SECOND ORDER CORRECTION ON ELECTRON CONTENT MEASUREMENTS WITH FARADAY ROTATION TECHNIQUE*

by

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Determination of the total electron content ($I = \int Ndh$) of the ionosphere by means of the application of Faraday rotation technique on signals received from satellite beacons has been performed since the launching of Sputnik I, by many investigators, see for instance the list of references in the paper by Garriott and Mendonça[1]. Most computations utilizing Faraday techniques has been done using single frequencies and lately two closely spaced frequencies such as the ones transmitted by the beacon satellites BE-B and BE-C in 40 and 41 MHz (see Fig. 1).

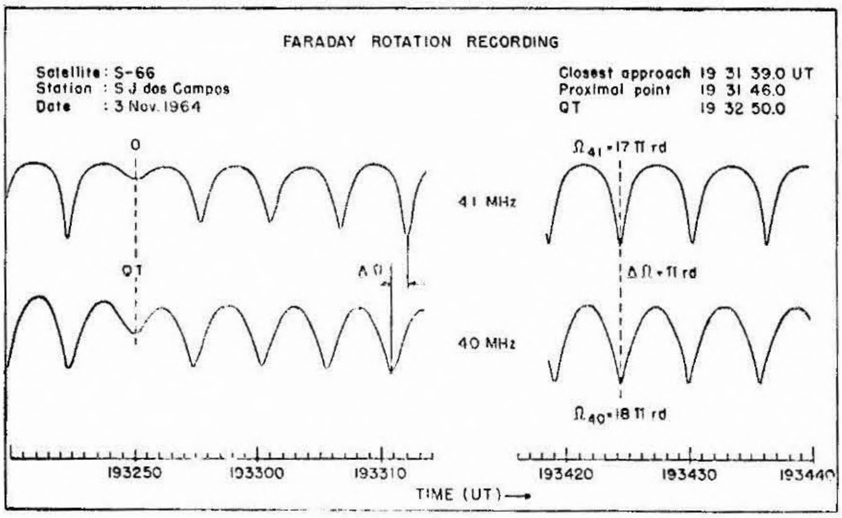

Fig. 1 — Recording of the Faraday rotation of polarization in the 40 and 41 MHz beacons of S-66, showing the quasi transverse (QT) propagation region (Ω = O) and a case with Ω_{40} = 18 $\Delta\Omega$.

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The first order relation

$$\Omega = (K/f^2)\,(H \cos\theta \sec\chi)^{o}\,I, \tag{1}$$

which is applicable only in restricted cases, including the absence of horizontal gradients, has been widely used for the determination of I. Ross[2] introduced a second order correction obtaining the relation

$$\Omega = (K/f^2)\,(H \cos\theta \sec\chi)\,(1 + \alpha)\,I_2 \tag{2}$$

where

$$\alpha = (X^*/2)\,[\beta + (\beta - 1)\,G] \tag{3}$$

The factor $G = \tan\theta\,(\tan\theta - Y/Y_L)$ is related to the geometry of the geomagnetic field (Fig. 2).

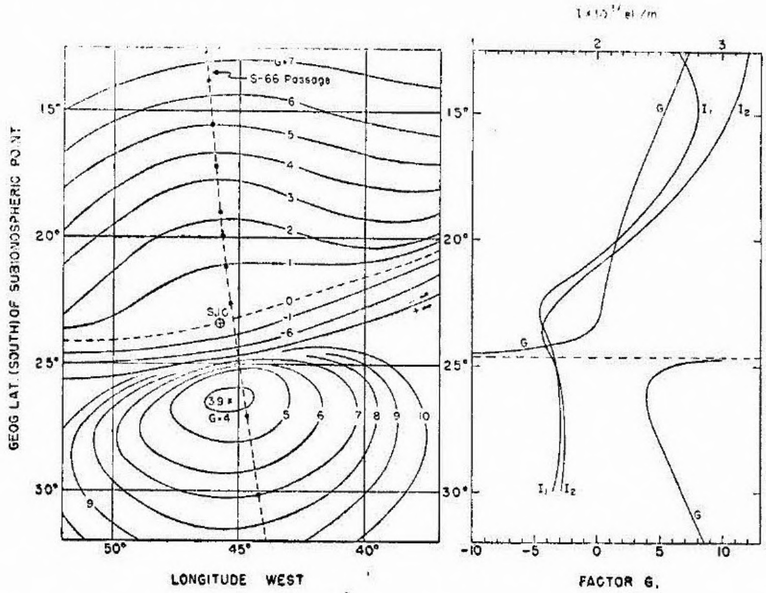

Fig. 2 — Plot of the factor G for ionospheric points at h = 350 km for the São José dos Campos station, including the track of a southbound passage of S-66 on 16 Oct. 1964 with QT at 215435 UT. Also shown are the values of G, I_1 (First order) and I_2 (Second order) for this particular passage of the satellite.

