Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **474,** 618–624 (2018) doi:10.1093/mnras/stx2785
Advance Access publication 2017 November 1

The two-component virial theorem and the acceleration–discrepancy relation

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Accepted 2017 October 24. Received 2017 October 4; in original form 2017 May 20

ABSTRACT

We revisit the 'two-component virial theorem' (2VT) in the light of recent theoretical and observational results related to the 'dark matter problem'. This modification of the virial theorem offers a physically meaningful framework to investigate possible dynamical couplings between the baryonic and dark matter components of extragalactic systems. In particular, we examine the predictions of the 2VT with respect to the 'acceleration–discrepancy relation' (ADR). Considering the combined data (composed of systems supported by rotation and by velocity dispersion), we find the following: (i) The overall behaviour of the 2VT is consistent with the ADR. (ii) The 2VT predicts a nearly constant behaviour in the lower acceleration regime, as suggested in recent data on dwarf spheroidals. We also briefly comment on possible differentiations between the 2VT and some modified gravity theories.

Key words: galaxies: fundamental parameters – galaxies: kinematics and dynamics – dark matter.

1 INTRODUCTION

A fundamental question currently spanning astrophysics, cosmology and particle physics is the 'dark matter problem'. It arose in the 1930s as a curious disagreement in the mass estimates of astronomical bodies, and persists today as one of the greatest unsolved problems in physics (Bertone & Hooper 2016; Gaskins 2016; Freese 2017). Dynamical mass estimates of galactic systems exceed those obtained from their luminous contributions. Such a discrepancy would, in principle, be solved by conjecturing the existence of some additional mass (undetectable in terms of electromagnetic emission): its contribution would then account for the gravitational binding of galaxies and clusters of galaxies.

This initial puzzle deepened in the past decades, with the improvement of observational techniques, bringing forth higher quality surveys and the gravitational lensing methods (Treu 2010), expanding our studies of extragalactic systems and large-scale structures. At the same time, the accuracy of cosmological parameters improved (Planck Collaboration XIII 2016a), requiring not only a 'non-baryonic dark matter' (non-baryonic DM), but also a dominant 'dark energy' (DE) component, in order to account for an apparent accelerated cosmic expansion of the Universe. These investigations established a 'standard model' of big bang cosmology: the 'Lambda cold dark matter' (ΛCDM) model, providing a relatively consistent picture to describe various independent properties of the Universe (for implications of recent *Planck* data regarding a few tensions with independent results, see Planck Collaboration XIII 2016a; and with respect to alternative scenarios, see Planck Collaboration XIV 2016b).

The virial theorem (VT; cf. Binney & Tremaine 1987) is one of the main methods to estimate the total mass of a galactic system. In Zwicky's pioneering paper (Zwicky 1937), the VT was used to estimate the total mass of the Coma cluster, under the assumption that this system is in stationary equilibrium, at least in a first approximation (as inferred from its regular, spherically symmetric distribution). This work not only marks the beginning of the 'missing mass problem', but also highlights the importance of the VT as a fundamental tool in extragalactic astrophysics, particularly in the context of this problem (for an historical account on virial mass estimates of galaxy clusters, see e.g. Biviano 2000). The application of the VT is not straightforward, as it requires an underlying hypothesis of stability of the system and well-understood observational selection effects (a pioneer study in this regard is given in Aarseth & Saslaw 1972). More recently, gravitational lensing mass estimates have shown compatibility with the idea that the overestimation of total virial masses can be attributed by the presence a dominant DM component (Treu 2010).

The VT refers to a global condition on the kinetic and potential energies of the system as a whole. But it is also a matter of great interest to gain further information on the detailed equilibrium

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requirements of a cluster composed of other subsystems, such as a less massive luminous component embedded in a dominant halo. In 1959, Limber (1959) first derived a more general, two-component VT form, in order to model a cluster embedded in an extended (nonviscous) gaseous background, interacting only through gravity. Clearly, Limber's 'two-component virial theorem' (2VT) can also be applied to DM haloes, by re-interpreting the 'extended gaseous background' as a DM halo. In this regard, Smith (1980) designated the 'Limber effect' the overestimation of the total mass, obtained from the application of the (usual) VT, for systems with an extended, unseen background.

