# Moisture Data Assimilation to the Global FSU Atmospheric Model

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#### Abstract

This paper shows the results of a data assimilation (DA) technique using artificial neural networks (NN) to obtain the analysis to the atmospheric general circulation model (AGCM) for the Florida State University, USA. The NN data assimilation is designed to emulate the initial condition from the Local Ensemble Transform Kalman Filter (LETKF). The DA techniques are coupled to the Florida State University Global Spectral Model (FSUGSM). The experiments are based on simulated observations data at each model grid localization and the FSUGSM 6-hours forecasts. This paper deals with the assimilation only for specific humidity observations. For the NN data assimilation, we adopted the Multilayer Perceptron (MLP) with supervised training. A self-configuration technique finds the optimal MLP topology parameters. The ANNs were trained with data from each month of years 2001, 2002, and 2003. And the data assimilation cycle at January 2005. The results demonstrate the effectiveness of the DA for atmospheric data assimilation, with similar quality of LETKF analyses. The major advantage of using ANN for DA is the computational performance, which DA with data assimilation is many times faster than LETKF.

**Keywords**: artificial neural network, data assimilation, LETKF, numerical weather prediction.

## 1. Introduction

In numerical weather prediction (NWP), models predict the time evolution of the atmospheric state by solving the system equations numerically. The state of atmosphere is described for a finite number of vertical levels and at a series of grid points by a set os state variables such as surface pressure,

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wind, temperature, specific humidity. Observations cannot be used directly to start model integration, but they most be modified in a dynamically consistent way to obtain a suitable dataset [4].

Model forecasts have limits to the predictability of the behavior of the atmosphere. It is because the chaotic dynamics are sensitive to the error in the initial state [1]. The model error is a source of uncertainty, like measures data. Even if the initial condition is well defined, the uncertanties can not affect the accuracy of forecasts.

Data assimilation (DA) is the process by which measurements and model predictions are combined to obtain an accurate representation of the state of the modelled system as its initial condition. Data assimilation techniques have been employed in the context of atmospheric and oceanic prediction, environmental and hydrological prediction, and ionosphere dynamics for some years. There are many different types of data assimilation algorithm, each varying in formulation, complexity, optimality and suitability for practical application. A useful overview of some of the most common data assimilation methods can be found in texts such as Daley [6] and Kalnay [7].

The ensemble Kalman filter (EnKF) [8] uses a probability density function associated with the initial condition, characterising the Bayesian approach [6], and represents the model errors by an ensemble of estimates in state space. The local ensemble transform Kalman filter (LETKF; [10]) is an EnKF-based scheme restricted to small areas (local) over a grid point. This method is implemented to this study.

The application of Artificial Neural Networks (NN) is an original Brazilian research from Laboratory of Mathematics and Applied Computational (LAC) of National Institute for Space Research (INPE). The experiments apply NNs to mimic other data assimilation methods and have shown consistent results with all implementations, see [12], [13], [14], [15], [17]. The main advantage to using NN is the speed-up of the data assimilation process.

This paper presents the approach based on NN to emulate the LETKF method of data assimilation. The method uses multilayer perceptron (MLP), referred as MLP-DA, trained with synthetic observation, simulating measurements for the surface and upper-air moisture variable. Humidity is the amount of water vapor in the air. Water vapor is the gaseous state of water and is invisible. Humidity (or moiture) indicates the likelihood of precipitation, dew, or fog. The specific humidity observations can obtained directly in meteorological stations or indirectly by remote sensors.

The experiment was conducted using the (*FSUGSM-Florida State Uni*versity Global Spectral Model) [20]. The grid of synthetic observations seeks to reproduce the dense network of observations. Here, a set of 12 NNs is employed to emulate the LETKF analysis, which is used, in training phase, as the target analysis. The analysis computed by the MLP-DA has been used to get the faster similar quality as the analysis produced by LETKF (see [19, <u>18</u>, 16]).

