AGU PUBLICATIONS

JGR

Journal of Geophysical Research: Space Physics

RESEARCH ARTICLE

10.1002/2015JA022206

Special Section:
Nature of Turbulence, Dissipation, and Heating in Space Plasmas: From Alfven Waves to Kinetic Alfven Waves

Key Points:
- A comparison of three-particle distributions
- Stable and unstable solutions
- For electrostatic case kappa and Maxwellian overlap

Correspondence to:
N. Rubab,
drnrubab@gmail.com

Citation:
Rubab, N., A. C.-L. Chian, and V. Jatenco-Pereira (2016), On the ordinary mode Weibel instability in space plasmas: A comparison of three-particle distributions, *J. Geophys. Res. Space Physics*, *121*, 1874–1885, doi:10.1002/2015JA022206.

Received 1 DEC 2015
Accepted 28 FEB 2016
Accepted article online 2 MAR 2016
Published online 25 MAR 2016

On the ordinary mode Weibel instability in space plasmas: A comparison of three-particle distributions

Nazish Rubab[1], Abraham C.-L. Chian[2,3,4,5], and Vera Jatenco-Pereira[6]

[1]Department of Space Science, Institute of Space and Technology, Islamabad, Pakistan, [2]School of Mathematical Sciences, University of Adelaide, Adelaide, South Australia, Australia, [3]Department of Biomedical Engineering, George Washington University, Washington, District Columbia, USA, [4]Institute of Aeronautical Technology and World Institute for Space Environment Research, São José dos Campos, Brazil, [5]National Institute for Space Research (INPE), São José dos Campos, Brazil, [6]Instituto de Astronomia, Geofísica e Ciências Atmosféricas, (IAG/USP), Departamento de Astronomia, Universidade de São Paulo, São Paulo, São Paulo, Brazil

Abstract Electromagnetic wave fluctuations driven by temperature anisotropy in plasmas are of interest for solar flare, solar corona, and solar wind studies. We investigate the dispersion characteristics of electromagnetic wave propagating perpendicular to the uniform magnetic field which is derived by using multiple particle distribution functions: Maxwellian, bi-kappa, and product bi-kappa. The presence of temperature anisotropy in which the parallel plasma kinetic energy density exceeding by a sufficient amount can lead to Weibel-like electromagnetic instability. A general description is made to calculate the growth/damping rates of Weibel-like modes when the temperature anisotropy and nonthermal features are associated with these distributions. We demonstrate that for the zeroth cyclotron harmonic, our results for bi-Maxwellian and bi-kappa overlap with each other, while the product bi-kappa distribution shows some dependence on parallel kappa index. For higher harmonics, the growth rates vanish and the damping prevails.

1. Introduction

The solar plasma environment is hot and weakly collisional, which is in a state far from thermal equilibrium [*Chian et al.*, 2014] as observed in situ in the solar wind through its nonthermal characteristics of velocity distribution function (VDFs). Many processes in a collisionless plasma lead to the development of particle temperature anisotropy to generate plasma instabilities which are often kinetic in nature. The free energy sources associated with the deviation from the thermodynamic equilibrium distribution function could also excite plasma waves. These instabilities can be driven by drifting bi-Maxwellian and bi-Lorentzian particle distributions. The free energy associated with nonequilibrium distribution functions is capable of driving instabilities through velocity or temperature anisotropy. In recent years a considerable interest is directed to nonpropagating waves and instabilities which provide a better explanation for the observed distribution function in many astrophysical and space phenomena [*Schlickeiser et al.*, 2011; *Jatenco-Pereira et al.*, 2014; *Rubab et al.*, 2009, 2010, 2011]. For example, *Lazar et al.* [2014] focused on the interplay between Weibel and firehose instabilities in solar flares and solar winds.

The presence of a background magnetic field (B_0) in astrophysical plasma creates direction-dependent transport processes that leads to plasma heating, deposition, acceleration [*Jatenco-Pereira and Opher*, 1989; *Jatenco-Pereira et al.*, 1994], and cooling acting preferentially either parallel or perpendicular to B_0. As a result, each plasma species display some degrees of temperature anisotropy in its VDF. Hence, kinetic anisotropies are essential part of solar atmosphere and interplanetary plasmas. It is well known that anisotropy in the average kinetic energy in homogeneous collisionless plasma can provide free energy for the excitation of Weibel instability (WI) [*Weibel*, 1959], which results in a purely growing electromagnetic mode in a plasma even in the absence of external magnetic field due to electron temperature anisotropy. The mechanism of this instability is that the fluctuating magnetic field induces a momentum flux due to electrons, and consequently, the spontaneous microscopic plasma (electron or ion) currents flowing in a plasma become amplified and cause a nonvanishing magnetic field to grow in the plasma to maximum amplitude. It also stabilizes via the dissipation of some free energy or by a rapid attenuation of resonant particle interactions.

The physical picture of Weibel instability in counterstreaming plasma was interpreted by *Fried* [1959]. The waves propagating perpendicular to the magnetic field in which the free energy resides in velocity anisotropy can destabilize the ordinary mode [*Tautz and Schlickeiser*, 2006]. Temperature anisotropy caused by particle clusters driven by the Lorentz force of a magnetic perturbation is responsible for the magnetic Weibel instability. The nonlinear effects often arise into the system when the wave reaches its highest level of amplification where the instability saturation is provided by flattening the distribution function [*Pokhotelov and Amariutei*, 2011]. On temporal scales longer than growth and relaxation times the fields generated are quasi-stationary. A number of previous works have demonstrated that Weibel instability can drive the ordinary mode (O mode) unstable in the presence of temperature anisotropy [*Furth*, 1963; *Hamasaki*, 1968; *Davidson and Wu*, 1970; *Bashir and Murtaza*, 2012; *Ibscher et al.*, 2012; *Ibscher and Schlickeiser*, 2014; *Treumann and Baumjohann*, 2014], in counterstreaming plasmas [*Bornatici and Lee*, 1970; *Stockem et al.*, 2006; *Tautz and Schlickeiser*, 2006; *Ibscher et al.*, 2013; *Ibscher and Schlickeiser*, 2013], or in beam-plasma systems [*Bret et al.*, 2005, 2010; *Yoon et al.*, 2007].

