A Comparison of Methods Used for Obtaining Electron Content from Satellite Observations

O. K. Garriott

Radioscience Laboratory, Stanford University, Stanford, California

F. de Mendonça

Comissão Nacional de Atividades Espaciais
São José dos Campos, São Paulo, Brazil

Abstract. Measurements of the effects of the ionosphere on the polarization and the Doppler shift of radio transmissions from earth satellites permit the electron content of the ionosphere to be calculated. Thirty-five passages of Transit 2A have been analyzed in a variety of ways to estimate the accuracy of the several methods. The most accurate method is a hybrid analysis using both Faraday and Doppler data simultaneously. Methods based on the rate of polarization rotation, on the number of rotations between two times, and on best-fitting polynomials to either the Faraday or the Doppler data are compared and their errors estimated.

1. Introduction

The radio transmissions of orbiting earth satellites have been found very useful in the determination of ionospheric electron content. Most authors have based their calculations on the observed polarization rotation (the Faraday effect) or on the small change in Doppler shift imposed by the ionosphere. Refraction measurements are also possible in theory [*Al'pert et al.*, 1958; *Weekes*, 1958; *Titheridge*, 1961], but they have not yet found widespread application.

Various methods have been used in the analysis of satellite recordings, and different assumptions are necessary in each. In this paper, 35 passages of Transit 2A are analyzed in a number of different ways. The results of each method are then compared for internal consistency or with other methods in order to estimate their individual accuracies. A similar procedure has been followed by *Burgess* [1963], who has compared several methods of analysis and has shown the results of his comparisons for one passage of Transit 4A. In section 2 are described the methods to be used in this paper: (1) two methods based on the observed rate of polarization rotation; (2) a 'rotation angle' method, obtained by measuring the change in the angle of polarization rotation between two times; (3) several methods using only differential Doppler data or only Faraday rotation data, which will obtain polynomial expressions for the electron content as a function of time with minimum mean-square error; (4) an independent method based on Doppler data alone; and (5) a hybrid Faraday-Doppler technique using both types of observational data. Finally, in section 3 these techniques are applied to the 35 passages of Transit 2A, and the results are considered.

The satellite records were obtained between July 23 and October 13, 1960, at Stanford University. The received 54-Mc/s signal was compared with a subharmonic of the received 324-Mc/s signal (using phase-locked receivers), and the small effect of the ionosphere was determined. Polarization rotation was measured by the amplitude fluctuation of the 54-Mc/s signal. These two types of observations and the satellite ephemeris provided by the Applied Physics Laboratory of the Johns Hopkins University are the basic input data necessary to calculate electron content.

2. Analysis Procedures

A. *Rotation-rate methods.* The total angle of polarization rotation is usually expressed as

$$\Omega = (K/f^2)H \cos\theta \sec\chi \cdot I \qquad (1)$$

where

Ω = angle of polarization rotation, radians.
f = wave frequency, cycles per second.
H = magnetic field intensity, ampere-turns per meter.

θ = angle between wave normal and magnetic field.
χ = angle between ray and the vertical.
$I \equiv \int_0^{h_s} N\, dh$, electron content.
$dh = ds/\sec \chi$, where ds is an element of length along the ray.
K = a constant, equal to 2.97×10^{-2} in mks units.

The equation assumes quasi-longitudinal propagation, high frequencies, and a single ray path for both modes, but these approximations are quite satisfactory at the mentioned frequencies. It is convenient for us to abbreviate this equation as

$$\Omega = \eta M I \tag{2}$$

where $\eta = K/f^2$ and $M \equiv H \cos \theta \sec \chi$. The factor M may be computed at any point in space about the observer as has been done by *Yeh and Gonzales* [1960]. In equation 2 the value of M should be determined near the height of the centroid of the electron density profile. Therefore, in all the work to follow, M was evaluated at a level approximately 50 km above the height of maximum density, which was determined from a true height analysis of an ionogram recorded near the time of the satellite passage.

Since the speed of a satellite is much higher than the drift velocities to be expected in the ionosphere, we can consider that N (and consequently I) is a function of position only, e.g., $N = N$ (height, latitude, longitude), $I = I$ (latitude, longitude). However, owing to the motion of the satellite, the coordinates of the points along the ray path are functions of time. Thus it is possible to consider I an explicit function of time. In this paper, any reference to the time variation of I or M is to be understood in this context.