#### 2. Data Assimilation

Considering a general nonlinear system with a n-dimensional state vector  $x^f$  and a m-dimensional observation vector  $y^o$  evolving according to

$$x_{k+1}^f = f(x_k^f, t_k) + q_k \tag{1}$$

$$y_k^o = h(x_k^f, t_k) + v_k \tag{2}$$

where  $q_k$  and  $v_k$  are Gaussian noise terms.

In atmospheric data assimilation, the state  $x_{k+1}^f$  got by forecasting step, and observations  $y_k^o$  of the current states are combined by filtering step. Filtering is applied to balance the predicted state by forecasting step and current observations. Forecasting is a step to predict a state  $x_{k+1}^f$  of a system from the last state by numerical weather prediction model. The forecasting procedure is a nonlinear process. From mathematical point, the assimilating process can be represented by

$$x_k^a = x_{k+1}^f + W[y_k^o - H(x_{k+1}^f)]$$
(3)

$$W = (HP^f H^T + R) \tag{4}$$

The analysis step, in the equation (3), where H is the observation operator. W is the weighting matrix, generally computed from the error covariance matrices  $P^f$  and R represent model and observations errors, respectively, and they can also be updated in data assimilation scheme;  $x_k^a$  is the analysis field with innovation that results the initial conditions to the model. Observations  $y_k^o$  are information that can obtain from various sources, mainly from satellites sensors.

DA is a mathematical technique that enables the use of model and observational resources, offering the potential to generate more accurate forecasts. According [2], the DA purpose is to reconstruct as accurately as possible the dynamical system state, using all available appropriate information, e.g. the analysis for atmospheric flow is based on observational data and a model of the physical system, with some background information on initial condition. A problem in atmospheric data assimilation lies in the large number of degrees of freedom of NWP models. Very large numerical dimensions are

required:  $10^7 - 10^9$  parameters to be estimated with  $2 \times 10^7$  observations per 24-hour period. The large number of degrees of freedom of covariance matrices involved can prohibit the implementation of the best assimilation method known, taking into account the short time period to compute the prediction.

#### 3. Artificial Neural Network (NN)

NN is a computational system with parallel and distributed processing that has the ability to learn and store experimental knowledge. NN consists of interconnected artificial neurons or nodes, which are inspired by biological neurons and their behaviour. The neurons are connected to others to form a network, which is used to model relationships between artificial neurons.

The neuron processing can be nonlinear, parallel, local, and adaptable. Each artificial neuron is constituted by one or more inputs and outputs, and has a function to define outputs, associated with a learning rule. The connection between neurons stores a weighted sum, called synaptic weight. In NN processing, the inputs are multiplied by weights; the result is applied to the activation function. This function activates or inhibits the next neuron. Mathematically, we can describe the  $i^{th}$  input with the following form:

input summation: 
$$u_i = \sum_{j=1}^p w_{ij} x_i$$
  
neuron output:  $y_i = \varphi(u_i)$  (5)

where  $x_1, x_2, \dots, x_n$  are the inputs;  $w_{i1}, \dots, w_{ip}$  are the synaptic weights;  $u_i$  is the output of linear combination;  $\varphi(\cdot)$  is the activation function, and  $y_i$  is the *i*-th neuron output, n is number of patterns, p is number of neurons. A feed-forward network, which processes in one direction from input to output, has a layered structure. The first layer of an NN is called the input layer, the intermediary layers are called hidden layers, and the last layer is called the output layer. Some parameters as number of layers and the quantity of neurons in each layer define the neural network topology, but other parameters are also need to be computed, such as learning ratio and momentum. These parameters are determined by the nature of the problem.

The multilayer perceptron (MLP) is the NN architecture used in this experiment; which the interconnections between the inputs and the output layer have at least one intermediate layer of neurons, a hidden layer [23]. MLP uses: the training phase (learning process) and the run phase (activation process). The training phase of the NN consists of an iterative process for adjusting the weights for the best performance of the NN in establishing the mapping of input and target vector pairs. The goal is to minimize the

error between the actual output  $y_i$  and the target output  $(d_i)$  of the training data.