The Weibel instability in magnetized/unmagnetized plasma has been investigated by *Zaheer and Murtaza* [2007a] using non-Maxwellian distribution functions and showed that the Weibel instability depends on the spectral indices of the distribution functions. In particular, they observed that by using the Lorentzian (kappa κ) and (r, q) distribution function (where r belongs to the shoulders and q to the high-energy tail of the profile of the distribution function), the Weibel instability shows interesting effects; i.e., the growth rates were suppressed for high-energy tails (small values of κ and q) and for positive values of r the instability prevails, while for negative r, the instability disappears and damping occurs. They also extended their results for the semirelativistic distribution functions and showed that in the presence of relativistic factor, the Weibel instability occurs even for very small temperature anisotropy [*Zaheer and Murtaza*, 2007b]. *Lazar et al.* [2008] and *Lazar et al.* [2010a] investigated in detail the parallel and perpendicular propagation for counterstreaming plasma for kappa distribution function by showing that the growth rates strongly depend on the spectral index; i.e., for parallel propagation the growth rate significantly reduces as compared to Maxwellian, while for perpendicular propagation the Weibel mode is significantly enhanced due to Lorentzian plasma. Moreover, these authors considered counterstreaming plasmas with temperature anisotropies using non-Maxwellian distribution functions by following the idea of *Davidson* [1983], which provide multiple source of free energy by employing bi-Maxwellian distribution function. The nonresonant modes could be stabilized from the kinetic effects arising from the transverse temperature of surrounding plasma. In the presence of uniform magnetic field the nonresonant mechanism of Weibel instability is influenced by the presence of temperature anisotropy and certainly the energy stored in it.

It is well known that space plasmas are collisionless and sufficiently dilute [*Schlickeiser*, 2002] where a power law (kappa, with single index) distribution function is needed to model superthermal kinetic anisotropies. However, a recently proposed product bi-kappa distribution function shows advanced flexibility by providing a general description of nonthermal and anisotropic features of gyrotropic velocity distribution functions with two discrete temperature and power indices as well, i.e., $\kappa_\parallel$ and $\kappa_\perp$. Product bi-kappa distribution function has not been applied so often and needs a special consideration as it decouples the particles dynamics in two principal directions and seems to be more realistic in modeling the gyrotropic velocity distribution functions with two discrete temperatures and spectral indices which allows a more realistic description to investigate different plasma modes. This new distribution function is expected to provide an excess of free energy which is expected to enhance the electromagnetic instability. In the limit of a very large spectral index $\kappa \rightarrow \infty$, these both distribution functions approach to bi-Maxwellian.

In this paper, we shall consider the instability driven by electron anisotropic distribution function, by employing bi-Maxwellian, bi-kappa, and product bi-kappa distribution functions for strong magnetic field limit; i.e., $b_e (= k_\perp v_\perp / \omega_{ce}) < 1$ which is usually valid for solar flare regimes. where isotropic ions ($T_{i\parallel} = T_{i\perp}$) due to inertia are considered to be a neutralizing positive background. We investigate the Weibel electromagnetic instability driven by energy anisotropy; i.e., plasma kinetic energy along the magnetic field lines exceeds the perpendicular kinetic energy for wave propagation perpendicular to the uniform magnetic field. For instance, *Lazar et al.* [2010a] studied the impact of product bi-kappa distribution on the parallel propagating O mode instability and found the dependence of $\kappa_\parallel$ on the threshold condition, while in our case for perpendicular propagation, the new threshold conditions are found to be dependent on both spectral indices ($\kappa_\parallel, \kappa_\perp$). We shall briefly discuss the Weibel instability driven by electron thermal anisotropy for a bi-Maxwellian plasma such that $T_{e\parallel} > T_{e\perp}$, which is the critical parameter for determining the onset and the saturation of the instability. The impact of anisotropic Lorentzian distributions (bi-kappa and product bi-kappa) on the growth rate of perpendicularly propagating ordinary mode including the higher harmonics effect is analyzed in the strong magnetic field limit.

The plan of the paper is as follows: In section 2, we introduce the plasma model and velocity distribution functions employed for the instability analysis. In section 3, dispersion relation of Weibel instability using three bi-Maxwellian, bi-Kappa and product bi-kappa distribution, the threshold conditions, and the analysis in the strong magnetic field limit will be discussed analytically. Finally, we shall discuss the obtained results to reach a final conclusion and summary in sections 4 and 5, respectively.

2. O Mode Weibel-Like Instability Perpendicular to B_0

In this paper, we shall present a comprehensive picture of anisotropic temperature driven non-Maxwellian Weibel instability in a magnetized plasma. In the Vlasov-Maxwell formalism, the general dispersion relation for the electromagnetic O mode instability propagating perpendicular to B_0 with $k_\parallel=0$, $\mathbf{k}=k_\perp \hat{e}_x$ in a homogeneous and collisionless plasma can be obtained as [*Davidson*, 1983],

$$\frac{c^2k_\perp^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2}\left(1 + \sum_{n=-\infty}^{\infty}\int \frac{n\omega_{ce}}{\omega - n\omega_{ce}} j_n^2(b_e)\frac{v_\parallel^2}{v_\perp}\frac{\partial f_e}{\partial v_\perp}\mathrm{d}^3\mathbf{v}\right), \tag{1}$$

where $\omega_{ce} = eB_0/m_e c$ is the electron cyclotron frequency and $j_n\,(b_e = k_\perp v_\perp/\omega_{ce})$ is the Bessel function of first kind. Equation (1) predicts an electromagnetic Weibel-like instability when the plasma kinetic energy along the magnetic field lines exceeds the perpendicular energy by a sufficiently large amount.