For the case of two closely spaced frequencies one has:

$$d\Omega \doteq - 2\Omega \, (df/f) \, (1 + 2\,\alpha) \tag{4}$$

or

$$\triangle\Omega = 2\Omega \, (n\text{-}1) \, (1 + 2\alpha)$$

Thus one may obtain the correction term $(1 + \alpha)$ from the measured values of Ω and $\triangle\Omega$:

$$\alpha = [1 - 2\,(n\text{-}1)\,\Omega/\triangle\Omega] \;/\; [4(n\text{-}1)\Omega/\triangle\Omega] \tag{5}$$

Combining equations (1), (2) and (5) we have

$$I_2 = (f^2\Omega/KM) \; 4\,\Omega\,(n\text{-}1)/[\triangle\Omega + 2\Omega(n\text{-}1)] \tag{6}$$

Hence, with precalculated values of M one can easily obtain values of electron content with the second order correction. The values of α obtained with this procedure (Fig. 3) are naturally much closer to reality than the ones calculated with models and equation (3). We have written a program for our small computer in which we feed the values of the satellite position, Ω, $\triangle\Omega$ and height of the ionospheric point, and obtain the output values of I_1 and I_2 for the sub-ionospheric point, including dip angle, the factor M and G. A few passages are plotted in Fig. 4.

Note that if $n = f_2/f_1 = 1.025$ in equation (5) we get

$$(1 + \alpha) = 0.500 + 10.0 \; \triangle\Omega/\Omega$$

and that $\alpha = 0$ when $\Omega = 20\,\triangle\Omega$. In this situation one has $I_1 = I_2$
At our low latitude station we have observed extreme cases in which a variation of π rd in $\triangle\Omega$ corresponded to a variation of only 6 π rd in Ω. The first order results are such that the over estimation in I for some areas tend to cancel the variation of electron content through the equatorial anomaly. Thus one should be cautious in drawing conclusions from the first order results. The full paper to be published will include comparisons between results obtained with Faraday and Doppler methods.

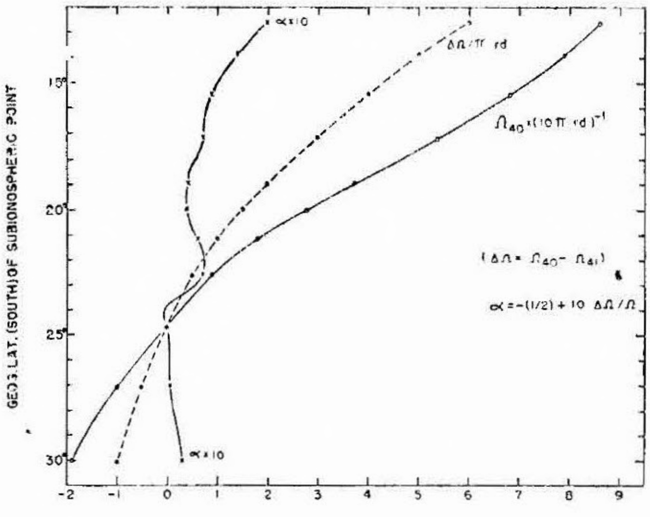

Fig. 3 — Plot of the values of α, $\triangle\Omega$ and Ω for the satellite passage of Fig. 2. Note that when $\triangle\Omega$ varies by π rd from 5 to 6 π rd, Ω varies only by 7 π rd from 79 to 86 π rd.

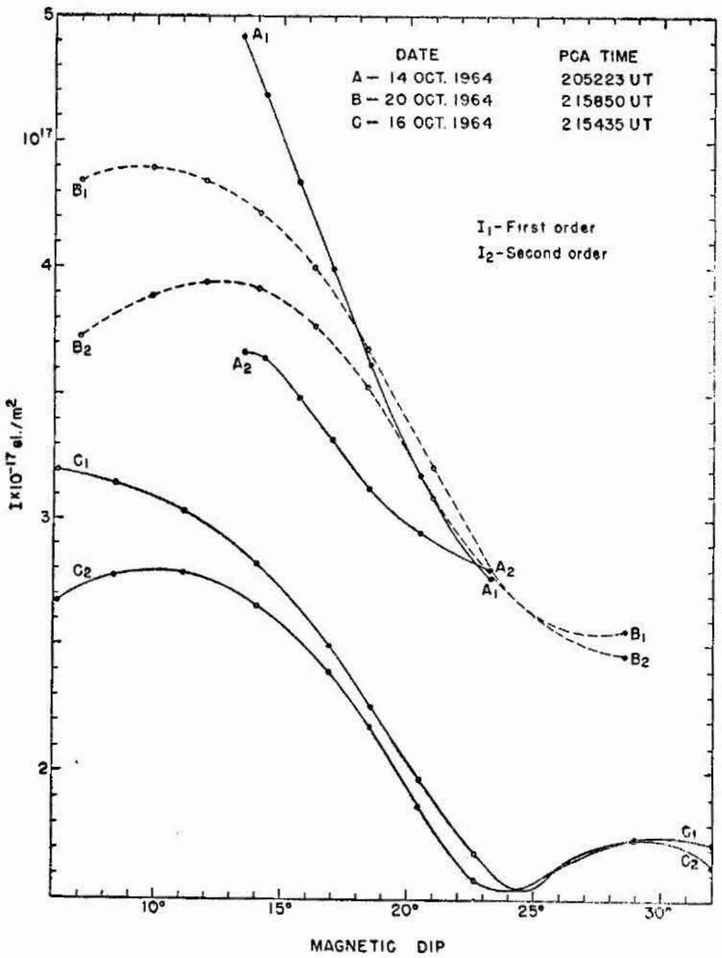

Fig. 4 — Plots of I_1 and I_2 for a few passages of the satellite S-66 (BE-B). Note the parts where $I_1 \approx I_2$, i.e., $\alpha \approx 0$ and also the tendency of over estimation in I_1 masking the equatorial anomaly in I.

Reference:

1) Garriott, O.K. and F. de Mendonça, J. Geophys. Res., **68**, 4917, Sept. 1963.

2) Ross, W., J. Geophys. Res., **70**, 597, Feb. 1965.