This training algorithm, here, is a supervised learning., e.g. the adjustments to the weights are conducted by back propagating of the error [23]. *Backpropagation* a well known strategy for training, and it is a version of the least mean square method, under studies for promoting improvements [24]. For a given input vector  $x_i$ , the output vector  $y_i$  is compared to the target vector  $d_i$ . If the difference is smaller than a required precision, no learning takes place; in the other hand, the weights are adjusted to reduce this difference. The purpose of the learning process is to minimize the output errors by adjusting the NN synaptic weights  $w_{ij}$ .

The generalization (or activation process) is the phase for which NN calculates the corresponding outputs, once it is trained and the MLP is ready to receive new inputs (different from training inputs). The set of weights is already defined during the learning phase and it is constant.

## 4. MPCA for NN configuration

The selection of appropriated NN topology is a complex task, and requires a great effort by an expert, identifying the best parameter set to solve the problems.

In the present study, an automatic tool is used to configure the parameters of NNs, identifying the best topology for given NNs. The methodology developed by [25] deals with self-configuration using a new meta-heuristic called the Multiple Particle Collision Algorithm (MPCA) ([26], [31]) to compute the optimal topology for an MLP. The Particle Collision Algorithm (PCA) starts with a selection of an initial solution, it is modified by a stochastic perturbation leading to the construction of a new solution. The new solution can or cannot be accepted. If the new solution is not accepted, the particle can be send to a different location of the search space (*scattering*), giving the algorithm the capability of escaping a local minimum. If a new solution is better than the new solution is adopted (*absorption*) ([26],[31]). The implementation of the MPCA algorithm is similar to PCA, but it uses a set of particles, where a mechanism to share the particles information is applied.

The main advantage in using this procedure to configure NN is the ability to define the near-optimal NN arquitecture, without needing the help of experts on the NN approach. Such approach avoids this time consuming and tiring process of trial and error to find the optimal neural network topology - see [27]. The MPCA is a stochastic optimization procedure. Therefore, several realizations are performed with MPCA. The same parameters to set up the MPCA are used to identify the best NN topology.

## 5. Local Ensemble Transform Kalman Filter (LETKF)

The EnKF originated as a version of the Extended Kalman Filter (EKF) [30]. The classical KF method, see[5], is optimal in the sense of minimizing the variance only for linear systems and Gaussian statistics. In the EnKF approach, an ensemble of state estimates can be used to calculate an approximate mean and error covariance matrix  $P_{n+1}^f$ .

The LETKF scheme is a model-independent algorithm to estimate the state of a large spatial temporal chaotic system [9]. It is a EnKF-based scheme. The term "local" refers to an important feature: it solves the Kalman filter equations locally for each mesh point, applying a cut-off radius of influence for each observation. The ensemble transform matrix, composed of the weights of the linear combination, it is computed for each local subset of the state vector independently. The basic idea of LETKF is to perform analysis at each grid point simultaneously using the state variables and all observations in the region centred at given grid point. The local strategy separates groups of neighbouring observations around a central point for a given region of the grid model [11].

The algorithm follows the sequential assimilation steps of classical Kalman filter [5], but it calculates the error covariance matrices with the ensemble mean of forecasting  $(\bar{x}^f)$ . Each member of the ensemble gets its forecast  $\{x_{n-1}^f\}^{(i)}$  :  $i = 1, 2, 3, \dots, k$ , where k is the total members at time  $t_n$  to estimate the state vector  $\bar{x}^f$  of the reference model. The analysis step determines a state estimate to each ensemble member:

$$\{x^a\}^{(i)} = \{x^f\}^{(i)} + \{[P^f H^T [HP^f H^T + R]^{-1}]] [y^o - H(\{x^f\}^{(i)})]\}.$$
 (6)

The matrices R and H represent the observation error and observation operator, respectively. The model covariance matrix  $P^f$  is associated with the forecast model  $x^f$ , updated at forecast step. The analysis  $\{x^a\}^{(i)}$  : i = $1, 2, 3, \dots, k$ , (eq. 6) is solving by getting the optimal weight (e.g. Kalman gain), melting observations  $y^o$ . The analysis step also updates the analysis covariance error matrix  $P^a$ , where the ensemble mean is expressed by:

$$\bar{x}^a \equiv k^{-1} \sum_{i=1}^k \{x^a\}^{(i)} .$$
(7)

The LETKF system runs with 40 members in this experiment. The LETKF scheme is performed, and we obtain the analysis for each member, the fore-cast average and analysis average of all members.