To obtain the rotation rate, equation 2 is differentiated with respect to time, giving

$$\dot{\Omega} = \eta[M\dot{I} + I\dot{M}] \tag{3}$$

in which dots imply a time derivative. The analysis of *Bowhill* [1958] shows that, for a flat earth and a uniform magnetic dip, the term $\dot{M}$ is a constant. Furthermore, when horizontal gradients in the ionosphere are neglected ($\dot{I} = 0$), the rotation rate becomes proportional to the electron content

$$I = \dot{\Omega}/\eta\dot{M} \tag{4*}$$

(Equations identified by an asterisk are those for which calculations of electron content have been made, and the calculated values are discussed in section 3.) Many authors [*Garriott*, 1960; *Hame and Stuart*, 1960; *Yeh and Swenson*, 1961] have used this expression or its near equivalent because of the ease with which the satellite observations are related to electron content. In the calculations to be shown in section 3, M was determined from a table of M values supplied by Mr. L. J. Blumle, at the Goddard Space Flight Center, and $\dot{\Omega}$ was determined by a five-point differentiation formula using the times of the Faraday nulls. Both terms are evaluated at the 'proximal point' of the passage, which will be defined more precisely in section 2C.

If the electron content is constrained to vary linearly with time, equation 3 may be differentiated to give

$$\ddot{\Omega} = \eta[2\dot{M}\dot{I} + I\ddot{M}] \tag{5}$$

Then equations 3 and 5 may be solved simultaneously (*) for values of I and $\dot{I}$ at the proximal point. This method has the considerable advantage of eliminating the restriction to a horizontally stratified ionosphere but the disadvantage of requiring the second derivative of Ω to be evaluated.

B. *Rotation-angle method.* Perhaps the most widely used method is based on the *change* in the rotation angle between two times. When horizontal stratification of the ionosphere is once again assumed, the change in the rotation angle between times t_1 and t_2 is

$$\Delta\Omega \equiv \Omega_1 - \Omega_2 = \eta(M_1 - M_2)I \tag{6*}$$

from which I can be calculated. The change $\Delta\Omega$ is π times the number of Faraday fades between t_1 and t_2. Numerous authors have used equations closely equivalent to (6), although some improvement is obtained when propagation approaches the transverse direction [*Garriott*, 1960; *Blackband*, 1960], when allowance is made for refraction and the higher-order terms in the expressions for the index of refraction [*Yeh*, 1960], and when path splitting is minimized by computer ray tracing [*Lawrence et al.*, 1963]. The principal disadvantage of this method is the neglect of horizontal gradients.

C. *Best-fitting polynomials.* If the time variation of electron content is represented by a power series, $I(t) = a_0 + a_1 t + a_2 t^2 \cdots$, the coefficients a_0, a_1, a_2 $\cdots$ may be evaluated so that the polynomial provides the least mean-square error when compared with the observed data. This method is similar to that of *de Mendonça* [1962] and has been developed more fully by *Burgess* [1963]. To obtain the coefficients several equations must be solved simultaneously, and matrix notation provides a convenient way to display the results and to formalize the method for machine computation. The method is applied to the Faraday rotation data as follows. Equation 2 may be written as

$$\Omega = \eta M(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots) \quad (7)$$

in which it should be remembered that M is also a function of time. The value of Ω at the proximal point which serves as the time reference is

$$\Omega_0 = \eta M_0 a_0 \quad (8)$$

Subtracting (8) from (7) leads to

$$(\Omega - \Omega_0)/\eta = (M - M_0)a_0 + (Mt)a_1 + (Mt^2)a_2 + (Mt^3)a_3 + \cdots \quad (9)$$

The left-hand side of (9) may be determined from observation; the right-hand side predicts a value dependent on the choices of a_0, a_1, $\cdots$. To select these coefficients, the mean-square error is first found by subtracting the right-hand side from the left-hand side, and summing up the errors at each of the n times at which equation 9 has been evaluated.

$$\epsilon = \frac{1}{n}\sum_1^n \left[\left(\frac{\Omega - \Omega_0}{\eta}\right) - (M - M_0)a_0 - (Mt)a_1 - \cdots\right]^2 \quad (10)$$

For least error, the coefficients are determined by simultaneous solutions of the set of equations obtained from

$$\frac{\partial \epsilon}{\partial a_0} = \frac{\partial \epsilon}{\partial a_1} = \frac{\partial \epsilon}{\partial a_2} = \cdots = 0 \quad (11)$$