In certain VT applications, constraints on the DM halo can be obtained. For example, when the stellar contribution to the gravitational field can be considered sufficiently small in comparison to the DM component, so that the former is primarily moving in the gravity field of the DM halo. Thus, for such a tracer stellar populations, the tensor VT (Binney & Tremaine 1987) could be used to constrain the DM halo in our Galaxy (Agnello & Evans 2012a) and in the dwarf spheroidal galaxy Sculptor (Agnello & Evans 2012b). By extending the tensor VT to subsystems, more information can be obtained about individual components, than that acquired by the application of the usual VT to the system as a whole (Brosche, Caimmi & Secco 1983; Caimmi, Secco & Brosche 1984; Caimmi & Secco 1992). For instance, the structural configuration of a component in equilibrium may be distorted by the tidal force induced by the other, introducing a length dependence on the baryonic subsystem induced by the DM halo (Marmo & Secco 2003). These, and possibly other dynamical effects, could provide, at least partially, a regulatory mechanism for explaining tight observational constraints, such as the 'Fundamental Plane' of elliptical galaxies (Djorgovski & Davis 1987; Dressler et al. 1987; Capelato, de Carvalho & Carlberg 1995; Dantas et al. 2000, 2003; D'Onofrio et al. 2016), including a general, combined observational effect comprising a large range in scales and different types of systems, the so-called 'Cosmic Metaplane' (Burstein et al. 1997; Dantas et al. 2000; Secco 2000).

A recent observational result of particular interest is the existence of a tight correlation between the radial acceleration derived from rotation curves of galaxies and the observed distribution of baryons [hereafter the 'acceleration–discrepancy relation' (ADR); McGaugh, Lelli & Schombert 2016; Lelli et al. 2017]. This empirical relation suggests a strong coupling between dark and baryonic components, possibly related to galaxy formation mechanisms. The 2VT provides a physically meaningful framework to investigate dynamical couplings between these components, based on their mutual equilibrium conditions. Note that the usual (one-component) VT does not address any systematic couplings between 'hidden' and baryonic masses, it just implies that a 'remainder' mass must be added, by contingency, to the dynamical equilibrium budget of the system. But the 2VT formulation indicates a correction that depends systematically on the dark component in which the baryonic mass is embedded.

In this paper, we revisit the 2VT to address these recent theoretical and observational results. Our paper is outlined as follows. In Section 2, we present the 2VT, in terms of a suitable expression to fit the data in the ADR space. In Section 3, we compare the 2VT predictions with data for late-type galaxies (LTGs) and dwarf spheroidals (dSphs) and discuss the 'flattening' behaviour of dSphs, as indicated in recent data. Finally, we briefly comment on possible differentiations between the 2VT and some modified gravity theories. In Section 4, we present our conclusions.

The usual, one-component, VT will be denoted here as '1VT' (cf. equation A1 in Appendix A). Our notation uses the index B to refer to the baryonic matter, D to refer to the DM component and ρ to refer to the respective average matter density within a given radius r.

2 THE TWO-COMPONENT VIRIAL THEOREM

In this section, we present the 2VT (Limber 1959; Dantas et al. 2000; Secco 2000) in a suitable form to be compared with the ADR (McGaugh et al. 2016; Lelli et al. 2017). In Appendix A, we review the 2VT as originally derived in Dantas et al. (2000) and present details on its derivation.

The 2VT provides a correction term to the 1VT, which accounts for the influence of a second component (the putative DM halo). In terms of acceleration variables, the observed acceleration, as predicted by the 2VT, is given by (cf. Appendix A)

$$g_{\mathrm{obs,2VT}} = g_B + R g_D. \tag{1}$$

where g_B is the radial acceleration of the baryonic component, g_D is the radial acceleration associated with the dark component (cf. equation A5) and R is a parameter depending only on the properties of the baryonic matter distribution, relating the projected (2D) to the 'physical' (3D) radii of the observed baryonic component. The 1VT is simply

$$g_{\mathrm{1VT}} = g_B. \tag{2}$$

The 2VT gives a simple linear correction to the 1VT, and the behaviour of the 2VT as seen in a log–log plot appears as curved line. In Appendix C, we present a brief illustration of the 2VT as seen in that plot, showing how variations in the parameters (g_D, R) affect it. Here, we highlight two relevant facts:

(i) The 2VT curve with a fixed (g_D, R) represents a parametrized family of baryonic–DM systems within an arbitrary baryonic mass range.

(ii) The 2VT predicts some discriminating characteristics in the log–log plane: *(i)* a bending departing from the 1VT and *(ii)* a nearly constant behaviour in the lower acceleration regime. The parameters (g_D, R) cannot modify these features, only allowing for an adjustment of the height of the asymptotic behaviour in the lower acceleration regime, or, alternatively, the point at which the 1VT is retrieved.