The ordinary mode is unstable transverse plasma mode with its electric field linearly polarized in the direction of B_0. In order to obtain Weibel-like instability, we assume a simple model and power law distribution functions that maintain the essential physics and at the same time able to highlight the effect of temperature anisotropy on the O mode Weibel-like instabilities. In order to accomplish our aim, we adopt bi-Maxwellian, bi-kappa, and product bi-kappa distribution functions which are the unperturbed anisotropic plasma distribution functions with an excess of parallel temperature, i.e., $T_{e\parallel} > T_{e\perp}$. The observed distribution functions may contain non-thermal characteristics such as counterstreaming and temperature anisotropies. In this analysis, we shall focus on the free energy temperature anisotropy which drives purely growing electromagnetic O mode instabilities and the lower bound on the growth rates. We consider the following three plasma distribution functions:

1. The anisotropic bi-Maxwell distribution function is given by

$$f_e(v_\perp^2, v_z) = (\frac{m_e}{2\pi T_{e\parallel}})^{1/2}(\frac{m_e}{2\pi T_{e\perp}})\exp\left(-\frac{m_e v_\perp^2}{2T_{e\perp}} - \frac{m_e v_\parallel^2}{2T_{e\parallel}}\right), \tag{2}$$

where $T_{e\parallel}(T_{e\perp})$ is parallel (perpendicular) temperature to magnetic field B_0.

2. The anisotropic generalized Lorentzian or kappa distribution

$$f_{\kappa e} = \frac{1}{\pi^{3/2}\theta_{e\parallel}\theta_{e\perp}^2}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-1/2)}\left(1 + \frac{v_\parallel^2}{\kappa\theta_{e\parallel}^2} + \frac{v_\perp^2}{\kappa\theta_{e\perp}^2}\right)^{-\kappa-1},$$

where κ is the spectral index and is constrained to $\kappa > 3/2$ and Γ is the Gamma function. This distribution function is normalized to unity such that $\int f_\kappa \mathrm{d}^3 v = 1$, which makes the reference that thermal speed θ is related to particle's temperature T which is given by

$$\theta_{e\parallel}^2 = \left(\frac{2\kappa-3}{\kappa}\right)\left(v_{te\parallel}^2\right); \quad \theta_{e\perp}^2 = \left(\frac{2\kappa-3}{\kappa}\right)\left(v_{te\perp}^2\right),$$

with

$$v_{te\parallel,\perp}^2 = \left(\frac{T_{e\parallel,\perp}}{m_e}\right).$$

3. Product bi-kappa distribution function

$$f_{\kappa_{\parallel,\perp}}(v_\parallel, v\perp) = \frac{1}{\pi^{3/2}\theta_{e\perp}^2\theta_{e\parallel}\kappa_\parallel^{1/2}}\frac{\Gamma\left(\kappa_\parallel+1\right)}{\Gamma(\kappa_\parallel+1/2)}\left(1+\frac{v_\parallel^2}{\kappa_\parallel\theta_{e\parallel}^2}\right)^{-\kappa_\parallel-1}\left(1+\frac{v_\perp^2}{\kappa_\perp\theta_{e\perp}^2}\right)^{-\kappa_\perp-1}.$$

This function is also normalized to unity $\int f d^3v = 1$, and the thermal velocities are given by

$$\theta_{e\parallel}^2 = \frac{2\kappa_\parallel - 1}{2\kappa_\parallel} v_{te\parallel}^2; \quad \theta_{e\perp}^2 = \frac{\kappa_\perp - 1}{\kappa_\perp} v_{te\perp}^2,$$

where $v_{te\parallel}^2 = 2T_{e\parallel}/m_e$, $v_{te\perp}^2 = 2T_{e\perp}/m_e$ and the spectral indices are constrained by $\kappa_\perp > 1$ and $\kappa_\parallel > 0.5$.

3. General Solution for Distribution Functions

3.1. Bi-Maxwellian Distribution Function

The linear dispersion relation for a collisionless anisotropic plasma by using Maxwellian distribution function in equation (1) can be obtained as [*Davidson*, 1983]

$$\gamma^2 = R(k_\perp)/I\left(k_\perp, \gamma^2\right), \tag{3}$$

where

$$I = 1 + \sum_{n\neq 0} \frac{T_{e\parallel}}{T_{e\perp}} \frac{\omega_{pe}^2}{\gamma^2 + n^2\omega_{ce}^2} \exp(-\lambda_e) I_n(\lambda_e), \tag{4}$$

$$R(k_\perp) = \omega_{pe}^2 \frac{T_{e\parallel}}{T_{e\perp}} \left[\left(1 - \frac{T_{e\perp}}{T_{e\parallel}}\right) - G((\lambda_e)\right], \tag{5}$$

where $G(\lambda_e) = \lambda_e \frac{2}{\beta_{e\parallel}}$, $\beta_{e\parallel} = \frac{8\pi n_0 T_{e\parallel}}{B_0^2}$, and $\lambda_e = \frac{k_\perp^2 T_{e\perp}}{m_e \omega_{ce}^2}$.

The general stability condition can be discussed on the basis that $R(k_\perp) > 0$. The dispersion relation admits purely growing solution of the Weibel instability for the wave numbers smaller than a cutoff value

$$k_\perp < k_{c1} = \frac{\omega_{pe}}{c}\left(\frac{T_{e\parallel}}{T_{e\perp}} - 1\right)^{1/2},$$

which shows that in the presence of a static magnetic field, the solutions are found for wave numbers smaller than cutoff values. It can also be observed that critical value depends on perpendicular temperature $T_{e\perp}$.

3.2. Small Gyroradius Approximation ($b_e < 1$)

In solar flares, the plasma instabilities are responsible for releasing free energy which comes through either the streaming of solar flares particles or the temperature anisotropy. The magnetic fields are assumed to be of the order of 10^2 G.