This set of equations can be represented in matrix form by

$$[\mathbf{F}][\mathrm{a}] = [\mathrm{g}] \quad (12)$$

with

$$[\mathbf{F}] = \begin{bmatrix} \Sigma(M - M_0)^2 & \Sigma M(M - M_0)t & \Sigma M(M - M_0)t^2 & \cdots \\ \Sigma M(M - M_0)t & \Sigma M^2 t^2 & \Sigma M^2 t^3 & \cdots \\ \Sigma M(M - M_0)t^2 & \Sigma M^2 t^3 & \Sigma M^2 t^4 & \cdots \\ \vdots & \vdots & \vdots & \cdots \\ & & & \cdots \end{bmatrix} \quad (13a)$$

$$[\mathrm{g}] = \begin{bmatrix} \Sigma(\Omega - \Omega_0)(M - M_0)/\eta \\ \Sigma(\Omega - \Omega_0)(Mt)/\eta \\ \Sigma(\Omega - \Omega_0)(Mt^2)/\eta \\ \vdots \end{bmatrix} \quad (13b)$$

The unknown coefficients are obtained by multiplying both members of equation 12 by the inverse of matrix [**F**], giving

$$[\mathrm{a}] = [\mathbf{F}]^{-1}[\mathrm{g}] \quad (14)^*$$

In the calculations described in section 3, slightly different matrices [**F**] and [g] were used to reduce the tendency of $[\mathbf{F}]^{-1}$ to approach singularity. The altered matrices were obtained by multiplying each term of equation 9 by $\sqrt{t/M}$, then proceeding through equations 10 to 14. Values of [a] were obtained for three power series, e.g., third-, second-, and first-degree polynomials with all higher powers set equal to zero.

A similar procedure may be followed for the evaluation of the Doppler observations. It can be shown readily that the reduction in phase path length due to propagation through the ionosphere is related to the electron content by

$$\Delta P = (40.3/f^2) \sec \chi \cdot I \quad \text{meters} \quad (15)$$

Again, χ should be evaluated near the centroid of the electron distribution, and we have used a height about 50 km greater than the height of maximum density in all cases. Dividing through by the free space wavelength λ and expressing

I as a polynomial, we obtain

$$(\Delta P/\lambda) = (40.3/cf) \cdot \sec \chi (b_0 + b_1 t + b_2 t^2 + \cdots) \qquad (16)$$

We may now define precisely the proximal point (which determines our time reference) as the point along the satellite path at which the phase path defect ΔP given by equation 15 is a minimum. This point is usually quite obvious on the record showing the beat (differential Doppler) between the 54-Mc/s signal and the (1/6) subharmonic of the 324-Mc/s signal. At the proximal point the beat frequency will go to zero and the relative phase shift of two signals will reverse its direction. This point is usually quite near the point of minimum geometrical range, but either a vertical component in the satellite velocity or horizontal ionospheric gradients may shift the two points apart by 30 seconds or even more in time. At the proximal point

$$(\Delta P/\lambda) = (\Delta P_0/\lambda) = (40.3/cf) \sec \chi_0 b_0 \quad (17)$$

Proceeding as before, (17) is subtracted from (16), giving

$$\frac{(\Delta P - \Delta P_0)}{\lambda} = \frac{(40.3)}{cf} [(\sec \chi - \sec \chi_0) b_0 + \sec \chi b_1 t + \cdots] \qquad (18)$$

The left-hand side of (18) may be determined from the satellite observations. Each cycle of phase change measured from the proximal point contributes 1 wavelength to equation 18 (a small correction must be included to account for the phase path change at 324 Mc/s).

The values of b_0, b_1, $\cdots$, are to be selected so that the right-hand side of (18) agrees as closely as possible with the observations. It is convenient to multiply both sides of (18) by $\cos \chi$ and then to make the substitutions $\beta = (cf)/40.3$ and $\varphi \equiv (\cos \chi_0 - \cos \chi)/\cos \chi_0$. Then the mean-square error is

$$\epsilon = \frac{1}{n} \sum_1^n \left\{ \left(\frac{\Delta P - \Delta P_0}{\lambda} \right) \beta \cos \chi - \varphi b_0 - b_1 t - b_2 t^2 - \cdots \right\}^2 \qquad (19)$$

A set of linear equations is obtained by forming the partial derivatives of (19), summarized as

$$[\mathbf{D}][\mathbf{b}] = [\mathbf{h}] \qquad (20)$$

in which

$$[\mathbf{D}] = \begin{bmatrix} \Sigma\varphi^2 & \Sigma\varphi t & \Sigma\varphi t^2 & \cdots \\ \Sigma\varphi t & \Sigma t^2 & \Sigma t^3 & \cdots \\ \Sigma\varphi t^2 & \Sigma t^3 & \Sigma t^4 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

$$[\mathbf{h}] = \begin{bmatrix} \Sigma\left(\frac{\Delta P - \Delta P_0}{\lambda}\right)\beta(\cos \chi)\varphi \\ \Sigma\left(\frac{\Delta P - \Delta P_0}{\lambda}\right)\beta(\cos \chi)t \\ \Sigma\left(\frac{\Delta P - \Delta P_0}{\lambda}\right)\beta(\cos \chi)t^2 \\ \vdots \end{bmatrix}$$