In other words, even with two free parameters, the 2VT presents a relatively 'rigid' form, implying that this model cannot be arbitrarily contrived to fit the data. This level of predictive power is an important feature of the 2VT.

3 OBSERVATIONAL COMPARISONS

In this section, we analyse the 2VT in terms of current observational data leading to the so-called ADR (McGaugh et al. 2016; Lelli et al. 2017).

3.1 The acceleration–discrepancy empirical relation

The ADR is an empirical best-fitting relation for the centripetal acceleration estimated from the rotation curves of rotationally supported galaxies, and it was first described by McGaugh et al. (2016):

$$\mathcal{F}(g_B) = \frac{g_B}{1 - e^{-\sqrt{g_B/a_M}}}. \tag{3}$$

Note that the above relation tends to a linear slope at high accelerations and $g_{obs} \propto \sqrt{g_B}$ at low accelerations.

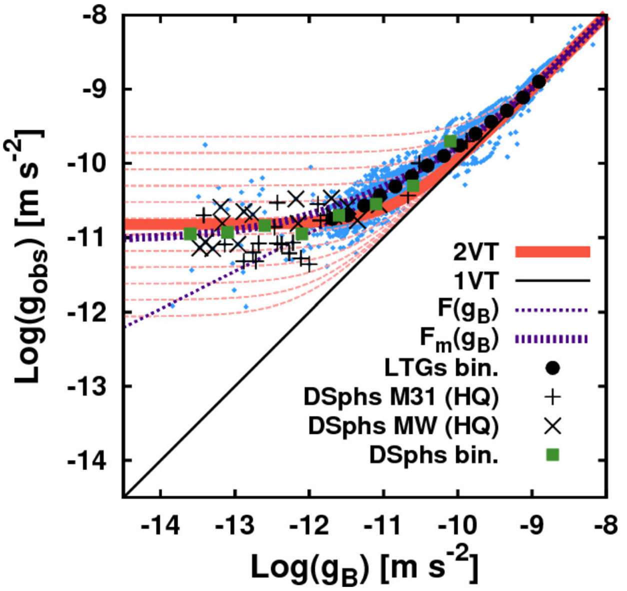

Figure 1. (Colour online). The acceleration obtained from the baryonic matter ($g_{\rm B}$) versus the observed acceleration ($g_{\rm obs}$). The 2VT predicts that $g_{\rm obs}$ obeys equation (1). We show a series of 2VT curves (thin dashed lines) covering most of the data, using different R values, in equal (dex) steps, from $R = 1.905$ (upper) to $R = 0.007$ (lower curve), with $g_{\rm D}$ fixed to the Milgrom a_M scale (see the main text). An 'eyeball' best 2VT curve for the data as a whole is also indicated (thick continuous line), with $R = 0.125$. Empirical relations expressed by equation (3) [labelled '$F(g_B)$'; thin dotted line] and equation (4) [labelled '$F_m(g_B)$'; thick dotted line], with the respective best-fitting parameters obtained in Lelli et al. (2017), are shown for comparison. Data is taken from Lelli et al. (2017) and corresponding binned data are shown in larger symbols, as indicated in the legend.HQ data for dSphs (M31 and Milky Way) are highlighted with different symbols. Small dots in the background represent the full data set for both LTGs and dSphs. The line of unit ($g_{\rm obs} = g_B$) expresses the 1VT expectation for baryons only.

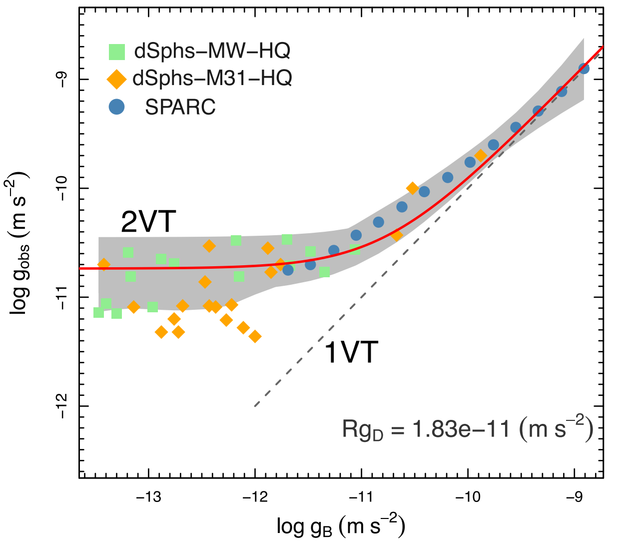

Figure 2. (Colour online). Same as Fig. 1, using binned LTG data and HQ dSphs data, with a 2VT best-fitting curve obtained from a linear regression method, including the 95 per cent confidence band around the regression fit (grey band), as explained in the main text. Fixing g_D to Milgrom's acceleration scale (equation 5) gives $R = 0.152$.

