EFFECTS OF THE INITIAL CONDITIONS IN A SWING-BY OF A CLOUD OF PARTICLES

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Abstract: *In this paper we perform a study of the effects of a close approach in the three-dimensional space between a planet and a cloud of particles, with the goal of understanding the dispersion of this cloud in terms of the variations of velocity, energy, angular momentum and inclination. It is assumed that the cloud is formed at the periapsis and the particles differ only by the magnitude of the velocity at this point. In this research we use the three-dimensional circular restricted three-body problem as the basic model for this close approach. A numerical algorithm is developed and implemented to study this problem and then it is applied to a cloud of particles, based in an analytical description of the close approach maneuver in the three-dimensional space. Analytical equations based in the patched conics approximation are used to calculate the variation in velocity, angular momentum, energy and inclination of the spacecraft that performs this maneuver.*

Key-Words: *astrodynamics, artificial satellites, orbital dynamics, swing-by.*

1 Introduction

The swing-by maneuver is a very popular technique used to decrease fuel expenditure in space missions. The most usual approach to study this problem is to divide the problem in three phases dominated by the "two-body" celestial mechanics. Other models used to study this problem are the circular restricted three-body problem (like in [1], [2] and [3]) and the elliptic restricted three-body problem ([4], [5] and [6]).

The goal of this paper is to use analytical equations for the variations of velocity, energy, angular momentum and inclination for a spacecraft that passes close to a celestial body. This passage, called swing-by, is assumed to be performed around the secondary body of the system. Among the several sets of initial conditions that can be used to identify uniquely one swing-by trajectory, the following five variables are used: Vp, the velocity of the spacecraft at periapsis of the orbit around the secondary body; two angles (α and β), that specify the direction of the periapsis of the trajectory of the spacecraft around M_2 in a three-dimensional space; rp the distance from the spacecraft to the center of M_2 in the moment of the closest approach to M_2 (periapsis distance); g, the angle between the velocity vector at periapsis and the intersection between the horizontal plane that passes by the periapsis and the plane perpendicular to the periapsis that holds $\vec{V}_p$.

It is assumed that the system has three bodies [7]: a primary (M_1) and a secondary (M_2) bodies with finite masses that are in circular orbits around their common center of mass and a third body with negligible mass that has its motion governed by the two other bodies. The result of this maneuver is a change in velocity, energy, angular momentum and inclination in the Keplerian orbit of the spacecraft around the central body.

Then, our goal is to study the change of the relative inclination of the orbits of this cloud of particles after the close approach with the planet, as well as the variations in velocity, energy and angular momentum. It is assumed that all the particles that belong to the cloud have all the same orbital elements, except by the magnitude of the velocity at the periapsis that will be varied in a short interval around a nominal value to represent a group of particles that have similar orbital elements.

2 Analytical Equations for fhe Swing-by in Three Dimensions

First, it is calculated the initial conditions with respect to M_2 at the periapsis [8]:

Position:

$$x_i = r_p \cos\beta \cos\alpha \tag{1}$$

$$y_i = r_p \cos\beta \, sin\alpha \tag{2}$$

$$z_i = r_p \; sin\beta \tag{3}$$

Velocity:

$$V_{xi} = -V_p sin\gamma \, sin\beta \cos\alpha - V_p \cos\gamma \, sin\alpha \tag{4}$$

$$V_{yi} = -V_p sin\gamma \, sin\beta \, sin\alpha + V_p \cos\gamma \cos\alpha \tag{5}$$

$$V_{zi} = V_p \cos\beta \sin\gamma \tag{6}$$

During the passage, it is assumed that the two-body celestial mechanics are valid and the whole maneuver takes place in the plane defined by the vectors $\vec{r}_p$ and $\vec{V}_p$. So, the vectors $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$, that are velocity vectors before and after the swing-by, respectively, with respect to M2 can be written as a linear combination of the versors associated with $\vec{r}_p$ and $\vec{V}_p$. Using $\vec{V}_\infty$ to represent both $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$, since the conditions are the same for both vectors and a double solution will give the values for $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$, we have:

$$\vec{V}_\infty = A\frac{\vec{r}_p}{r_p} + B\frac{\vec{V}_p}{V_p} \tag{7}$$

Which means that:

$$\begin{aligned}\vec{V}_\infty = {} & A(\cos\beta\cos\alpha, \cos\beta\, sin\alpha, sin\beta) + \\ & + B(-sin\gamma\, sin\beta\cos\alpha - \cos\gamma\, sin\alpha, sin\gamma\, sin\beta\, sin\alpha + \\ & + \cos\gamma\cos\alpha, \cos\beta\, sin\gamma)\end{aligned} \tag{8}$$