In a strong magnetic field limit, equation (1) can be written as

$$I(k_\perp, \gamma^2) = 1 + \sum_{n\neq 0} \frac{T_{e\parallel}}{(T_{e\perp})^{1-n/2}} \frac{\omega_{pe}^2}{\gamma^2 + n^2\omega_{ce}^2} A_n, \tag{6}$$

where $A_n = \alpha_1 \left(\frac{k_\perp}{2\omega_{ce}^2}\right)^n$, $\alpha_1 = \frac{\Gamma\left(1+\frac{n}{2}\right)}{\Gamma(n+1)}$, and

$$R(k_\perp) = \omega_{pe}^2 \frac{T_{e\parallel}}{(T_{e\perp})^{1-n/2}} A_n \left[1 - A_n^{-1}\left(\frac{(T_{e\perp})^{1-n/2}}{T_{e\parallel}} + G(\lambda_{en})\right)\right], \tag{7}$$

which is the O mode Weibel instability for n harmonics where

$$G(\lambda_{en}) = \alpha_1 \frac{2^{n+1}}{\beta_{e\parallel}} \lambda_{en},$$

$$\lambda_{en} = \frac{k_\perp^{2-n}(T_{e\perp})^{1-n/2}}{m_e^{1+n/2}\omega_{ce}^2}.$$

It can be seen that when $n = 0$, $T_{e\perp}/T_{e\parallel} < 1$, then $A_n = 1$, and by employing $\beta_{e\parallel} >> 1$, we arrive at the electrostatic Weibel instability in an unmagnetized plasma, which means that purely growing modes are associated even with zeroth cyclotron harmonics, i.e., $n = 0$. For sufficient large plasma beta $\beta_{e\parallel}$, purely electromagnetic mode exists propagating perpendicular to B_0.

In this case the dispersion relation admits purely growing solutions of the Weibel type for wave numbers smaller than a cutoff value

$$k_\perp < k_{c2} = \frac{\omega_{pe}}{c}\left(A_n \frac{T_{e\parallel}}{\left(T_{e\perp}\right)^{1-n/2}} - 1\right)^{1/2},$$

which shows that the cutoff value is now dependent on the cyclotron harmonics and for $n=0$, $A_n=1$, we get $k_{c1}=k_{c2}$.

3.3. Bi-Kappa Distribution Function

However, by taking anisotropic distributions to study ordinary mode instability, we employ bi-kappa distribution function which is very common in almost collisionless environment and to be far from equilibrium. The statistical behavior of space plasma that generates steady energetic particle distributions is generally believed to have long-range interactions in competition with Coulomb collisions which is strictly valid for short range interactions. Owing to large numbers of particles in a Debye sphere, these systems are generally collisionless, which are typical for solar and astrophysical scenarios, for example, for typical parameters for solar wind plasma, the values of $\kappa \geq 2$, which has been measured for electron velocity distribution function [*Štverák et al.*, 2009] and significantly enhanced nonthermal tails ($\kappa \sim 4.1$–5.8) in the above loop top regions [*Oka et al.*, 2015]. After applying the limit $\kappa \to \infty$, we are able to retrieve the Maxwellian counterpart.

Now by inserting the kappa distribution function in equation (1), the ordinary mode Weibel instability in a Lorentzian plasma then turns out to be

$$R(k_\perp) = \omega_{pe}^2 \frac{\theta_{e\parallel}^2}{\theta_{e\perp}^{2-n}} A_{\kappa n}\left[1 - (A_{\kappa n})^{-1}\left(\frac{\theta_{e\perp}^{2-n}}{\theta_{e\parallel}^2} - G(\lambda_{en})\right)\right], \quad (8)$$

where $A_{\kappa n} = \alpha_2 \left(\frac{k_\perp}{2\omega_{ce}^2}\right)^n$, $\alpha_2 = \kappa^{n/2}\frac{\Gamma(\kappa-1/2-n/2)}{\Gamma(\kappa-1/2)}\frac{\Gamma\left(1+\frac{n}{2}\right)}{\Gamma(n+1)}$, $G(\lambda_{en}) = \alpha_2^{-1}\frac{2^{n+1}}{\beta_{e\parallel}}\lambda_{en}$, and $\lambda_{en} = \frac{k_\perp^{2-n}\theta_{e\perp}^{2-n}}{\omega_{ce}^2}$.

To compare the two distribution function (bi-Maxwellian and bi-kappa), we observe the dependence of κ on the dispersion characteristics, which leaves no difference between two distributions when $n=0$. Therefore, for $n=0$, $\alpha_2 = 1$ and $\beta_{e\parallel} > 1$, the Weibel mode becomes electrostatic to a very good approximation, irrespective of the values of kappa.

It can also be noticed that for $\kappa = 3, 4, 5\ldots$, there is a corresponding odd value of $n = 5, 7, 9\ldots$ for which $\alpha_2 = 0$, i.e., $\kappa = 3, n = 5$ and $\kappa = 4, n = 7$, and so on. For $\alpha_2 = 0$, we get $R(k_\perp) = 0$, which leads toward a stable solution.

The threshold condition by employing kappa distribution is given by

$$k_\perp < k_{c3} = \frac{\omega_{pe}}{c}\left(A_{\kappa n}\frac{\theta_{e\parallel}^2}{\theta_{e\perp}^{2-n}} - 1\right)^{1/2},$$

which is dependent on $A_{\kappa n}$ and $\theta_{e\perp}^{2-n}$, and we may notice that by putting $n = 0$, the cutoff wave number k_{c3} does not depend on the Lorentzian index and we get the classical result of ordinary mode electromagnetic instability [*Davidson*, 1983] for Maxwellian plasma, i.e., $A = A_n = A_{\kappa n}$. For $n = 1$, superthermality can take part through α_2; therefore, it is necessary to retain sufficient numbers of cyclotron harmonics for kappa dependent purely growing solutions.