## 5. Florida State University Global Spectral Model (FSUGSM)

The two DA methods (MLP-DA and LETKF) are applied to FSUGSM, an atmospheric general circulation model. The computer model is a global three-dimensional primitive-equation system, simulating atmospheric dynamics for the entire global circulation [28]. The dynamical processes are the six primitive equations to forecast atmospheric motion: vorticity, divergence, thermodynamic, continuity, hydrostatic, and moisture, which are expanded in their spectral form. The nonlinear terms are calculated on a Gaussian grid using a transform method. Details, physical parametrizations, equations and numerical methods can be found in [21] and [20].

The vertical coordinates are defined on sigma ( $\sigma = p/p_0$ ) surfaces, where  $p_0$  is the surface pressure and p is the layer pressure. The horizontal coordinates are latitude and longitude on real space. The FSU model is global with spectral resolution T63L27 (horizontal truncation of 63 numbers of waves and 27 vertcal levels), the gaussin grid corresponding to a regular grid with 192 zonal points (longitude), 96 meridian points (latitude) (approximately  $1.875^{\circ} \times 1.875^{\circ}$ ), and 27 unevenly spaced vertical levels.

The prognostic variables for the model input and output are the absolute temperature (T), surface pressure  $(p_s)$ , zonal wind component (u), meridional wind component (v), and an additional variable (specific humidity q).

#### 6. MLP-DA for FSUGSM

DA process generates a model state that is consistent with the observed data, which can be used as an initial condition for next model prediction period; this run is called the DA cycle. The LETKF and MLP-DA are tested with synthetic observations simulating specific humidity (all layers) at the model grid point localization. The NN topology for this experiment is a set of multilayer perceptrons, configured by MPCA tool. The experiment consist of DA cycles with MLP-DA and LETKF to obtain the results and comparing their effectiveness, see [22].

One strategy used to collect data and to accelerate the processing of the MLP-DA training was to divide the entire globe into four regions: for the Northern Hemisphere,  $90^{\circ}$  N and two longitudinal regions of  $180^{\circ}$  each; for the Southern Hemisphere,  $90^{\circ}$  S and two longitudinal regions of  $180^{\circ}$  each.

This division provides the same size for each region, and the same number of observations as illustrated by Fig. (). This regional division is applied only for the MLP-DA. The MLP-DA scheme is developed with a set of twelve NNs, e.g. four regions with three layers with specific humidity variable (q) vectors. Firstly, we run the FSUGSM to generate the fields and then, collect the synthetic observations based on the model grid localizations. The observational grid is a regularly distributed in the dense network; it has (45 x 96 x 27) points for the upper-air q. The grid localization is every other latitude/longitude grid point of the FSUGSM native grid of (96 x 192 x 27), (see Fig. ). The next step is to perform the LETKF analysis-forecast cycle for obtaining the datasets with: the FSUGSM 6h-forecast and runs the observations routine. The input and target vectors are collected from forecasts and analyses averages from LETKF results.

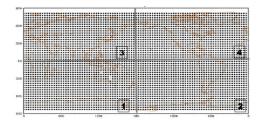


Figure 1 - Observations localizations, divides in four regions of global area, each is  $(90^{\circ} \times 180^{\circ})$  size. The dot points represent stations.

In this experiment, the MPCA runs with input vectors of q dataset from simulated observations and from FSUGSM model. The target vector used is q from LETKF analysis. The MLP training phase, with supervised algorithm, for four NNs to each region data and to each set of three layers variable (each layers has nine values). The datasets collected, with specific humidity values, train a set of MLP with:

- 1. Four input nodes, one node for the synthetic observation vector and other for the 6-hours forecast model vector, a node for grid point horizontal coordinate and a node for grid point vertical coordinate. The vectors values represent individual grid points for a single variable with a correspondent observation value;
- 2. One output node for the analysis vector results. In the training algorithm, the MLP-DA computes the output and compared it with the analysis vector of LETKF results (the target data). The vectors represent the analysis values for one grid point.

each.