With A, B constants that follows the relations:

$A^2 + B^2 = V_\infty^2$, where V_∞ can be obtained from $V_\infty^2 = V_p^2 - \frac{2\mu}{r_p}$, that represents the conservation of energy of the two-body dynamics. A second requirement for $\vec{V}_\infty$ is that it makes an angle δ with $\vec{V}_p$, where δ is half of the total rotation angle described by the velocity vector during the maneuver (angle between $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$). This condition can be written as:

$$\vec{V}_\infty \bullet \vec{V}_p = V_\infty V_p \cos\delta \tag{9}$$

where the dot represents the scalar product between two vectors.
From the two-body celestial mechanics it is known that:

$$\sin\delta = \frac{1}{1+\frac{r_p V_\infty^2}{\mu_2}} \tag{10}$$

Using the equation for $\vec{V}_\infty$ as a function of $\vec{r}_p$ and $\vec{V}_p$, we have:

$$\vec{V}_\infty \bullet \vec{V}_p = \left(A\frac{\vec{r}_p}{r_p} + B\frac{\vec{V}_p}{V_p} \right) \bullet \vec{V}_p = BV_p = V_\infty V_p \cos\delta \tag{11}$$

So, $B = V_\infty \cos\delta$, because $\vec{r}_p \bullet \vec{V}_p = 0$ (at the periapsis $\vec{r}_p$ and $\vec{V}_p$ are perpendicular) and $\vec{V}_p \bullet \vec{V}_p = V_p^2$.

Then, since $A^2 + B^2 = V_\infty^2 \Rightarrow A^2 = V_\infty^2 - B^2 = V_\infty^2 - V_\infty^2 \cos^2\delta = V_\infty^2 (1-\cos^2\delta) = V_\infty^2 \sin^2\delta \Rightarrow A = \pm V_\infty \sin\delta$.

From those conditions, we have:

$$\begin{aligned}\vec{V}_\infty^- = {} & V_\infty \sin\delta(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ & + V_\infty \cos\delta(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \\ & -\sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{12}$$

$$\begin{aligned}\vec{V}_\infty^+ = {} & -V_\infty \sin\delta(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ & + V_\infty \cos\delta(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \\ & -\sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{13}$$

For M_2, its velocity with respect to an inertial frame ($\vec{V}_2$) is assumed to be:

$$\vec{V}_2 = (0, V_2, 0) \tag{14}$$

By using vector addition:

$$\begin{aligned}\vec{V}_i = {} & \vec{V}_\infty^- + \vec{V}_2 = V_\infty sin\delta(\cos\beta\cos\alpha, \cos\beta\, sin\alpha, sin\beta) + \\ & + V_\infty \cos\delta(-sin\gamma\, sin\beta\cos\alpha - \cos\gamma\, sin\alpha, -sin\gamma\, sin\beta\, sin\alpha + \\ & + \cos\gamma\cos\alpha, \cos\beta\, sin\gamma) + (0, V_2, 0)\end{aligned} \tag{15}$$

$$\begin{aligned}\vec{V}_0 = {} & \vec{V}_\infty^+ + \vec{V}_2 = -V_\infty \sin\delta(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + \\ & + V_\infty \cos\delta(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, -\sin\gamma\sin\beta\sin\alpha + \\ & + \cos\gamma\cos\alpha, \cos\beta\sin\gamma) + (0, V_2, 0)\end{aligned} \tag{16}$$

where $\vec{V}_i$ and $\vec{V}_0$ are the velocity of the spacecraft with respect to the inertial frame before and after the swing-by, respectively.