The inclusion of dSphs data (Lelli et al. 2017), however, seem to imply a flattening of the above relation, specially for the ultra-faint dSphs. The present data are not conclusive, but this flattening behaviour seems to be favoured at this time, and suggested a modification of the relation above to (Lelli et al. 2017)

$$\mathcal{F}_m(g_B) = \frac{g_B}{1 - e^{-\sqrt{g_B/a_M}}} + \hat{g} e^{-\sqrt{g_B a_M/\hat{g}^2}}, \quad (4)$$

as a better fit to the data, for the tendency of dSphs to deviate from the original ADR (equation 3).

3.2 The 2VT and the ADR

In Fig. 1, we reproduce the suggested ADR best-fitting forms, equations (3) and (4), together with a series of 2VT curves, equation (1), for illustrative purposes. We provisionally fix g_D to Milgrom's acceleration scale (Milgrom 1983a),

$$a_M \equiv 1.2 \times 10^{-10} [ms^{-2}], \quad (5)$$

and use different values of R, covering most of the data (represented by small dots in the figure; binned data is indicated by larger symbols). Detailed fits are presented below (see also Appendix C).

The data was taken from (Lelli et al. 2017), which includes rotationally supported systems (LTGs from SPARC data) and dSphs, which are supported by velocity dispersion. We indicate the subset of the 'high quality' (HQ) data which, among other cases regarding both the quality of the velocity dispersion determination as the sphericity of the dSphs, excludes all those strongly affected by the tidal forces produced by their host galaxies (see Lelli et al. 2017 for details). It is interesting to compare our figure with fig. 3 of McGaugh et al. (2016) and figs 10–12 of Lelli et al. (2017).

LTGs are preferentially located nearby the transition scale a_M between the Newtonian and MONDian ('MOdified Newtonian Dynamics'; Milgrom 1983a,b,c) regimes, whereas dSphs are mainly located at lower acceleration regimes (see also Sec. 3.5 below). The ultrafaint dSphs particularly contributes to the data in the lower acceleration region. As Fig. 1 shows, a band of 2VT curves cover in the right sense the ADR of rotationally-supported galaxies, including the low-acceleration flattening region, indicated by ultrafaint dSphs.

We also provide a quantitative fit to the 2VT (Fig. 2), using the binned data for LTGs and the HQ subset dSphs data, obtained with linear regression, a procedure that allows us to find the value for the parameter $\mathcal{G} \equiv Rg_D$ (cf. equation 1), with respective standard errors, minimizing the sum of the squares of the data points distances to the curve given by the 2VT. We ran the code LM under the stats package in R (R Core Team 2014). The code should be able to determine the best-fitting parameter, regardless of the initial guess. However, to avoid convergence problems, we created a broad grid that encloses reasonable values for the parameter $\mathcal{G}$ (following approximately the ranges obtained in Fig. 1). We also computed the residual standard deviation at each point on the grid to find the best parameters choice. The best-fitting value found was $\mathcal{G} = 1.83 \times 10^{-11}$ (p-value: $p = 3 \times 10^{-4}$); fixing g_D to Milgrom's acceleration scale (equation 5) gives $R = 0.152$.

We also present a brief analysis based on the full data for (unbinned) LTGs and dSphs (Fig. 3), based on the corresponding density distributions in their data. A possible bimodality in the dSphs data is indicated (the separation of modes is represented by a horizontal line in the figure). A best-fitting curve was obtained separately for each individual mode (above and below this line), giving two different sets for the best-fitting parameter $\mathcal{G} \equiv Rg_D$, as indicated in the legend of Fig. 3. The 95 per cent confidence upper limits are

