be written only in terms of axes of the body as

$$\mathbf{\Omega} = \dot{\xi} \sin \phi \mathbf{i} - \dot{\phi} \mathbf{j} + \dot{\xi} \cos \phi \mathbf{k}$$
(23)

The relative velocity can also be expressed as

$$\mathbf{v} = \dot{r}\mathbf{i} + \mathbf{\Omega} \times (r\mathbf{i}) \tag{24}$$

$$\mathbf{v} = \dot{r}\mathbf{i} + r\dot{\xi}\cos\phi\mathbf{j} + r\dot{\phi}\mathbf{k} \tag{25}$$

Comparing 21 and 25 we finally obtain the kinematic equations of motion



Figure 2: (a) Planet-fixed and local horizon frames for atmospheric flight and (b) External force resolved in the wind axes.¹⁶

$$\dot{r} = v \sin \gamma \tag{26}$$

$$\dot{\xi} = \frac{v\cos\gamma\cos\zeta}{r\cos\phi} \tag{27}$$

$$\dot{\phi} = \frac{v\cos\gamma\sin\zeta}{r} \tag{28}$$

To derive the dynamic equations we start from the Newton's second law

$$\mathbf{F} = m\mathbf{a}_I = m\frac{d\mathbf{v}_I}{dt} \tag{29}$$

Choosing the wind axes to express the forces on the body Figure 2b and doing the appropriate transformation to perform \mathbf{a}_I in the wind axes we finally get

$$\dot{\gamma} = \frac{T\sin\alpha_T}{m\nu} + \left(\frac{\nu}{r} - \frac{\mu_E}{r^2\nu}\right)\cos\gamma + \frac{L}{m\nu} + \cos\phi\left[2\omega_E\cos\zeta + \frac{\omega_E^2r}{\nu}(\cos\phi\cos\gamma + \sin\phi\sin\gamma\sin\zeta)\right]$$
(30)

$$\dot{v} = \frac{T\cos\alpha_T}{mv} - \frac{\mu_E}{r^2}\sin\gamma - \frac{D}{m} + \omega_E^2 r\cos\phi(\cos\phi\sin\gamma - \sin\phi\cos\gamma\sin\zeta)$$
(31)

$$\dot{\zeta} = -\frac{v}{r} \tan\phi \cos\gamma \cos\zeta + 2\omega_E \cos\phi \tan\gamma \sin\zeta - \frac{\omega_E^2 r}{v\cos\gamma} \sin\phi \cos\phi \cos\zeta - 2\omega_E \sin\phi$$
(32)

where

 α_T - angle of attack (deg)

 ω_E - Earth rotation (rad/s)

Eqs. (26-28) are the kinematical equations of motion and Eqs. (30-32) are the dynamic equations. With the integration of the system of differential equations, the vector position and the vector velocity of the vehicle can be determined by the following equations

$$\mathbf{r}(r,\phi,\xi) = r(\cos\phi\cos\xi\mathbf{I} + \cos\phi\sin\xi\mathbf{J} + \sin\phi\mathbf{K})$$
(33)

$$\mathbf{v}(v, \gamma, \zeta) = v(\sin\gamma \mathbf{i} + \cos\gamma\sin\zeta \mathbf{j} + \cos\gamma\cos\zeta \mathbf{k})$$
(34)

It's known that if one have a inertial vector position and a velocity vector of a given body in orbit, the orbital elements (or Keplerian elements) can be readily determined. Thus to get the orbital elements, it's necessary to perform some matrix rotation to obtain the desired inertial vectors.

2.4.1 Optimization

In order to obtain the maximal payload capacity of a given launch vehicle, and consequently, make the access to space cheaper, trajectory optimization techniques has been for decades a subject of intense research. The trajectory optimization can be categorized basically into direct and indirect methods. In the referred papers Betts¹ and Rao¹² was made a comprehensive survey about both methods. In the direct method, the problem is characterized by a set of parameters which define the control law. Perhaps the most popular software representing this category is the POST (Program to Optimize Simulated Trajectories).² This problem is a typical Non Linear Programming Problem (NLP) and can be solved using classical Gradient-based methods (deterministic methods) such as Sequential Quadratic Program (SQP) or by heuristic methods. Presumably because of the possibility of solving very complex problems with a minimum effort of mathematical analysis, this method is preferred for most of the researches and will be also considered in this paper.