As with the Faraday data, solutions for

$$[\mathbf{b}] = [\mathbf{D}]^{-1}[\mathbf{h}] \qquad (21)^*$$

were obtained for third, second, and first polynomials.

D. *Doppler method.* This method, like the one just described, uses only the differential Doppler data; it is approximately the same as that used by *de Mendonça* [1962]. The electron content is assumed to vary linearly with time ($b_2 = b_3 = \cdots = 0$), and the value of $(dI/dt) = b_1$ is obtained by calculating the *change* in electron content between two times. After $\dot{I}$ has been established, the best-fitting value of I_0 is determined by minimizing the mean-square error. The necessary equations are obtained below.

Two times (t_j and t_k) are established at which the zenith angles (χ) of the satellite are the same. (In the calculations shown in section 3 we have arbitrarily selected these times approximately ± 50 seconds on either side of the time of minimum χ.) Equation 16 is then written for each of these times, and their difference is taken.

$$\left(\frac{\Delta P_j - \Delta P_k}{\lambda}\right) = \left(\frac{\sec \chi}{\beta}\right) b_1 (t_j - t_k) \qquad (22)$$

The left-hand side is the number of cycles of phase shift between t_j and t_k (corrected for the ionospheric effect at 324 Mc/s). From this measurement b_1 can be calculated. The value of b_1 should be expected to fluctuate somewhat, depending on the time interval chosen, owing to the irregularities in the ionosphere and to the higher-order terms in $I(t)$ that were neglected.

To evaluate b_0 we find the mean-square error to be

$$\epsilon = \frac{1}{n}\sum_1^n\left\{\left(\frac{\Delta P - \Delta P_0}{\lambda}\right)\beta \cos\chi_0 - \left(\frac{\cos\chi_0}{\cos\chi} - 1\right)b_0 - \left(\frac{\cos\chi_0}{\cos\chi}\right)b_1 t\right\}^2 \quad (23)$$

Forming $(\partial\epsilon/\partial b_0) = 0$, we solve for b_0 as

$$b_0 = \frac{\Sigma\dfrac{(\Delta P - \Delta P_0)\beta \cos\chi_0}{\lambda}\left(\dfrac{\cos\chi_0}{\cos\chi} - 1\right) - \Sigma\left(\dfrac{\cos\chi_0}{\cos\chi}\right)\left(\dfrac{\cos\chi_0}{\cos\chi} - 1\right)b_1 t}{\Sigma\left(1 - \dfrac{\cos\chi_0}{\cos\chi}\right)^2} \quad (24)^*$$

E. *Hybrid Faraday-Doppler method.* This method uses both types of data simultaneously and has the great advantage that it is not necessary to assume horizontal stratification of the ionosphere. It was first described by *Burgess* [1962] and was improved upon by *Golton* [1962]. A somewhat more general approach was made by *de Mendonça and Garriott* [1962] in analyzing the same data that are discussed in this paper. Reference may be made to these papers for a detailed description of the methods. In applying this method no assumptions are necessary that are likely to introduce an error greater than a few per cent in the final value of electron content.

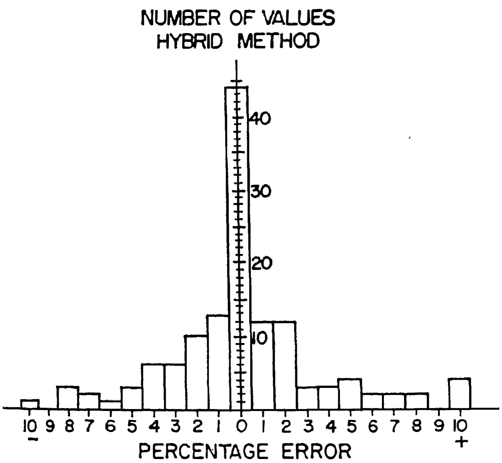

Fig. 1. The percentage error for each calculation made with the hybrid Faraday-Doppler equations. The 'correct' value of electron content was assumed to be near the mean of all measurements made on an individual passage, and the percentage error was calculated from this value. The number values in each 1 per cent interval are shown in the histogram, which illustrates the internal consistency of the method.