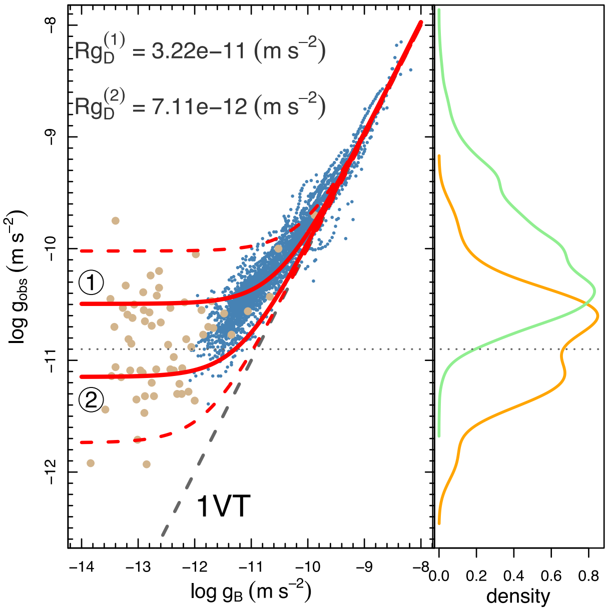

Figure 3. (Colour online). Left-hand panel: same as Fig. 2, using the full (unbinned) LTG (small dots) and dSphs (circles) data, with detailed 2VT best fits using a linear regression method, including the 95 per cent confidence band around the regression fit (dashed curves), as explained in the main text. Right-hand panel: density distributions in the data, shown separately for LTGs and dSphs, with a possible bimodality detected in the dSphs data (indicated by an horizontal dashed line). best-fitting values are shown in the legend.

the upper envelope for the regression fit obtained in the upper mode, and the lower envelope in the lower mode. The p-value of the F test for mode 1 is $p < 10^{-4}$, whereas the fitting for mode 2 does not converge easily, giving a high p-value, $p = 0.1873$. In other words, the 2VT fits mode 1 very well, but not mode 2. In this latter case, the 2VT curve in the high acceleration range misses the large volume of LTGs data in order to contemplate the lower acceleration data for dSphs.

It is interesting to note that mode 2 is mainly composed of M31 dwarf spheroidal data. Indeed, Walker et al. (Walker et al. 2010, and references therein) finds a systematic difference between M31 and MW dSphs velocity dispersions at a given half-light radius, with the former having lower velocity dispersions than the latter. It is not clear whether this effect is the result of some systematic bias or an intrinsic signature of different formation processes in these systems. A more detailed analysis of this possible bimodality in the context of the 2VT is left for a future work.

3.3 The 'flattening' behaviour of Sphs

In section 8.3.2 in Lelli et al. (2017) ('New physics in the dark sector?'), some considerations were given in order to account for the 'flattening' ADR behaviour in dSphs. In particular, the internal gravitational field of some of these systems could be 'contaminated' with that of their host galaxy, even though the 'quality cut' mentioned previously was adopted in the data. An attempt to bring the dSphs to the extrapolated behaviour of the LTGs (their fig. 14) is not in accordance with the standard MOND modifications for external field effects and seem to be not well understood in the MOND framework (Lelli et al. 2017; see also the next section).

The 2VT offers a natural explanation for this 'flattening', since the correction term to the observed acceleration does not depend on g_B, but on the product Rg_D (equation 1), which is fixed for a given family of baryonic–DM systems. That is, the 2VT cannot be much contrived, as illustrated in Appendix C. Therefore, the somewhat 'mysterious' dwarf spheroidal 'flattening' behaviour, if observationally confirmed, is a prediction of the 2VT, and such a prediction cannot be fine-tuned.

One possibility is that this flattening is due to tidal effects from the host (Caimmi & Secco 1992). In this case, the VT should be formulated with the presence of a 'third component' (host) in supposed equilibrium with the coupled baryonic–DM system. However, the resulting corrections would increase in complexity and become cumbersome, except if assuming some special cases and/or symmetries (Caimmi & Secco 1992). Evidently, it would have to be assumed that the dSph system is indeed in virial equilibrium with the host. Given the good agreement of the 2VT with the data, we consider that any contribution of a host within the gravitational radius of the baryonic system is negligible, or otherwise may contribute only to disperse of the data around the flattening region (low acceleration limit). Disentangling such possible contributions from others, such as dissipative effects (Ribeiro & Dantas 2010), is difficult to address at this point. LTGs, being located at higher acceleration regimes, are not affected by such considerations.

3.4 Systems with variable (g_D, R)

The structural parameter R depends on the type of galaxy, given their different structural equilibrium configurations, reflected on their different shapes. The value of g_D may also vary depending on the DM halo contribution within the baryonic gravitational radius, which may have different scalings for different systems. In the 2VT prediction (equation 1), these variations should be independent, but in a more general formulation (see Appendix B; Limber 1959), the equivalent to our R parameter would depend on the DM halo distribution as well. These variations would produce a spread around a given unique 2VT curve (i.e. around a family baryonic–DM systems), which would tend to be more obvious in the very low acceleration regime, where the 2VT curve admits a dominance of the DM halo (evidently, depending on the combined product, Rg_D).