3.4. Product Bi-Kappa Distribution Function

In order to model and demonstrate the space plasma dynamics and associated instabilities with non-Maxwellian anisotropic distribution functions, the dispersion characteristics of plasma waves when the equilibrium (unperturbed) state of the plasma is described by a product bi-kappa in a highly anisotropic plasma is given by

$$R(k_\perp) = \omega_{pe}^2 \frac{\theta_{e\parallel}^2}{\theta_{e\perp}^{2-n}} \acute{A}_{\kappa n}\left[1 - \acute{A}_{\kappa n}^{-1}\left(\frac{\theta_{e\perp}^{2-n}}{\theta_{e\parallel}^2} + G(\lambda_{en})\right)\right], \quad (9)$$

where

$$\acute{A}_{\kappa n} = \alpha_3 \left(\frac{k_\perp}{2\omega_{ce}^2} \right)^n,$$

$$\alpha_3 = \kappa_\parallel \kappa_\perp^{n/2} \frac{\Gamma(\kappa_\perp + 1 - n/2)}{\Gamma(\kappa_\perp + 1)(\kappa_\parallel - 1/2)} \frac{\Gamma\left(1 + \frac{n}{2}\right)}{\Gamma(n+1)},$$

and

$$G(\lambda_{en}) = \alpha_3^{-1} \frac{2^{n+1}}{\beta_{e\parallel}} \lambda_{en},$$

$$\lambda_{en} = \frac{k_\perp^{2-n} \theta_{e\perp}^{2-n}}{\omega_{ce}^2}.$$

The cutoff value for product bi-kappa distribution can take the form

$$k_\perp < k_{c4} = \frac{\omega_{pe}}{c} \left(\acute{A}_{\kappa n} \frac{\theta_{e\parallel}^2}{\theta_{e\perp}^{2-n}} - 1 \right)^{1/2},$$

which is now dependent on $\acute{A}_{\kappa n}$ and $\theta_{j\perp}^{2-n}$, and unlike kappa distribution function, in which for $n = 0$, $A_{\kappa n} = A_n = 1$, the product bi-kappa shows some dependence on $\kappa_\parallel$, i.e., $\acute{A}_{\kappa n} = \kappa_\parallel/(\kappa_\parallel - 1/2)$. For $\kappa_\parallel >> 1/2$, $\acute{A}_{\kappa n}$ turns out to be unity and the results for three distribution functions coincide. While for non zero harmonics, in addition to $\kappa_\parallel$, the power index perpendicular to the magnetic field $(\kappa_\perp)$ will also control the growth characteristics through α_3.

4. Results and Discussion

In this paper, we have examined the effect of superthermal population in the form of bi-kappa and bi-product kappa distribution on the dispersion characteristics of perpendicularly propagating O mode Weibel instability and hence its comparison with the classical bi-Maxwellian plasma. By assuming an excess of parallel temperature, $T_{e\perp}/T_{e\parallel} < 1$, i.e., $T_{e\perp} < T_{e\parallel}$, the instability exists for strongly magnetized plasma, for example, solar flares and coronal regions where $B_0 \sim 10^2 – 5 \times 10^2$ G, $n = 10^9$ cm^{-3}, $T = 10^6 - 10^7$ K.

A large population of superthermal particles exist in the Sun's explosive and energy release phenomena, i.e., solar flares/solar winds, etc., where electrons are accelerated to ∼10 MeV due to magnetic reconnection in the coronal current sheet. In the low-density regions (∼10^9 cm^{-3}), for example, above coronal loop top regions exhibit non-Maxwellian nature of particles whose velocity distributions are not correctly described by the classical Maxwellian distribution functions but rather by bi-kappa or product bi-kappa distribution functions due to collisionless environment and long-range interactions. In the dense regions (> 10^{10} cm s^{-3}), the nonthermal tail is suppressed due to Coulomb collisions which relaxes the system to its classical Maxwellian nature [*Oka et al.*, 2015]. The presence of a significantly larger number of suprathermal particles can drastically change the rate of resonant energy transfer between particles and plasma waves which in turn modify the growth (damping) rate and the excitation conditions for plasma waves.

A graphical representation is made to aid the comparison of the three distributions. Figure 1 shows the behavior of perpendicularly propagating O mode instability including the higher cyclotron harmonics effects $n = 0, 1, 2$ by employing (a) bi-Maxwellian, (b) bi-kappa, and (c) product bi-kappa distribution functions. By setting the temperature ratio $T_{e\perp}/T_{e\parallel} = 0.4$ and for zeroth cyclotron harmonic, i.e., $n = 0$, we may observe that the Maxwellian and kappa distributions show the same behavior and the growth rates are insensitive to the form of the distribution function, while the instability is enhanced drastically by using product bi-kappa distribution function. Our results are in perfect agreement with *Lazar et al.* [2010b] in a strong magnetic field limit, where temperature anisotropy-driven O mode Weibel instability remains insensitive to the form of kappa distribution in contrast to streaming filamentation instability which is enhanced by superthermal tails. It can be deduced that for perpendicular propagation, the electrostatic Weibel instability is enhanced with product bi-kappa distribution function. Further, we have observed that for higher harmonics, i.e., $n = 1, 2$, the instability disappears and instead damping occurs for the all three distributions. For the product bi-kappa