Methodology

The method applied within the framework of this work is based on the Master thesis of Silva.¹⁵ Here a polynomial control function is used to model the flight profile. Four parameters are optimized in order to get the minimum payload mass which are the coast time duration and three parameters of the polynomial control function. A code from a NASA report⁷ written in FORTRAN is transcript here to C++ language and used to solve the problem.

$$\beta = \begin{cases} \pi/180, & \text{if } t \le t_v, \\ b_0 - b_1(t - t_v) + b_2(t - t_v)^2, & \text{if } t_v < t \le t_{f1}. \\ b_3 - b_4(t - t_{tf1}) + b_5(t - t_{f1})^2, & \text{if } t_{f1} < t \le t_{bf}. \end{cases}$$

where

$$b_{0} = \frac{\pi}{2}$$

$$b_{1} = \frac{\beta_{1}\pi}{180(t_{f1}-t_{v})}$$

$$b_{2} = -\frac{b_{0}-\beta_{2}\pi/180-b_{1}(t_{f1}-t_{v})}{(t_{f1}-t_{v})^{2}}$$

$$b_{3} = \frac{\beta_{2}\pi}{180}$$

$$b_{4} = \frac{\beta_{3}\pi}{180(t_{bf}-t_{f1})}$$

$$b_{5} = \frac{b_{4}-b_{5}(t_{bf}-t_{f1})}{(t_{c}-t_{c}+v^{2})^{2}}$$

And the parameters t_v , t_{f1} and t_{bf} are, respectively, the vertical flight time, first stage burn time and overall flight time. Vertical lift-off is necessary for safety issues, i.e., the launch vehicle must follow a vertical ascent for a few seconds until the vehicle is safely away from the launch pad. The parameters β_1 , β_2 and β_3 are the set of optimization control parameters. The optimization programming problem can be then stated as follows

Find
$$\mathbf{X} = \begin{bmatrix} t_{coast} \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$
 which maximize the payload mass $F(\mathbf{X}) = m_p$

subject to the constraints at orbit injection

$$r = R_e + h_f \tag{35}$$

$$\gamma = 0 \tag{36}$$

$$v_I = \sqrt{v^2 + (\omega_E r \cos \phi)^2 + 2v\omega_E r \cos \phi \cos \gamma \cos \zeta} = \sqrt{\frac{\mu}{r}}$$
(37)

To avoid unrealistic rocket flight simulation and to preserve the payload integrity or crew safety in case of manned missions some path constraints should be included: dynamic pressure, bending load, axial acceleration, heat flux and angle of attack.

3. Program Structure

In this section is presented the structure of the programming tool. As aforementioned stated, a modular approach using object-oriented programming (OOP) is chosen and to allow a better visualization of the codes it is used UML diagrams. The UML is used to visualize the code and the communication between objects enabling a high degree of abstraction. In Figures 3 and 4 are presented UML diagrams, respectively, for a liquid rocket engine and a launch vehicle. The configuration of these diagrams were conveniently chosen to represent the L75 rocket engine and the VLS-alfa. From the diagrams we can see some parameters and functions of each component and the relationship between them. In order to make the diagrams clear, some parameters and functions are omitted. The rocket engine is compound of objects of the following components (classes): *Turbompump*, *ThrustChamber*, *Valves*, *GasGenerator* and *LiquidPropellant*. These objects together with specific impulse, thrust force, mixture ratio and pressure drops in the lines form the parameters of the engine. In Figure 4 it is showed which parameters define the launch vehicle and the interactions with the environment.



Figure 3: UML - Liquid Rocket Engine Model

4. Results

This section presents the simulation of the future brazilian launch vehicle VLS-alfa and the the analysis of the effect of changes in parameters of the L75 rocket engine on the stage mass, engine mass and launcher performance.



Figure 4: UML - Launch Vehicle Model

4.1 Flight Simulation

It is known that the VLS-alfa will replace the last two stages of the former VLS by a single liquid upper stage. Then since the VLS-alfa is an improvement of the former VLS, we will take the VLS as a reference vehicle and both vehicle will be simulated. The upper stage of the VLS-alfa presumably will perform a coast phase, so the L75 is supposed to support restart capability. The mission is to launch a satellite into a reference circular orbit of 500 km of altitude from the Alcântara Launch Center (2°22'39.52"S, 44°23'57.71"W). The parameters of the vehicles are given in Tables 1 and 2.

VLS	m_p (kg)	m_s (kg)	I_{sp} (m/s)	t_b (s)	C_d
1st Stage	28900	6200	257.9	62.826	3.82
2nd Stage	7140	1680	279.1	62.087	1.6552
3rd Stage	4370	1330	270.74	58.267	0.0
4th Stage	820	170	281.85	74.546	0.0

Table 1. Date: brazilian lounab vahiala VI S

Table 2: Data: brazilian launch vehicle VLS-Alfa

VLS-alfa	m_p (kg)	m_s (kg)	I_{sp} (m/s)	t_b (s)	C_d
1st Stage	28900	6200	257.9	62.826	3.82
2nd Stage	7140	1680	279.1	62.087	1.6552
3rd Stage (before costing)	5800	987.422	315.0	243.657	0.0
3rd Stage (after coasting)	1100	987.422	315.0	46.219	0.0

As the mission of the VLS-alfa is still not totally defined, the propellant mass of the upper stage had to be estimated. Thus, an amount of 6900 kg was conveniently defined. From this value was taken an amount of 1100 kg for the phase after coasting, i.e, for orbit injection. In Figure 5 the altitude and relative velocity profiles for both vehicles are presented and in Figure 6 we can see the ground track of the launch vehicle VLS-alfa.