3.5 A note on the 2VT and modified gravity theories

A broad alternative scenario to the DM paradigm considers that at large scales gravity should be modified (Calmet & Kuntz 2017). A proposal in this direction, initially applied to galaxies and clusters of galaxies, was provided by Milgrom in the 80's, as a modification of Newton's law in the extremely weak field regime (Milgrom 1983a,b,c; cf. reviews in Famaey & McGaugh 2012; Milgrom 2015). There are currently several implementations of MOND, but the basic mechanism is a departure from Newtonian dynamics below the critical acceleration, a_M, equation (5). A connection of MOND with cosmology was also considered by Milgrom (1999), where this acceleration scale could arise from a vacuum effect.

Another testable proposal has been recently given by Verlinde, who states that gravity itself might arise as an entropic force, so that space–time emerges from a microscopic substratum (Verlinde 2011, 2016). Verlinde's proposal has a close connection with previous works, aiming to derive general relativity from thermodynamics (Jacobson 1995; Padmanabhan 2010). A question was then brought forth concerning a possible retrieval of MOND-like behaviour from entropic arguments (see Milgrom & Sanders 2016, and references therein), so that these ideas have been taken, separately or combined, as possible alternative candidates to the DM paradigm.

MOND has been subjected to a large number of observational tests throughout many years (see reviews above), and a few tests for Verlinde's theory have been made (e.g. Pardo 2017, Hossenfelder 2017, and references therein), but results seem premature at this point, specially due to simplifications adopted in the theory. A covariant proposal for Verlinde's theory, along with some clarifications, has recently been made, which may advance the theory and favour cleaner observational testing (Hossenfelder 2017).

The ADR is a particularly interesting test for these alternative theories, especially on the possibility of differentiating DM- from non-DM-based models. Qualitatively, it is possible that MOND reproduces the ADR (see e.g. discussion in Lelli et al. 2017), although it is unclear at this point whether the behaviour of dSphs at the lower acceleration region can be accounted for. Therefore, at this point, a detailed prediction of MOND for that specific lower acceleration region ($g_B < -12$ dex) is necessary for a clear comparison with the 2VT prediction; however, to the knowledge of the authors, such a prediction is not yet available.

As for Verlinde's theory, the main phenomenological result has been derived from spherical symmetry (equation 7.47 in Verlinde 2016):

$$\rho_D^2(r) = \left[4 - \bar{\beta}_B(r)\right] \frac{a_0}{8\pi G} \frac{\rho_B(r)}{r}, \tag{6}$$

where $a_0 = 6a_M$, and $\bar{\beta}_B(r)$ is the slope parameter, $\bar{\beta}_B(r) = -\mathrm{d}\log\rho_B(r)/\mathrm{d}\log r$, and subscript D refers to the resulting 'apparent' DM effect. By the use of equations (A5), (A6) of Appendix A into equation (6), the latter can be written in terms of acceleration variables as

$$g_{(\mathrm{D,VEGT})} = \left[4 - \bar{\beta}_B(r)\right]^{1/2} \sqrt{a_M g_B}. \tag{7}$$

Hence, on general grounds and using simplifying assumptions, Verlinde's theory predicts a correction $g_{(\mathrm{D,\,VEGT})}$ for the observed acceleration g_{obs} that depends on $\sqrt{a_M g_B}$, similarly to MOND (e.g. Lelli et al. 2017), with prefactors that may differ. On the other hand, the 2VT predicts that this correction depends on the product Rg_D (cf. equation 1), that is, it is not a function of g_B, as in the former theories, but a function of the baryonic matter distribution (R) and the acceleration scale (g_D within r_B) associated with the DM component of the system.

This difference in the functional dependence of the correction term for the observed acceleration imposes a qualitative distinction on the form of the predicted ADR. In Verlinde's theory and in MOND, obtaining a nearly constant behaviour in the lower acceleration regime requires a functional tuning (in terms of g_B) in their respective correction terms. On the other hand, the 2VT already predicts an asymptotically constant behaviour in that regime, independent of g_B, and this asymptotic form cannot be arbitrarily contrived (cf. Appendix C). Therefore, it is important that modified gravity theories present clear and specific predictions for the lower acceleration regime in order to be possible to differentiate them for DM-based models, such as the 2VT.