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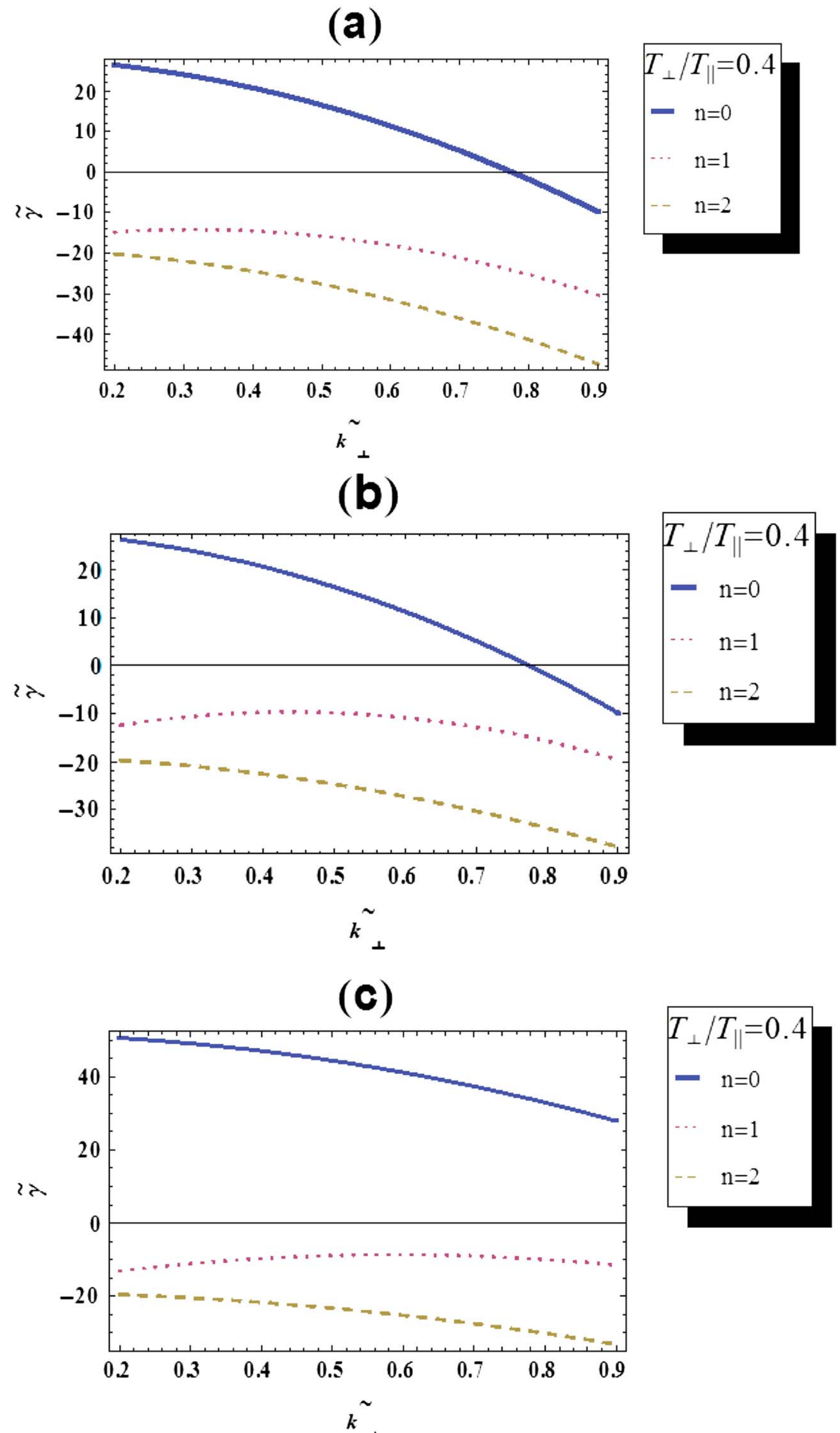

Figure 1. Growth rates of Weibel instability for three non-Maxwellian distributions: (a) bi-Maxwellian, (b) bi-kappa, and (c) product bi-kappa.

distribution damping rates are observed to be slightly higher for $n = 1$ as compared to bi-Maxwellian and kappa distributions, while at $n = 2$ and even higher harmonics, the damping decrement remains same for three distributions. We can conclude here that purely growing modes are associated with zeroth cyclotron harmonics, while all higher harmonics ($n = 1, 2 \ldots$) comprise of almost zero growth rates which means that instability is inhibited for higher harmonics and hence damping prevails.