From those equations, it is possible to obtain expressions for the variations in velocity, energy and angular momentum. They are:

$$\Delta\vec{V} = \vec{V}_0 - \vec{V}_i = -2V_\infty \sin\delta(\cos\alpha\cos\beta, \cos\beta\sin\alpha, \sin\beta) \tag{17}$$

which implies that:

$$\Delta V = \left|\Delta \vec{V}\right| = 2V_{\infty}\sin\delta \tag{18}$$

$$\Delta E = \frac{1}{2}\left(V_0^2 - V_i^2\right) = -2V_2V_{\infty}\cos\beta\sin\alpha\sin\delta \tag{19}$$

For the angular momentum ($\vec{C}$) the results are:

$$\begin{aligned}\vec{C}_i &= \vec{R}\times\vec{V}_i = d\,V_{\infty}(0, -\sin\beta\sin\delta + \cos\beta\cos\delta\sin\gamma,\\ &\frac{V_2}{V_{\infty}} + \cos\alpha\cos\delta\cos\gamma + \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma)\end{aligned} \tag{20}$$

$$\begin{aligned}\vec{C}_0 &= \vec{R}\times\vec{V}_0 = d\,V_{\infty}(0, \sin\beta\sin\delta - \cos\beta\cos\delta\sin\gamma,\\ &\frac{V_2}{V_{\infty}} + \cos\alpha\cos\delta\cos\gamma - \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma)\end{aligned} \tag{21}$$

Where $\vec{R} = (d,0,0)$ is the position vector of M_2.

Then:

$$\Delta\vec{C} = \vec{C}_0 - \vec{C}_i = 2d\,V_{\infty}\sin\delta(0, \sin\beta, -\cos\beta\sin\alpha) \tag{22}$$

and

$$\left|\Delta\vec{C}\right| = 2dV_{\infty}\sin\delta\left(\cos^2\beta\sin^2\alpha + \sin^2\beta\right)^{1/2} \qquad 23)$$

Using the definition of angular velocity $\omega = \frac{V_2}{d}$ it is possible to get:

$$\omega\Delta C_Z = -2V_2V_{\infty}\cos\beta\sin\alpha\sin\delta = \Delta E \tag{24}$$

For the inclination, the results are the following:

$$\mathrm{Cos}(i_i) = \frac{C_{iZ}}{\left|\vec{C}_i\right|} = \frac{1}{\sqrt{1 + \left(\dfrac{\sin\beta\sin\delta + \cos\beta\cos\delta\sin\gamma}{\dfrac{V_2}{V_{\infty}} + \cos\alpha\cos\delta\cos\gamma + \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma}\right)^2}} \tag{25}$$

$$\mathrm{Cos}(i_o) = \frac{C_{oZ}}{\left|\vec{C}_o\right|} = \frac{1}{\sqrt{1 + \left(\dfrac{\sin\beta\sin\delta - \cos\beta\cos\delta\sin\gamma}{\dfrac{V_2}{V_{\infty}} + \cos\alpha\cos\delta\cos\gamma - \cos\beta\sin\alpha\sin\delta - \cos\delta\sin\alpha\sin\beta\sin\gamma}\right)^2}} \tag{26}$$

where $\vec{C}_i$ and $\vec{C}_o$ are the initial and final angular momentum, respectvely, i_i and i_o are the initial and final inclinations, respectevely, and the subscript Z stands for the z-component of the angular momentum.

4 Results

With those equations available, the given initial conditions (values of r_p, v_p, α, β, γ) are varied in any desired range and the effects of the close approach in the orbit of the spacecraft are studied.

Figures 1 to 16 show the results. It was assumed that a satellite explodes when passing by the periapsis in a given position. In those examples, this position is given by $\alpha = 30°$, $\beta = 45°$. Then, a reference value was used for the direction of the velocity: $\gamma = 60°$. Two different values were used for the velocity at periapsis ($v_p = 4.0$ and $v_p = 4.5$) and two different values were used for the periapsis distance ($r_p = 1.5\ r_J$, $r_p = = 5.0\ r_J$), all of them expressed in canonical units. The vertical axis shows the difference between the value (inclination, velocity, energy and angular momentum) of every single particle and a reference value, that is the value that would exist if no explosion occurred, assumed to be the value of the particle that remains with the nominal values of γ. The horizontal axis shows the value of γ, in radians.