The parameters (see Table (1)), according the MPCA tool results, is used to make the MLP-DA topology for q analysis.

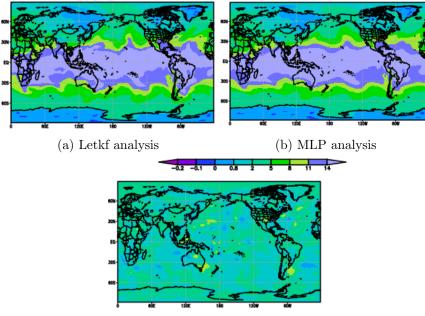
Table 1: Parameters of MPCA topology found to 12 MLPs for q variable, with one hidden layer.

| NETWORK         | NEURONS        | LEARNING RATE | MOMENTUM RATE |
|-----------------|----------------|---------------|---------------|
| (var/reg/layer) | (Hidden Layer) | $\eta$        | $\alpha$      |
| qq0101          | 09             | 0.424676      | 0.735560      |
| qq0102          | 07             | 0.695070      | 0.836189      |
| qq0103          | 09             | 0.128201      | 0.987913      |
| qq0201          | 09             | 0.091828      | 0.621134      |
| qq0202          | 10             | 0.247087      | 0.997031      |
| qq0203          | 09             | 0.068212      | 0.994036      |
| qq0301          | 05             | 0.601685      | 0.447649      |
| qq0302          | 06             | 0.543795      | 0.980525      |
| qq0303          | 10             | 0.852829      | 0.909061      |
| qq0401          | 06             | 0.517619      | 0.949778      |
| qq0402          | 10             | 0.355837      | 0.975882      |
| qq0403          | 10             | 0.438510      | 0.995963      |

After configuration of the MLPs, we perform the follow processes:

*Training* - The MPCA process begins after collecting the input/target vectors for whole period (one month for three years). The MPCA stops the training process when finds the best fitness. The training is performed with combined data from January at 2001, 2002, and 2003.

Activation - This process is, indeed, the data assimilation process. The MLP-DA results in a global analysis field. The MLP-DA activation is entering by input values (only 6 hours forecast, observations and coordinates) at each grid point once, with no data used in the training process. The procedure is the same for all MLPs, but each region and layer has different connection weights, which was obtained in training phase. The MLP-DA is performed for one-month cycle, e.g. 124 analysis-forecast cycles. The assimilation cycle begins as soon as the 6hs-forecast and observations are ready. It starts at 00:00 UTC at January 01, 2005, with FSUGSM model producing a 6-hours forecast and with observations produced previously, then, MLP-DA generate the initial condition to next FSUGSM. The process is repeated at each



(c) Differences analysis

six synoptic hours and generates analyses and 6 hours forecasts up through 31 January 2005 1800UTC.

## 7. Results

The input and output values of prognostic variable (q) are processed on grid model points for time integrations to an intermittent forecasting and analysis cycle for both DA methods. The results show the comparison of analysis fields, generated by the MLP-DA and the LETKF. Each NN is independent, and each one has different weights, different parameters, with different outputs. The twelve MLPs are gathered for a global analysis.

Figure (2) presents the global specific humidity fields in kilograms by kilograms (Kg/Kg) generated from assimilation cycle at 04/Jan/2005 - 12UTC. The differences between analysis from the MLP-DA and LETKF in surface fields are displayed in Fig. 1c. The differences are in the interval (2, -2 Kg/kg), and some points reached (-6 kg/kg) (on the Equator) or (6 kg/kg) (on de oceania).

Figure (3) presents the global specific humidity (q) at level 500 hPa fields in kilograms by kilograms (Kg/Kg) generated from assimilation cycle at 04/Jan/2005 - 12UTC. For evaluation of the analysis impact on the pre-



Figure 2: Surface Specific Humidity (Kg/Kg) Fields to 04/01/2005 at 12 UTC. (a) LETKF analysis (b) MLP-DA analysis (c) difference between LETKF and MLP-DA analyses.

diction, we can see Figs. (3c), and (3d), showing the difference between the analyses. The difference are in the interval (2 , -2 Kg/kg) in some points close to the South pole.