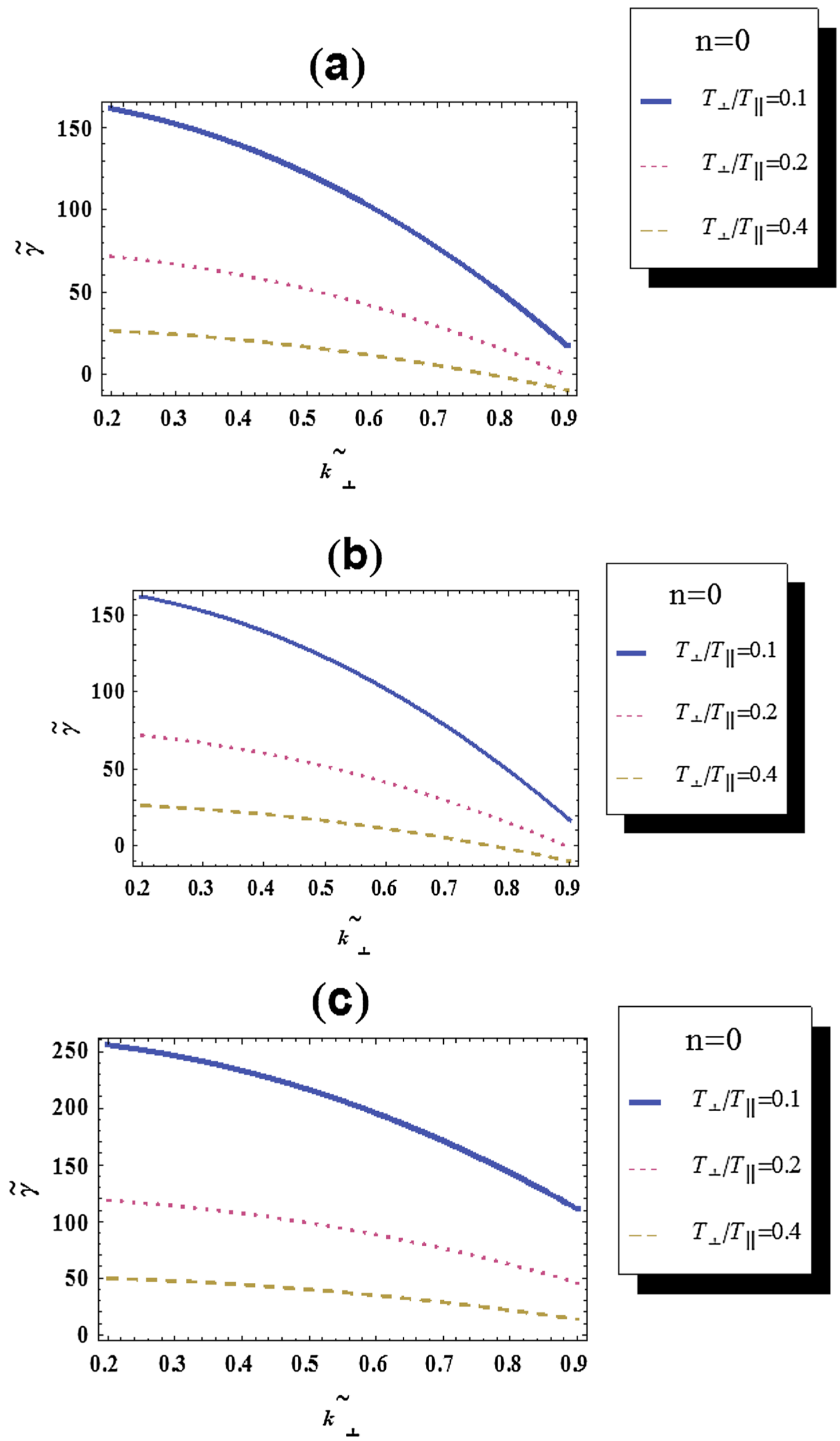

Figure 2. Growth rates of Weibel instability for three non-Maxwellian distributions: (a) bi-Maxwellian, (b) bi-kappa, and (c) product bi-kappa at different temperature ratios at $n = 0$.

Further, we make a comparison in Figures 2 and 3 for different temperature ratios by setting cyclotron harmonics $n = 0, 1$. It is obvious from the graphical representation that by increasing the temperature ratio the instability growth rate reduces drastically. It can also be noticed that growth rates are enhanced for $T_{e\perp}/T_{e\|} = 0.1$ for product bi-kappa function as depicted in the Figure 2a. For $n = 1$, growth rates for kappa distribution are slightly higher than Maxwellian and product bi-kappa when $T_{e\perp}/T_{e\|} = 0.1$ as depicted in Figure 2b. In the limit $\kappa_\| = \kappa_\perp \to \infty$, our results approach to the classical bi-Maxwellian distribution. It can

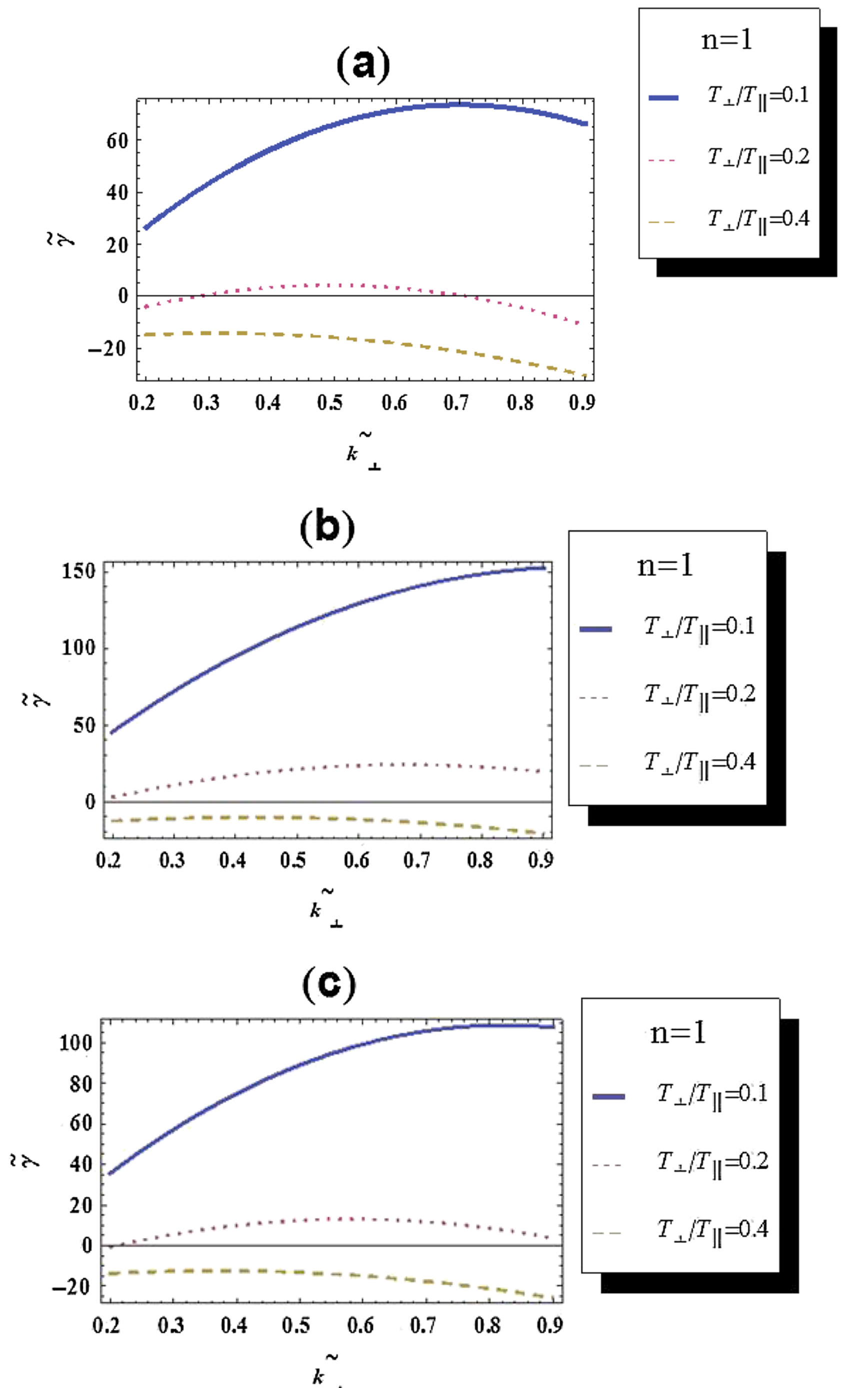

Figure 3. Growth rates of Weibel instability for three non-Maxwellian distributions: (a) bi-Maxwellian, (b) bi-kappa, and (c) product bi-kappa for $n = 1$.

also be observed that when $T_{e\perp}/T_{e\|} = 0.4$, the growth rates vanish and instead damping occurs for three distributions. We have also found that Weibel instability can grow only for a critical temperature ratio $T_{e\perp}/T_{e\|} \geq 0.2$ with $n = 1$, in our case. It also provides a cutoff for Weibel instability to grow beyond this critical value and hence damping occurs.