3. Calculations of Electron Content

Since the hybrid method is expected to be the most accurate of the analyses described above, it will be examined first for internal consistency by performing the calculations based on different sections of the data in a single satellite passage. These 'subsets' were usually spaced about ± 1, ± 2, and ± 3 minutes on either side of the proximal point; in a number of other cases the data were located asymmetrically about the proximal point. From each subset, the value of electron content *at the proximal point* was calculated. From the group of answers so obtained pertaining to a single passage, a value near the average of the group was arbitrarily assigned as the 'correct' value of electron content at the proximal point. Then the per cent deviation of each value was calculated from the assumed correct value of electron content. This procedure was repeated for each passage considered. The histogram showing the number of values in each 1 per cent error interval is presented in Figure 1.

Clearly, the method shows very good internal consistency in that the calculated value of electron content at the proximal point is largely independent of the data segment used in the calculation. More than 80 per cent of the values lie within ± 4 per cent of the correct value. When values obtained from asymmetric data sets are considered separately, the results are nearly the same as shown in Figure 1, although occasionally a larger deviation is encountered. Excluding these sets, more than 85 per cent of the values fall within ± 4 per cent of the correct value.

The same assumed correct values will now be used to estimate the accuracy of the other

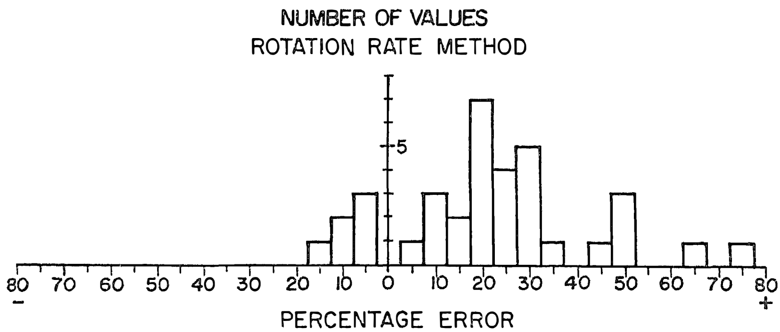

Fig. 2. The number of values in each 5 per cent error interval for the rotation-rate method of equation 4. One value was obtained for each of the 35 passages. The error was measured from the assumed 'correct' value previously obtained.

methods. It should be noted that, in all the comparisons to follow, the electron content has been evaluated at the same time in each satellite passage, e.g., the proximal point. The error in each of the two rotation-rate methods (section 2A) has been calculated. Figure 2 is a histogram showing the number of values in each 5 per cent error interval for the case in which horizontal gradients are neglected (equation 4). The figure shows that a systematic error is involved in the use of (4) so that the average of the values is about 20 per cent too high. The ionosphere above Stanford usually exhibits a decreasing electron density toward the north, and a gradient in this direction should result in an overestimation of electron content when the gradient is ignored, as the calculations have confirmed. The distribution is much broader as well, only 65 per cent of the values lying within ±15 per cent of the mean.

It might be expected that calculations made by solving equations 3 and 5 simultaneously would be appreciably better than those shown in Figure 2, since a gradient in the electron content would be allowed. Actually, they are so much worse that they are not even presented in

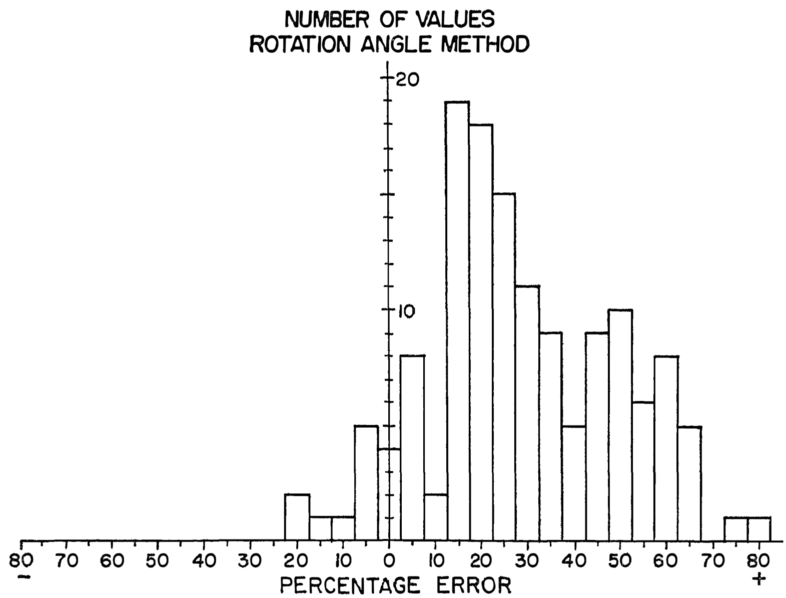

Fig. 3. The number of values in each 5 per cent error interval for the rotation-angle method of equation 6. A greater total number of values is obtained than was shown in Figure 2, because each passage yields several estimates of electron content.