Figure 5: Relative velocity and altitude profile



Figure 6: Ground track for VLS-alfa

4.2 Influence of Engine Parameters on the Launcher Performance

To study the influence of mixture ratio and chamber pressure on the payload mass the propellant mass is maintained constant. Thus we can choose between two approaches, i.e., overall thrust force fixed or overall propellant mass flow fixed. If we choose to fix the thrust force, then the burn time of the stage will vary and the simulation of the set of equations from Section 2.3 will be performed with Equation 19. In the second approach, we use Equation 20 to simulate the system of equations and we have a variable thrust but a constant burn time of the propellant of the stage. The nominal parameters of the L75 are given in Figure 7. Considering the first approach, the influence of the engine parameters on the launcher performance can be seen in the payload variation in Figure 8. From the figure we can see that the nominal parameters r_c and p_c could be changed to obtain a better payload mass. Although the payload gain seems to be not so significant, just a few kilograms heavier, since the engine is still in development, an adjust in the design parameters should be considered. In Figures 9 and 10 is presented the influence of mixture ratio and chamber pressure on the engine and stage mass.



Figure 7: L75 - Data



Figure 8: Payload Mass x Engine Parameters

5. Conclusions

This paper presented a modeling of engine cycle and a launch vehicle using object-oriented programming. It was presented models for engine components and stage mass, engine performance and ascent trajectory of the launcher. To allow a better visualization of the codes, the Unified Modeling Language (UML) was used. The future brazilian



Figure 9: Engine Mass x Engine Parameters



Figure 10: Stage Dry Mass x Engine Parameters

launch vehicle VLS-alfa was used as case study and since it is an improvement of the former VLS, the latter was used as a reference vehicle to simulate the ascent trajectory. To assess the influence of the engine parameters on the vehicle performance, the L75 engine was simulated for different values of mixture ratio and chamber pressure. The results showed that the original design parameters can be changed to achieve a better performance, i.e., a heavier payload could be obtained.

References

- [1] BETTS, J. T. Survey of numerical methods for trajectory optimization. *Journal of Guidance, Control, and Dynamics* 21, 2 (1999), 193–207.
- [2] BRAUER, G. L., CORNICK, D. E., AND STEVENSON, R. Capabilities and applications of the program to optimize simulated trajectories (POST). Tech. rep., NASA CR 270, 1977.
- [3] CASTELLINI, F. *Multidisciplinary Design Optimization for Expendable Launch Vehicles*. PhD thesis, Politecnico Di Milano, Milano, 2012.
- [4] ERNST, R. Liquid rocket analysis (lira) development of a liquid bi-propellant rocket engine design, analysis and optimization tool. Master thesis, Delft University of Technology, Holand, 2014.
- [5] FELBER, R. Einige biträge zur optimierung von chemischen raketen triebwerken.
- [6] HUMBLE, R. W., HENRY, G. N., AND LARSON, W. J. Space Propulsion Analysis and Design. McGrawHill, New York, 1995.
- [7] JACOB, H. G. An engineering optimization method with application to stol-aircraft approach and landing trajectories. Tech. rep., Ames Research Center, 1972.

- [8] MANSKI, D., GOERTZ, C., AND H., S. Cycles for earth-to-orbit propulsion. Journal of Propulsion and Power 14, 5 (1998), 588–604.
- [9] MANSKI, D., AND MARTIN, J. A. Optimization of the propulsion cycles for advanced shuttles part 2: Performance model methodology. AIAA/SAE/ASME/ASEE 26th Joint Propulsion Conference (1990).
- [10] MANSKI, D., AND MARTIN, J. A. Evaluation of innovative rocket engines for single-stage earth-to-orbit vehicles. *Journal of Propulsion* 7, 6 (1991), 929–937.
- [11] MCBRIDE, B. J., AND S., G. Computer program for calculation of complex chemical equilibrium compositions and applications. Tech. rep., NASA, 1996.
- [12] RAO, A. V. AAS/AIAA a survey of numerical methods for optimal control. *Journal of Guidance, Control, and Dynamics* (2009).
- [13] SCHLINGLOFF, H. Astronautical Engineering. Ingenieurbüro Dr. Schlingloff Publications, Bad Abach, 2005.
- [14] SCHMUCKER, R. H. A simple performance calculation method for LH2/LOX engines with different power cycles. Tech. rep., Marshall Space Flight Center - NASA TM X-64749, Alabama, 1973.
- [15] SILVA, C. S. C. Simulação de sistemas de motores foguetes a propelentes líquidos. Master thesis, Instituto Tecnonológico de Aeronáutica, São José dos Campos, Brazil, 1995.
- [16] TEWARI, A. Atmospheric and Spaceflight Dynamics. Birkhäuser, Boston, 2007.
- [17] ZANDBERGEN, B. Simple mass and size estimation relationships of pump fed rocket engines. Tech. rep., Delft University of Technology, 2013.