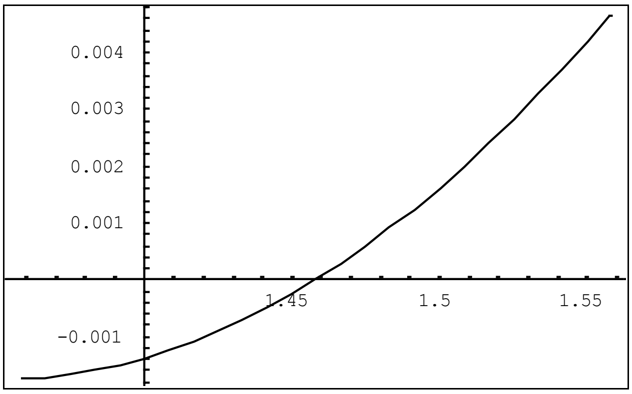

Figure 1. Variation in Inclination (rad) for $r_p = 1.5\ r_J$ and $v_p = 4.0$

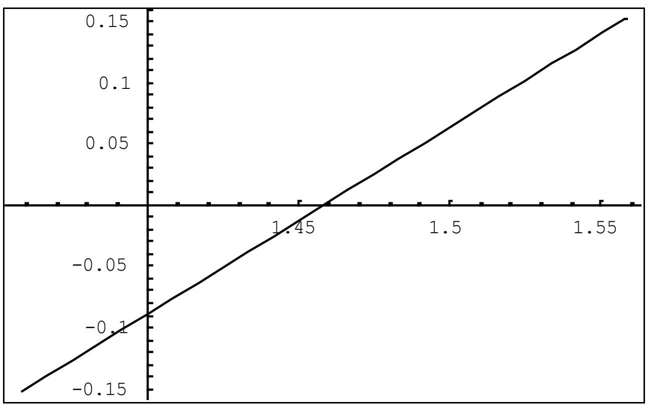

Figure 2. Variation in Velocity for $r_p = 1.5\ r_J$ and $v_p = 4.0$

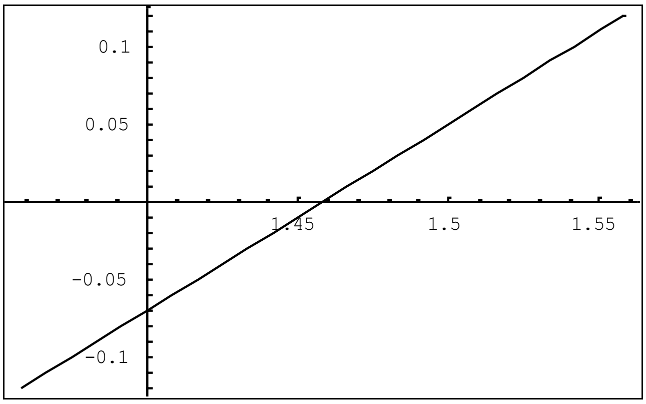

Figure 3. Variation in Angular momentum for $r_p = 1.5\ r_J$ and $v_p = 4.0$

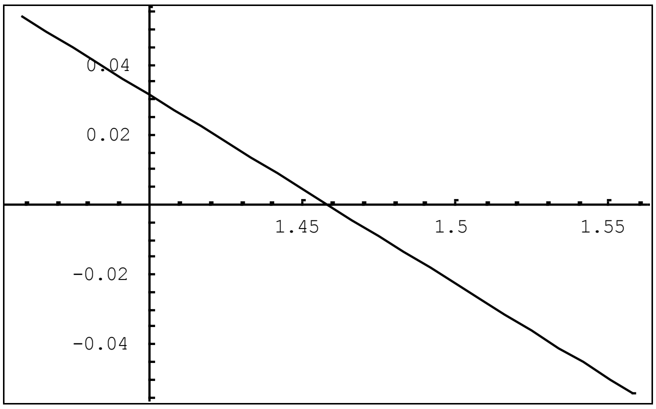

Figure 4. Variation in Energy for $r_p = 1.5\ r_J$ and $v_p = 4.0$

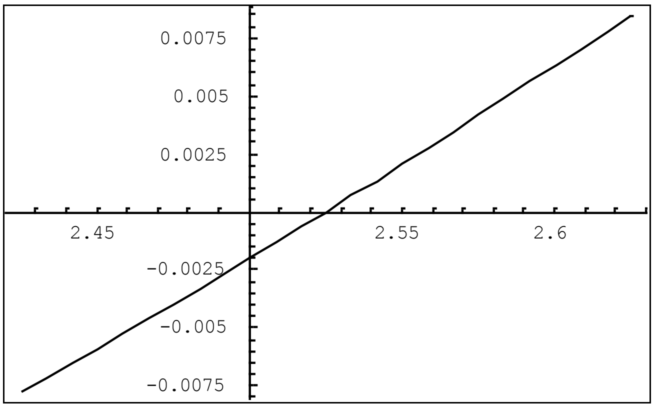

Figure 5. Variation in Inclination (rad) for $r_p = 1.5\ r_J$ and $v_p = 4.5$