4 CONCLUSION

In this paper, we have revisited the 2VT with the aim of identifying discerning features and predictions for the behaviour of gravitational systems in relation to the recent ADR findings. Our main conclusions are as follows:

(1) *The 2VT follows approximately the ADR*, considering LTG and dSphs data. The inferred coupling between dark and baryonic components from this relation seems to arise from their mutual equilibrium conditions. The detailed behaviour of this coupling depends on the structural distribution of these components. Our work did not offer predictions on the detailed forms of such final equilibrium states, which may include a variation of the parameters (g_D, R), and also dissipative mechanisms leading to different structural configurations. Indeed, the coupling between DM and baryons is a complex issue, and our previous study of this matter in the context of the 2VT indicates complementary contributions of dissipation and DM to the origin of scaling relations in astrophysical systems (Ribeiro & Dantas 2010). However, the overall behaviour of the 2VT curve is remarkably consistent with the ADR.

(2) *The 2VT predicts some of the main features of the ADR, such as a bending and a nearly constant behaviour in the lower acceleration regime*. The parameters (g_D, R) cannot disrupt these features, allowing only for an adjustment of the 'height' of the asymptotic behaviour in the lower acceleration region, or alternatively the point at which the 2VT departures from the 1VT.

(3) *The somewhat 'mysterious' dwarf spheroidal 'flattening' behaviour, if confirmed, would indicate that the 2VT provides a consistent physical description of this phenomenon* via the dynamical equilibrium of such systems, highly dominated by a DM component.

Finally, we point out that the 'rigidity' of the 2VT curve is an important discriminating factor, implying that *this model cannot be arbitrarily contrived to fit the data*. This level of predictive power is an important feature of the 2VT, which could, for instance, serve as a means to distinguish DM theories from non-DM (e.g. emergent gravity) theories of equilibrium systems.

ACKNOWLEDGEMENTS

We thank the referee for useful suggestions. CCD thanks Sabine Hossenfelder for her comments on an earlier draft. ALBR thanks the support of ConselhoNacional de DesenvolvimentoCientífico e Tecnológico (CNPq), grant 309255/2013-9.

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APPENDIX A: REWRITING THE 2VT

The (usual) virial theorem (hereafter 1VT) states that, for the *whole* system, the mean square velocity of the baryonic component is given by Binney & Tremaine (1987)

$$\langle v^2 \rangle_B = \frac{GM_{\rm v}}{r_B}, \tag{A1}$$

where $M_{\rm v}$ is the estimated virial mass of the system, and r_B is the gravitational radius of the baryonic component.. It is clear that, if the virial mass is larger than the observed mass, $M_{\rm obs}$ (inferred from the observed stellar and gaseous surface mass distributions), then there is a 'hidden mass' (or DM mass) given by the difference

$$M_{\rm (D,1VT)} = M_{\rm v} - M_{\rm obs}. \tag{A2}$$

The 2VT gives a correction to the 1VT, and was first derived by Limber (1959) (cf. Appendix B). A simplified form of the 2VT has previously been shown to reproduce in a broad sense the scaling relations of systems at various scales (Dantas et al. 2000):

$$\langle v^2 \rangle_B = \frac{GM_B}{r_B} + \frac{4\pi}{3} G\rho_D \langle r^2 \rangle_B, \tag{A3}$$

where the baryonic average square radius is defined by

$$\langle r^2 \rangle_B \equiv \frac{\int r^2 \rho_B(r) {\rm d}V}{\int \rho_B(r) {\rm d}V}. \tag{A4}$$

In the equations above, $\rho_B(r)$ is the mass density of the baryonic component, whereas ρ_D is the mean density of the DM halo within the region containing the baryonic component, M_B is the total baryonic mass within the gravitational radius.

Note that the 1VT does not address any systematic couplings between 'hidden' and baryonic masses, it just implies that a 'remainder' mass must be added, by contingency, to the dynamical equilibrium budget of the system. But in the 2VT formulation, the mean square velocity of the baryonic component *must be corrected in a way that it depends systematically on the dark component in which the former is embedded*. This description of the coupling of the baryonic and DM halo is a fundamental advantage of the 2VT formulation.

For spherically symmetric systems, the gravitational acceleration scales for the baryonic matter and DM are, respectively,

$$g_{\rm x} = \frac{GM_{\rm x}(< r_B)}{r_B^2}, \tag{A5}$$

with x either referring to B or D; r_B is the gravitational radius of the baryonic component. In the case of axisymmetric systems, like LTGs, flattened gravitational potentials associated with finite mass systems present a Keplerian circular speed at large galactocentric distances. In this limit, we assume that equation (A5) is approximately satisfied.