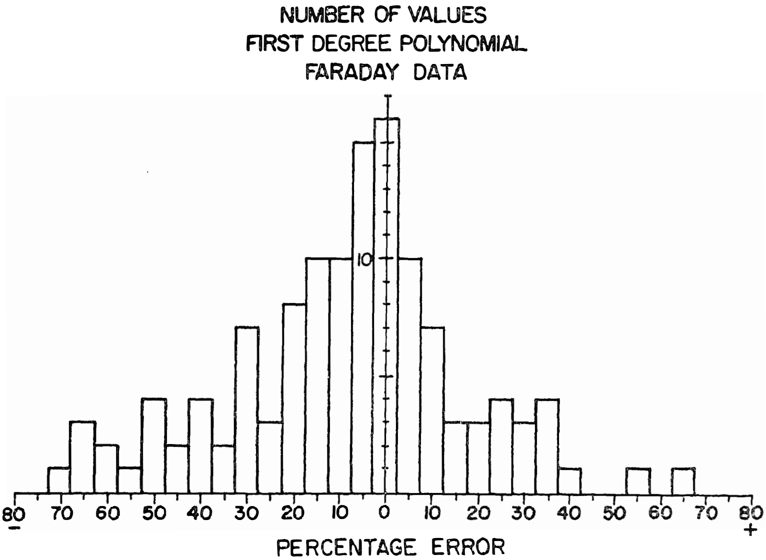

Fig. 4. The number of values in each 5 per cent error interval based on a best-fitting linear polynomial obtained with the Faraday data alone (14).

a figure. The difficulty seems to lie in the determination of $\ddot{\Omega}$. Five-point differentiation formulas were used to obtain $\dot{\Omega}$ and then again to find $\ddot{\Omega}$, and it is likely that some improvement could be achieved in a more refined analysis. Some error is certainly involved in scaling the times of the Faraday nulls, and the error is amplified by the double differentiation. Perhaps even more important are the large-scale irregularities in the ionosphere [*Little and Lawrence*, 1960]. They slightly shift the regular period of the Faraday fading and thereby contribute to the very erratic estimates of $\ddot{\Omega}$. These results suggest that little success can be expected from calculations requiring an accurate estimate of $\ddot{\Omega}$.

The results of the rotation-angle calculation (section 2B, equation 6) when compared with the correct values of electron content obtained by the hybrid method are shown in Figure 3. A larger total number of values is shown here and in most of the following figures than is shown in Figure 2, because each satellite passage contributes three or four values, one for each data subset used in the calculations. Again, the values appear to be too high by about 25 per cent and have appreciable scatter. Only 50 per cent of the results are within ±10 per cent of the mean, and only 65 per cent are within ±20 per cent of the mean value. The overestimation is consistent with a decreasing electron content toward the north as in the rotation-rate analysis. Some improvement should be expected both in the positive bias and in the scatter of the results when the additional precautions that some authors have used are incorporated (section 2B). However, the method clearly suffers from the necessity of neglecting horizontal gradients.

Next, the results of the polynomial methods will be described. Figure 4 shows the number of values in each 5 per cent error interval based on the Faraday fading data alone when the electron content is limited to a linear time variation, e.g., $I = a_0 + a_1 t$. It is observed that nearly all the systematic error has been removed and that the scatter of the results is comparable with that shown in Figures 2 and 3. About 60 per cent of the data are within ±15 per cent of the mean. This method therefore appears superior to the other, much more widely adopted, Faraday analysis techniques. However, when the polynomial expression for I is allowed to have third- or even second-degree terms, the errors become very large and the results are quite useless. Although this may to some extent be due to scaling inaccuracies and relatively few data points (only 10 to 30 Faraday nulls, usually), ionospheric irregularities too are believed to play a major part in the wide scatter of the results.