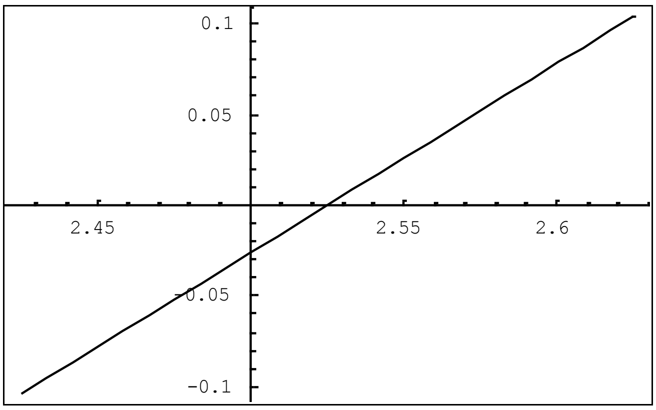

Figure 6. Variation in Velocity for $r_p = 1.5\ r_J$ and $v_p = 4.5$

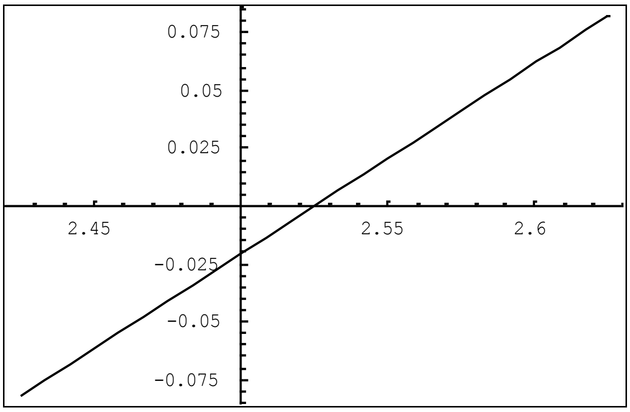

Figure 7. Variation in Angular momentum for r_p = 1.5 r_J and v_p = 4.5

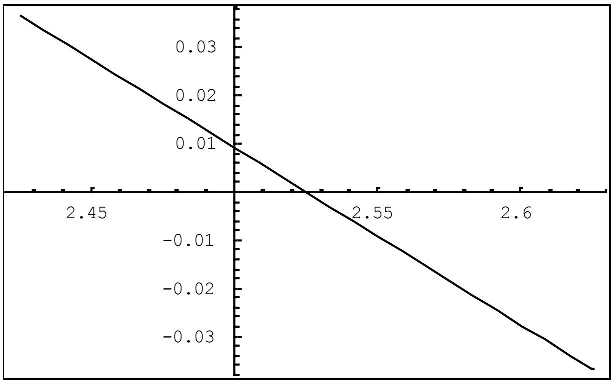

Figure 8. Variation in Energy for r_p = 1.5 r_J and v_p = 4.5

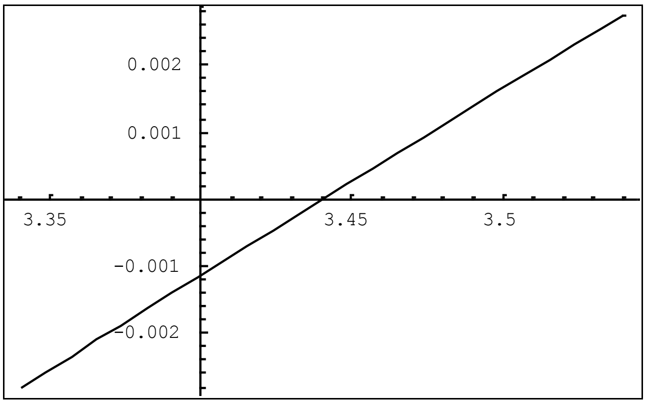

Figure 9. Variation in Inclination (rad) for r_p = 5.0 r_J and v_p = 4.0

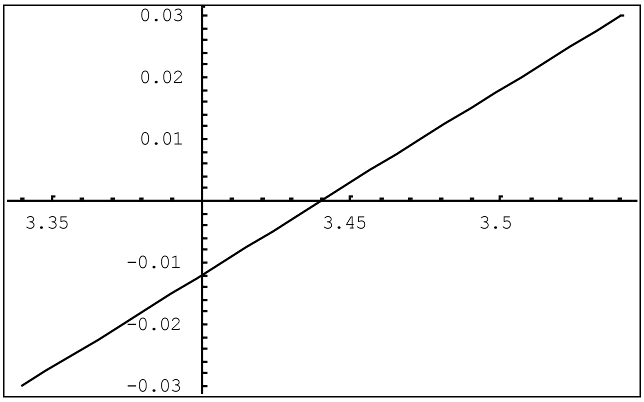

Figure 10. Variation in Velocity for r_p = 5.0 r_J and v_p = 4.0

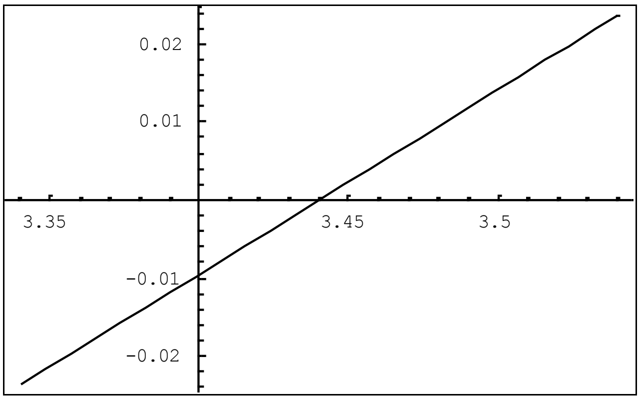

