Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Numerical Experiments for Time Integration of 2D Burgers' Equations

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1 Introduction

Numerical solution of partial differential equations requires the choice of a time integration method capable of simulating the evolution of a problem. While traditional methods are usually categorized into explicit and implicit, each with their own sets of advantages and disadvantages, a more recent approach is the combination of both types into the so called IMEX schemes. These were designed to solve equations containing fast and slow time-scales in such a way that the slow terms can be solved explicitly, while the fast terms are solved implicitly, mitigating the disadvantages of each individual scheme. In this work, the finite difference approach is used to solve a two-dimensional, viscous Burgers' system in a series of numerical experiments using each of the aforementioned time integration schemes.

2 Numerical Solution of 2D Burgers' Equations

Burgers' equation is one of the fundamental partial differential equations in fluid mechanics [2], and can be used to model several physical phenomena. Its known analytical solution [1] allows evaluation of the numerical experiments. When considering two dimensions (x, y) and two velocity components (u, v), Burgers' equation is written as a system of two coupled PDEs. In matrix form, it reads:

$$\begin{bmatrix} u \\ v \end{bmatrix}_t = - \begin{bmatrix} u_x & u_y \\ v_x & v_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \frac{1}{R} \begin{bmatrix} \nabla^2 & 0 \\ 0 & \nabla^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{1}$$

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which can be interpreted as:

$$\Phi_t = f(\Phi) + \frac{1}{R} g(\Phi) \tag{2}$$

where $f(\Phi)$ is a slow, convective term and $g(\Phi)$ is a fast, diffusive term.

We implement three numerical solutions for Equation 1 – one explicit (FTCS), one implicit (Crank-Nicolson), and one IMEX (Adams-Bashforth for f, Crank-Nicolson for g) – in order to compare their results. Partial results for a test case, presented in Table 1, show that both explicit and implicit schemes offer good results and converge to minimal errors as spatial resolution increases. The proposed IMEX scheme, however, does not converge under the same conditions. It is not yet clear nor obvious why this occurs, and further analysis is required.

Table 1: L^1 norm of u in 2D Burgers' system for proposed test case.

Number of cells	*Explicit*	*Implicit*	*IMEX*
$(4, 4)$	1.3726 E-009	1.3727 E-009	1.3455 E-009
$(8, 8)$	4.5893 E-010	4.5902 E-010	1.2062 E-009
$(16, 16)$	1.2700 E-010	1.2710 E-010	2.6562 E-008
$(32, 32)$	3.2953 E-011	3.3056 E-011	3.4057 E-007

3 Final Remarks

IMEX schemes are a promising way to increase computational efficiency in the solution of PDEs. Because the stability conditions are softened, the computational performance of such methods can be improved by using larger time steps when compared to a purely explicit scheme. However, their implementation is not as simple as it first appears, and more work is required to evaluate why results are not converging for a problem that other, simpler schemes can solve easily.

Acknowledgments

The authors would like to thank CAPES for the financial support on this work.

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