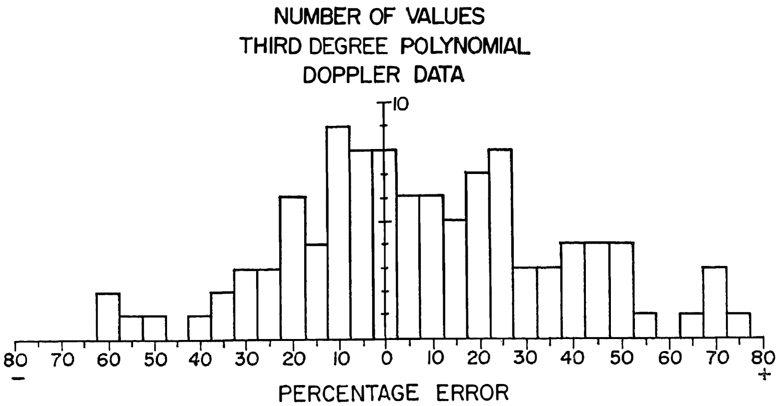

Fig. 5*a*. A histogram similar to that in the previous figures, obtained from a polynomial of the form $I = b_0 + b_1t + b_2t^2 + b_3t^3$, using differential Doppler data (equation 21).

Section 2C also described the method of best fitting a polynomial to the differential Doppler data. For these calculations the number of beats or wavelengths of phase change was tabulated approximately every 5 or 10 seconds. Thus, more data points were involved in the summations representing the elements of the matrices in equation 21 than were involved in the Faraday method, equation 14. Figures 5*a*, *b*, and *c* show the number of values in each 5 per cent error interval for polynomials of third, second, and first degree, respectively. A small positive systematic error occurs in all the figures, but only 5 or 10 per cent. The scatter of the data consistently improves as the degree of the polynomial is reduced. In Figure 5*c* almost 80 per cent of the data fall within ±15 per cent of the mean. Just as with the Faraday data, too many degrees of freedom in $I(t)$ degrades the accuracy with which the electron content is determined. When the values obtained from data sets located asymmetrically about the proximal point are plotted by themselves, the results are almost identical to Figure 5*c*.

An additional Doppler method was described in section 3D, and an expression for electron content was obtained in equation 24. The results of this method are shown in Figure 6. Just as in the preceding method, there is a positive bias of 5 or 10 per cent, and about 70 per cent of the data fall within ±10 per cent of the mean. These last two methods should correlate very closely, since they both permit a linear time variation of electron content only. Their values, compared for each passage, differ by an average of only 1 per cent, and they have a rms deviation of only 3 per cent.

4. Discussion

It has not been possible to verify the accuracy of the hybrid method directly, because no more accurate methods are available with which to compare it, but its internal consistency has been demonstrated and the lack of unrealistic approximations encourages considerable confidence.

The most commonly employed methods of reducing Faraday data (rotation rate and rotation angle) have been known to suffer from the neglect of horizontal gradients. The results shown above imply a systematic error of about +20 per cent for these methods when used at latitudes near Stanford (43° geomagnetic). It is likely that some improvement is possible by manually estimating the rotation rate from a careful plot of the fading record rather than using a computer, by selecting the times for rotation-angle calculations in order to minimize the error resulting from horizontal gradients, and by using ray-tracing methods when evaluating the required geometrical quantities. Nevertheless, the best-fitting linear polynomial is found to give reasonably good values without the above complexities, although a computer is still required if very large amounts of data are to be handled.

Both Doppler methods are found to be reasonably good and nearly free from systematic error as long as the electron content is restricted to a linear time variation.

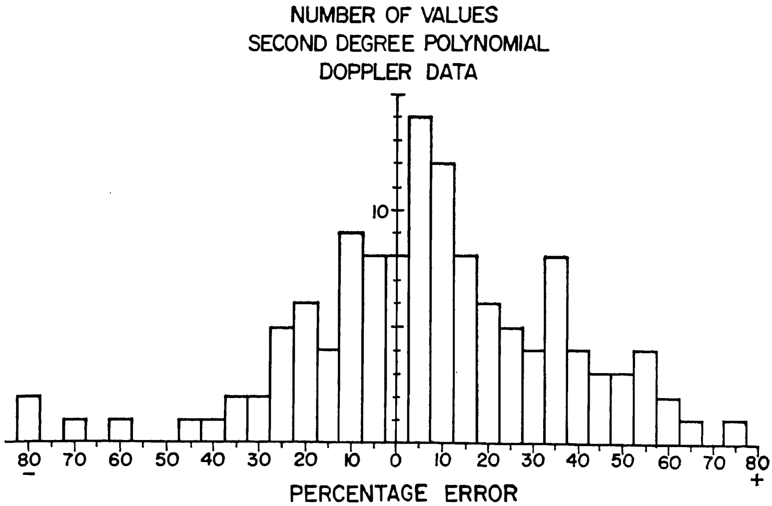

Fig. 5*b*. Similar to Figure 5*a*, except that a polynomial of the form $I = b_0 + b_1t + b_2t^2$ was used.