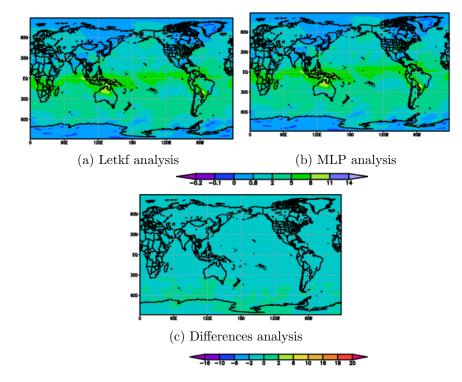


Figure 3: Specific Humidity (Kg/Kg) Fields in 500 hpa to 04/01/2005 at 12 UTC. (a) LETKF analysis (b) MLP-DA analysis (c) difference between LETKF and MLP-DA analyses.

The difference between the analysis and the control field (used to obtain simulated observations) is showing in Fig (4). These error maps for both assimilation methods are very similar. From this consideration, we can asseverate that both analysis will be to produce similar predictions. The error maps for other meteorological variables have similar behaviour (not shown).

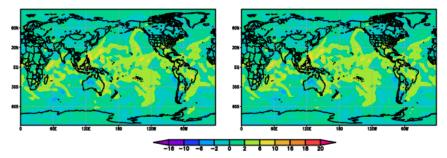


Figure 4: Specific Humidity (Kg/Kg) Fields in 500 hpa to 04/01/2005 at 12 UTC. Differences between LETKF and MLP-DA analyses.

Table 2: Total running time of 124 cycles of complete data assimilation (analysis and forecasting) for January/2005.

| Execution of 124 cycles | MLP-DA (hour:min:sec) | LETKF (hour:min:sec) |
|-------------------------|-----------------------|----------------------|
| Analisys time           | 00:02:29              | 11:01:20             |
| Ensemble time           | 00:00:00              | 15:50:40             |
| Parallel model time     | 00:27:20              | 00:00:00             |
| Total time              | 00:29:49              | 26:52:00             |

The computational efficiency for DA for both methods is shown in Table (2). For the adopted resolution, the MLP-DA is 266 times faster than LETKF producing analyses with same quality. Based on the same Table, the total time to run 124 cycles to January/2005, (to run analysis, obtain observation and run the model to obtain 6-hours forecasts) is 55 times faster than LETKF cycles.

#### 8. Conclusion

The MLP-DA data assimilation cycle is composed by 6-hours forecast from the FSUGSM model and set of observations to compute the global analysis field. The comparison in Table 2 is the data assimilation cycles for the same observations points and the same model resolution to the same time simulations. LETKF and MLP-DA executions are performed independently. Considering the total execution time of those 124 cycles (01/01/2005-31/01/2005) simulated with assimilated multilevel humidity variables (e.g. the other variables are the same from forcast fields to create a initial condition (analysis) dataset), the computational performance of the MLP-DA data assimilation, is better than that obtained with the LETKF approach.

Observations data are informative to understand weather behaviour.

The volume of the observations is increasing exponentially. In addition, the model resolution is enhancing. This scenario represents a huge challenge: our computers and algorithm are unable for processing all observation available for the operation window time. Therefore, strategies for reduction of observation dimension are implemented. These data come from the global meteorological network and satellites, by countries around the world.

The challenge of numerical weather prediction (NWP) is faced with new data volume acquisition for the data assimilation process: high spectral resolution meteorological satellites, data from radio occultation of low orbit satellite, environmental satellites (trace gases, aerosols, reactive gases), satellite measuring clouds and precipitation. The evolution of model resolutions in the horizontal and vertical coordinates is a challenge too, at the same time, physical parametrizations are improving, and there is a tendency for coupling systems: atmosphere, ocean, and land in a variety of applications. These challenges are dependent of computer algorithms and data assimilation techniques that supports these challenges.

The artificial neural networks can be a possible technique to address these challenges.

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