Figure 11. Variation in Angular momentum for r_p = 5.0 r_J and v_p = 4.0

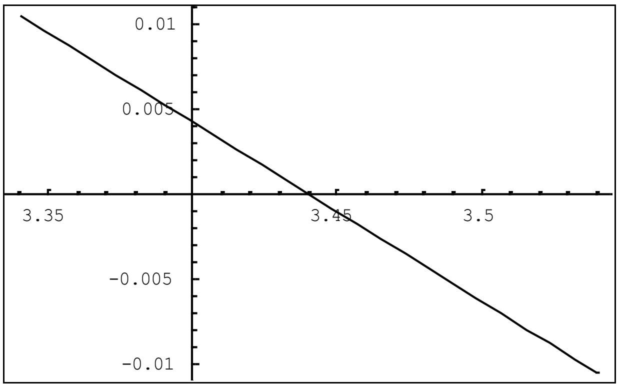

Figure 12. Variation in Energy for r_p = 5.0 r_J and v_p = 4.0

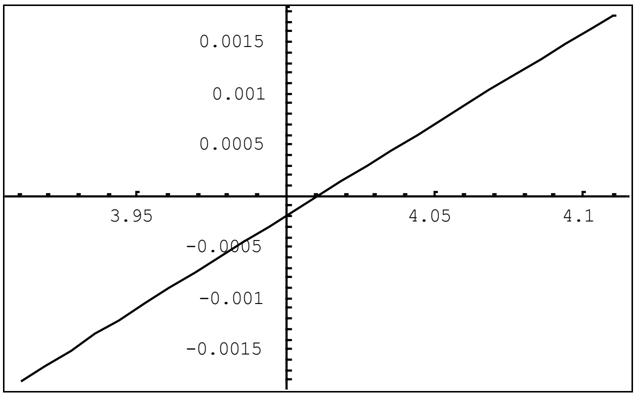

Figure 13. Variation in Inclination (rad) for r_p = 5.0 r_J and v_p = 4.5

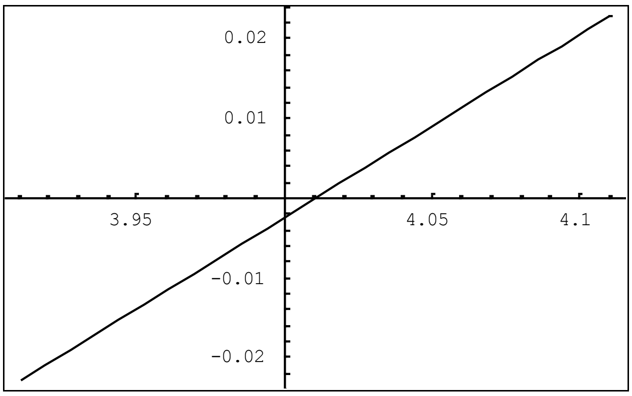

Figure 14. Variation in Velocity for r_p = 5.0 r_J and v_p = 4.5

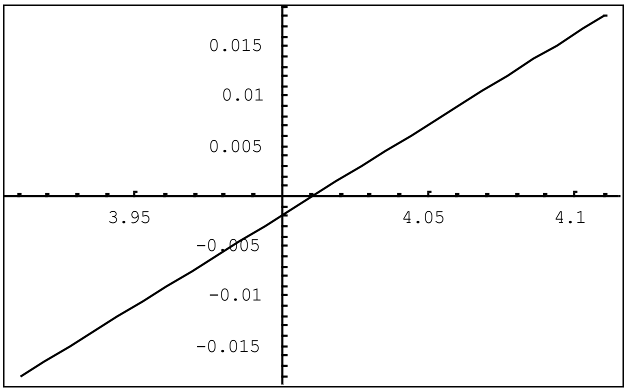

Figure 15. Variation in Angular momentum for r_p = 5.0 r_J and v_p = 4.5

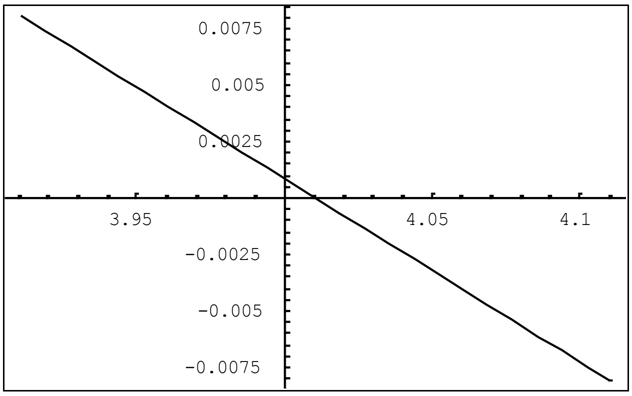

Figure 16. Variation in Energy for r_p = 5.0 r_J and v_p = 4.5

5 Conclusion

In this paper, analytical equations based in the patched conics approximation were used to calculate the variation in velocity, angular momentum, energy and inclination of a cloud of particles that performs a swing-by maneuver. The results show the distribution of those quantities for each particle of the cloud. Those results can be used to estimate the position of each individual particle in the future.

6 Acknowledgment

The authors are grateful to the FAPESP and CAPES for the finantial supports.

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