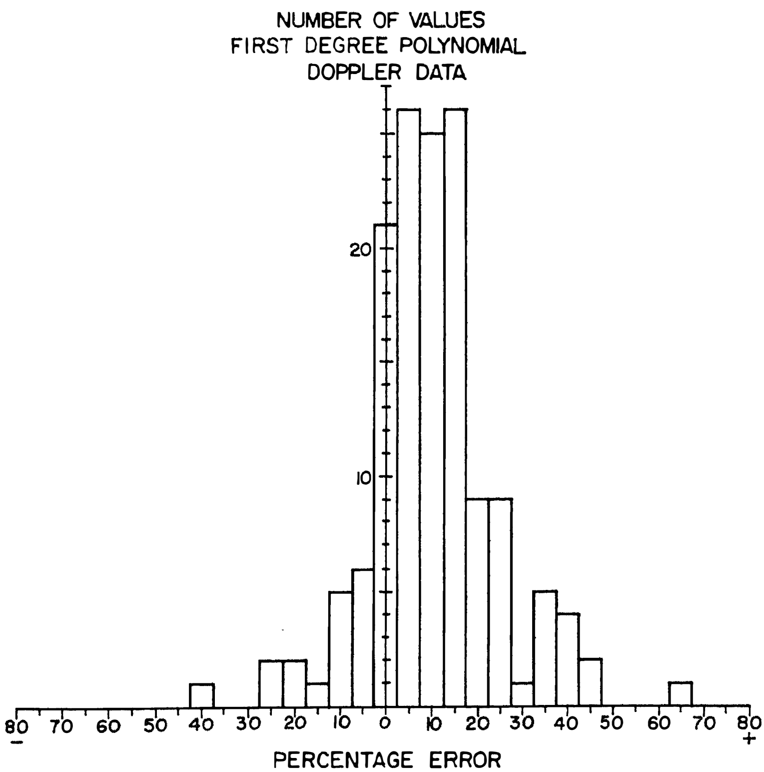

Fig. 5*c*. Similar to Figure 5*b*, except that a polynomial of the form $I = b_0 + b_1t$ was used.

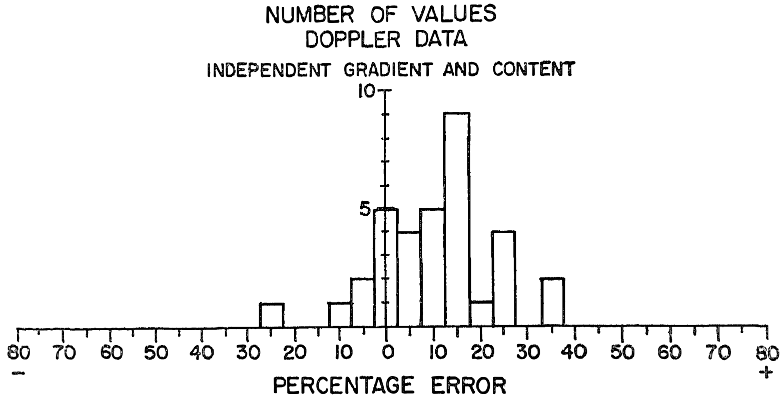

Fig. 6. Similar to Figure 5c, except that the linear gradient was calculated from (22) and the value of electron content was obtained from (24).

These results differ from those found by *Burgess* [1963] principally in that we have found the best-fitting polynomial methods to be relatively satisfactory as long as only first-degree polynomials are used. Burgess also considered a method in which (dI/dt) was estimated independently, analogous to the method we have described in section 2D. Although he found a relatively large error in one case, we have obtained somewhat small average errors for our complete data set. It is difficult to understand the reasons for these differences, but they may be attributable to the selection of one unfortunate satellite passage for his analysis. His results may also have been obtained with polynomials of too high a degree, which is found to lead to erratic values of electron content.

Still another method holds considerable promise, although there has been no opportunity to employ it as yet except with moon echoes. It is a 'differential Faraday' method [*Daniels*, 1957; *Evans*, 1957] in which the angular difference in the planes of polarization at two closely spaced frequencies is measured. Forthcoming radio beacon satellites are expected to provide the necessary frequencies, and relatively good accuracy should be obtained.

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