In Figure 4, we have plotted growth rates as a function of $\beta_{e\|}$ for $T_{e\perp}/T_{e\|} = 0.4$ and $n = 0, 1, 2$ for product bi-kappa. For $n = 0$ and $\beta_{e\|} \geq 2$, the growth rate increases and then remains same for all vales of $\beta_{e\|}$. It can also

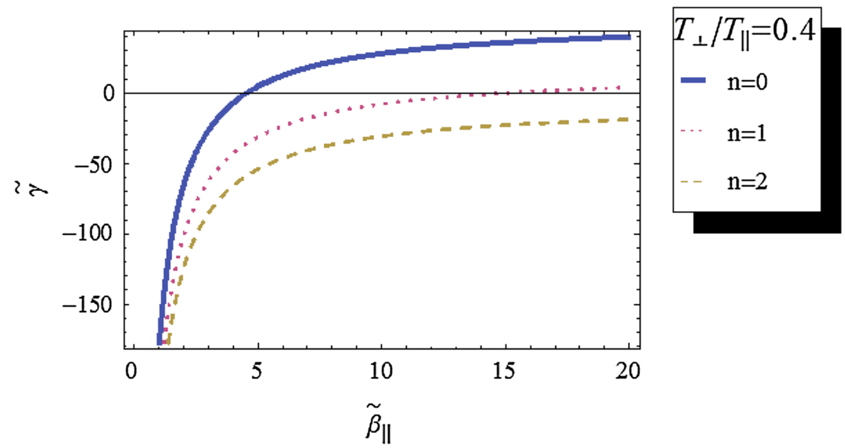

Figure 4. Growth rates of Weibel instability for three non-Maxwellian distributions: (a) bi-Maxwellian, (b) bi-kappa, and (c) product bi-kappa as a function of plasma beta.

be noticed that for $n = 1, 2$, the plasma becomes stable which shows that stronger magnetic fields suppress the instability to a large extent.

5. Summary

In this paper, a comparative analysis of the Weibel instability with an excess of parallel temperature predicts that all high harmonics are stable (damping rates) so it is reasonable to keep sufficient number of harmonics for various temperature anisotropies, while purely growing instability is associated with zeroth harmonic which remains insensitive to the kappa distribution function, and the results coincide with Maxwellian distribution, while higher harmonics solutions show some dependence on kappa distribution. In contrast, product bi-kappa distribution that seems to provide an excess of free energy depends on $\kappa_\parallel$ and is expected to influence the electromagnetic instability for zeroth cyclotron harmonics ($n=0$), and for higher cyclotron harmonics the contribution of $\kappa_\perp$ can also be observed. *Lazar et al.* [2010c] studied the effect of product bi-kappa distribution on parallel propagating O mode Weibel instability and showed that product bi-kappa distribution is highly susceptible to Weibel instability. Our analysis for perpendicular propagation also highlights the enhanced growth rates due to product bi-kappa distribution for $n=0$, but for higher cyclotron harmonics bi-kappa distribution leads over other distributions toward enhanced growth rates.

In this study, we used the plasma parameters close to solar flare source for illustration. It is proposed that a sufficient number of cyclotron harmonics are required to study the superthermal effect in order to describe the perpendicularly propagating O mode Weibel instability. We have also observed that all high harmonics are stable (zeroth growth or damping) for $T_{e\perp}/T_{e\parallel} = 0.4$, while for $T_{e\perp}/T_{e\parallel} \leq 0.2$, and $n = 1$; purely growing first harmonic instability can exist for three distributions.

The presence of superthermal particles in a kappa distribution effectively changes the rate of resonant energy transfer between plasma particles and by the action of plasma wave oscillation which transfers free energy to the wave which in turn generates instabilities and has been studied extensively by *Lazar et al.* [2011]. By employing such distribution functions the plasma stability can either be enhanced or degraded. In the present investigations, the particle anisotropy can be introduced by including temperature and kappa index anisotropies. To accomplish our task, we made comparison of three distributions and investigated that product bi-kappa has a leading role toward enhanced wave growth for zeroth cyclotron harmonics ($n=0$), while at higher cyclotron harmonics ($n = 1$), bi-kappa distribution leads over product bi-kappa distribution. The growth rate by taking bi-Maxwellian distribution function was observed to reduce in comparison to product bi-kappa distribution. In the limit $\kappa_\parallel = \kappa_\perp \rightarrow \infty$, our results approach to bi-Maxwellian distribution function.

Lazar et al. [2012] modeled space plasma dynamics by using anisotropic kappa distribution functions, and a comparison with contour plots was made to measure electron distribution at different heliospheric distances in a high-speed solar wind. The results showed some slanted and highly anisotropic tails along the magnetic field direction which look similar to the product bi-kappa to a certain extent. In their conclusion the instability constraint by employing bi-kappa distribution were not well fitted in the slow solar wind with the halo limits, but at the same time the analysis was well described for slow/fast solar wind by incorporating product bi-kappa model. In the present scenario, we intentionally kept our model simple to emphasize the effect of superthermal particles (bi-kappa and product bi-kappa) on the Weibel instability and its comparison with bi-Maxwellian distribution function by including higher harmonics effects. It is believed that the growth rates could be enhanced if the particle streaming and density gradients are introduced.

The relevance of our study is to demonstrate the fact that a theoretical description requires an appropriate kappa model to study a physical process in solar flares and solar winds and generates magnetic field in plasma heating process. In closing, we wish to remark that our results may also apply to perpendicular propagation of ordinary wave excited by the cross field ion drift in the magnetotail region of planetary magnetospheres [*Lui et al.*, 2008] supported by in situ satellite observations, which is relevant for the understanding of chaotic nature of solar-terrestrial environment and the onset of substorms in planets and exoplanets [*Chian et al.*, 2006, 2010].

Acknowledgments
A.C.L.C. thanks the hospitality of George Washington University, University of Adelaide, and ITA and the support of Fulbright-CAPES, CNPq, and CAPES-PVE. V.J.P. thanks the support of FAPESP (2013/10559-5). There are no new data in this paper.

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