In terms of acceleration scales and mean densities, within r_B,

$$\rho_{\rm x} = \frac{3g_{\rm x}}{4\pi G r_B}. \tag{A6}$$

The following assumptions and definitions were used (further simplifications are described in Appendix B):

(i) We define a structure parameter,[1] R, depending only on the properties of the baryonic matter distribution, such that $\langle r^2 \rangle = Rr_B^2$.

(ii) For rotationally supported galaxies, we make the correspondence $\langle v_{\rm circ}^2 \rangle_B \rightarrow \langle v^2 \rangle_B$, where $v_{\rm circ}$ is the circular velocity at radius r_B.

Given item (i) above, we rewrite equation (A3) as

$$\langle v^2 \rangle_B = \frac{GM_B}{r_B} + \frac{4\pi}{3} G\rho_D R r_B^2. \tag{A7}$$

On the other hand, the observed baryonic centripetal acceleration is

$$g_{\rm obs} = \langle v^2 \rangle_B / r_B. \tag{A8}$$

Hence, expressing equation (A7) in terms of accelerations (using equations A5, A6 and A8), we write the 2VT prediction as:

$$g_{\rm obs,2VT} = g_B + Rg_D. \tag{A9}$$

APPENDIX B: RELATION TO LIMBER'S 2VT EQUATION

The original modification of the VT proposed by Limber (equation. 21 in Limber 1959) for a model of a cluster of galaxies (here labelled by 'B') with an extended, non-dissipative gaseous background (here labelled by 'D'), is given by

$$\langle v^2 \rangle_B = \frac{GM_B}{r_B}\left[1 + (C_{BD} + D_{BD})\frac{M_D}{M_B}\right], \tag{B1}$$

Our formulation of the 2VT (equation A7) is a particular case of Limber's 2VT above. We assumed that inside the baryonic region defined by r_B the DM halo was sufficiently extended, so that the

[1] This definition is slightly different than the one adopted in Dantas et al. (2000).

average DM density inside that region did not depend on the galactocentric distance (equation 2 of Dantas et al. 2000). A similar approximation can be done in equation (25) of Limber's paper Limber (1959) for the coefficient C_{BD}. A second assumption is that the spatial distribution of both components is similar, which implies that Limber's coefficient D_{BD} is approximately zero (equation 26 of Limber 1959). With these approximations, our parameter R depends only on the properties of the baryonic matter distribution (cf. Appendix A), whereas in a more general formulation it would have to be re-written in terms of $C_{BD} + D_{BD}$.

APPENDIX C: CHARACTERISTICS OF THE 2VT CURVE IN THE LOG–LOG PLANE

The 2VT (equation 1), parametrized by a fixed value of (g_D, R), represents a family of baryonic–DM systems in an arbitrary baryonic mass range. The correction for the observed acceleration, predicted by the 2VT, is given by the combined product Rg_D, and it is not possible to obtain separated estimates for R and g_D. Here, we illustrate how the 2VT curve is affected by these quantities, by fixing one and varying the other, and vice-versa.

In Fig. c1, left-hand panel, we fix g_D to Milgrom's acceleration scale (equation 5) and vary the parameter R, whereas in the right-hand panel, we fix $R = 0.125$ and vary g_D. As expected, both parameters produce similar effects on the 2VT. Clearly, the height of the asymptotic part of the 2VT in the lower acceleration region is solely regulated by the departure from the 1VT. It is important to notice that the parameters (g_D, R) cannot disrupt this rigid form of the 2VT, allowing only for an adjustment of the height of the asymptotic behaviour of the curve or, alternatively, the point at which the 2VT departures from the 1VT.

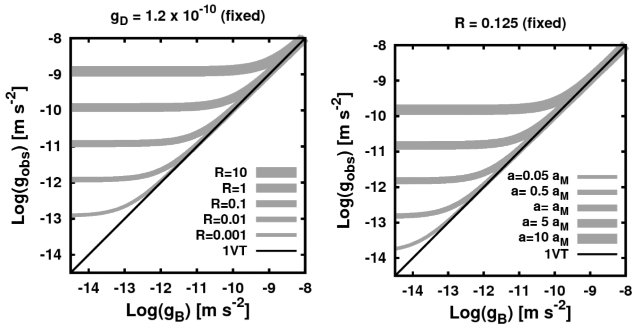

Figure C1. The acceleration associated with the baryonic matter (g_B) versus the observed acceleration predicted by the 2V (equation 1). Left panel shows 2VT curves with g_D fixed and varying R values, whereas the right panel shows the other way around (their corresponding values are indicated in the legends). These panels illustrate how these parameters affect the departure from the 1VT (the line of identity) in a similar fashion.

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